Stochastic Calculus with respect to Gaussian processes

Stochastic integration with respect to Gaussian processes, such as fractional Brownian motion (fBm) or multifractional Brownian motion (mBm), has raised strong interest in recent years, motivated in particular by applications in finance, Internet traffic modeling and biomedicine. The aim of this work to present an anticipative stochastic calculus with respect to a large class of Gaussian processes, denoted $\mathscr{G}$, that contains, among many other processes, Volterra processes (and thus fBm) and also mBm.

Built using White Noise Theory, this stochastic calculus includes a definition of a stochastic integral, Itô formulas (both for tempered distributions and for functions with sub-exponential growth), a Tanaka Formula as well as a definition, and a short study, of (both weighted and non weighted) local times of elements of $\mathscr{G}$. Depending on the remaining time, some applications to finance will be presented.

In order to build this stochastic calculus, a white noise derivative of any Gaussian process G of $\mathscr{G}$ is defined and used to integrate, with respect to G, a large class of stochastic processes, using Wick products. A comparison of our integral *wrt* elements of $\mathscr{G}$ to the ones provided by Malliavin calculus in [1] and by Itô stochastic calculus is also made. Moreover, one shows that the stochastic calculus with respect to Gaussian processes provided in this work generalizes the stochastic calculus originally proposed for fBm in [4, 3, 2] and for mBm in [6, 5, 7]. Likewise, it generalizes results given in [8] and some results given in [1]. In addition, it offers alternative conditions to the ones required in [1] when one deals with stochastic calculus with respect to Gaussian processes.

Keywords: Gaussian processes, Stochastic Analysis, White noise theory, Wick-Itô and Hitsuda-Skorohod integrals, It and Tanaka formulas, fractional and multifractional Brownian motions.

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