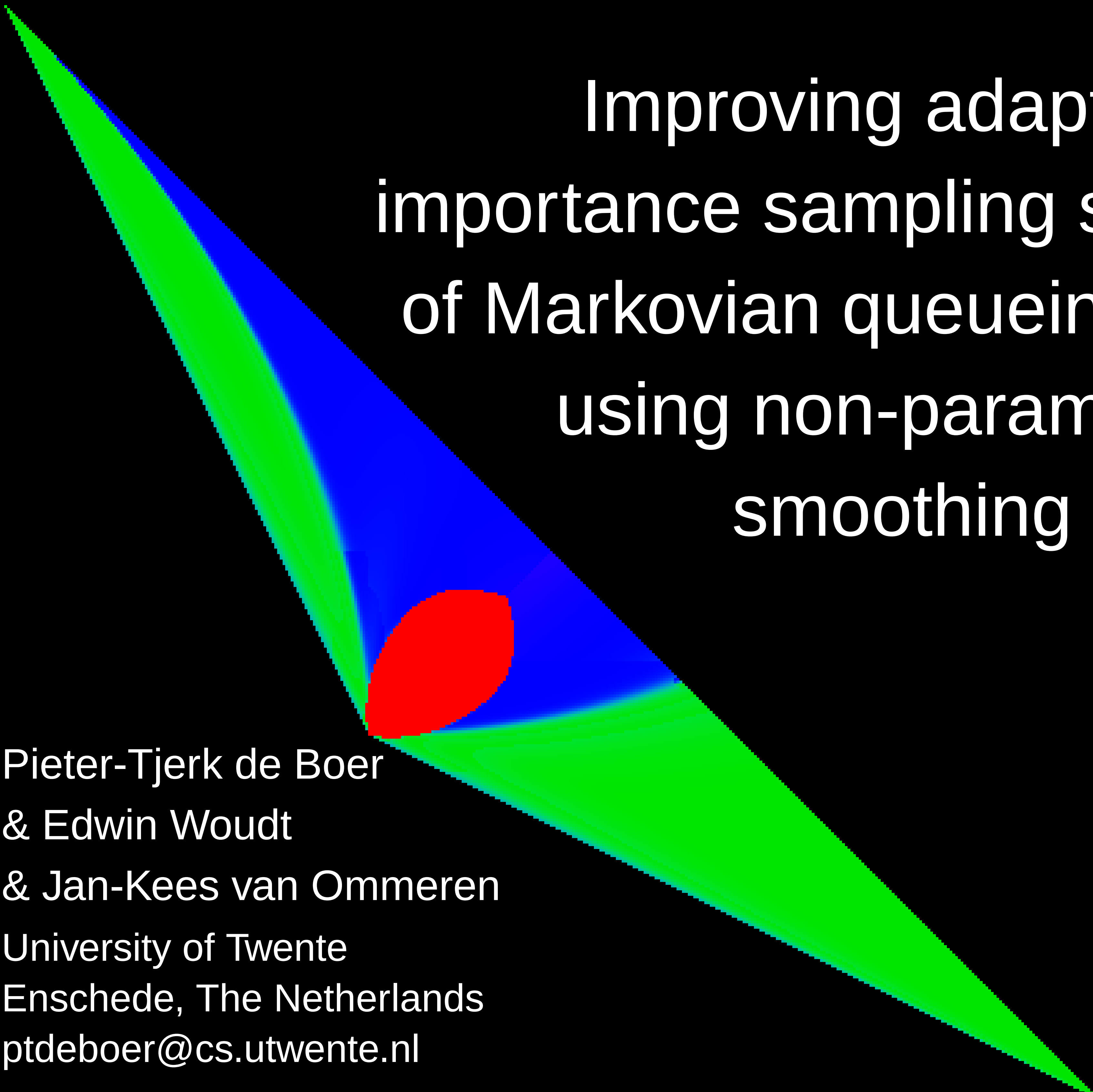
Improving adaptive importance sampling simulation of Markovian queueing models using non-parametric smoothing



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Back in 1999...

- Reuven was just starting the cross-entropy method.
- I was working towards my PhD at the University of Twente, on rare-event simulation.
- Reuven visited the UT during summer ...
- ... and introduced me to the cross-entropy method.
- We experimented with it on Markovian queueing networks, using state-*in*dependent tilting to estimate overflow probabilities.

Later in 1999...

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  - Local average
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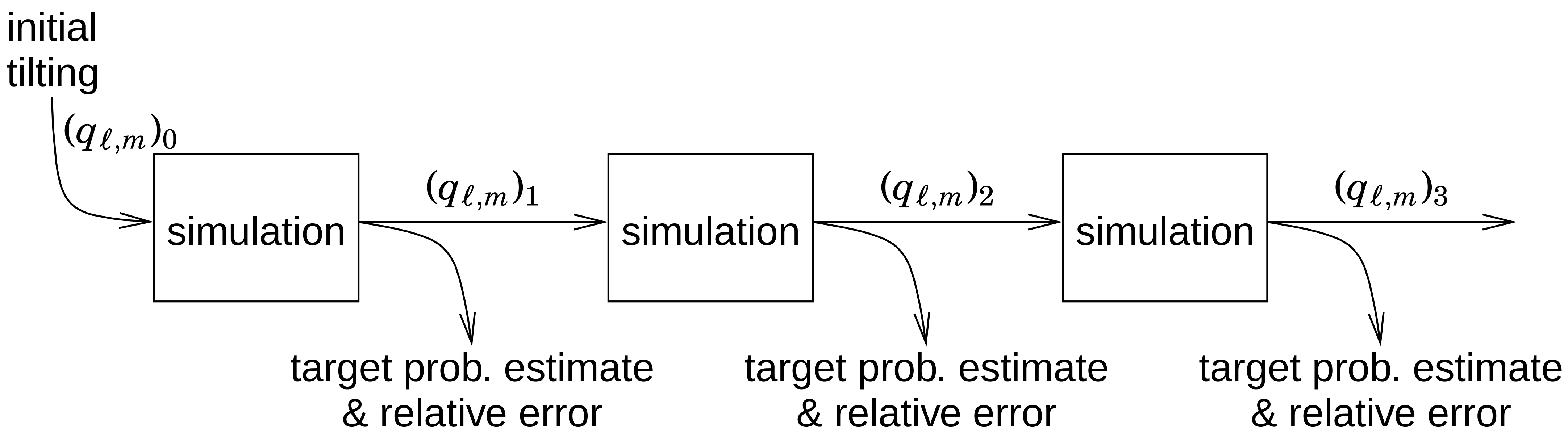
  - **2006/2007: non-parametric smoothing**

Iterative procedure



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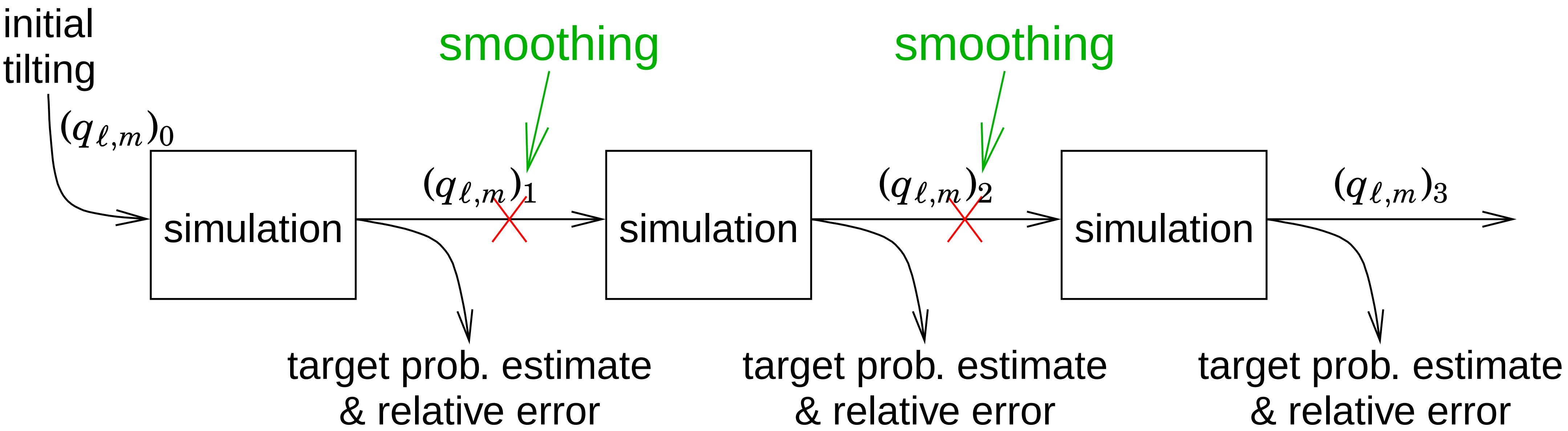
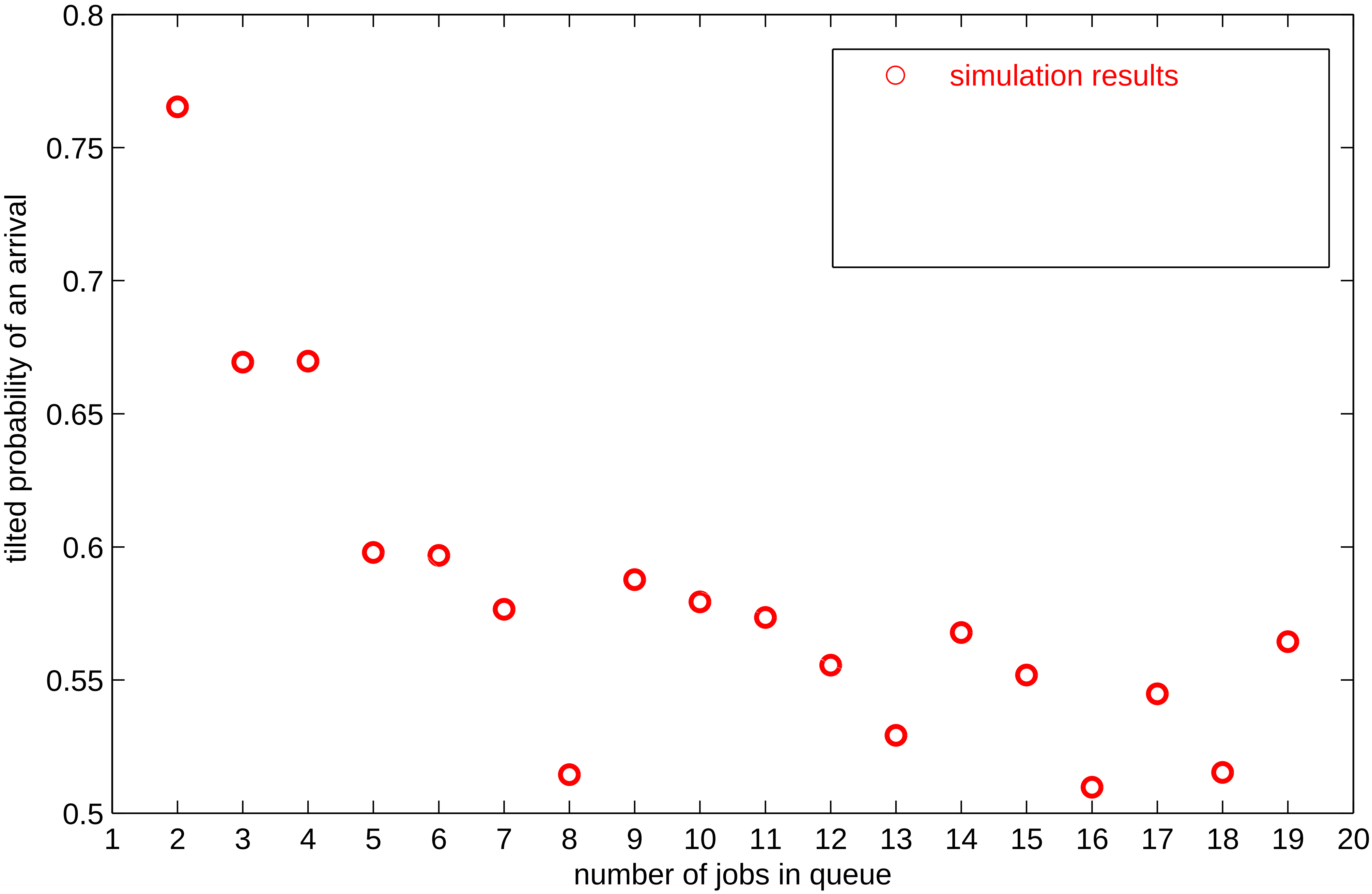


M/M/1/20 example



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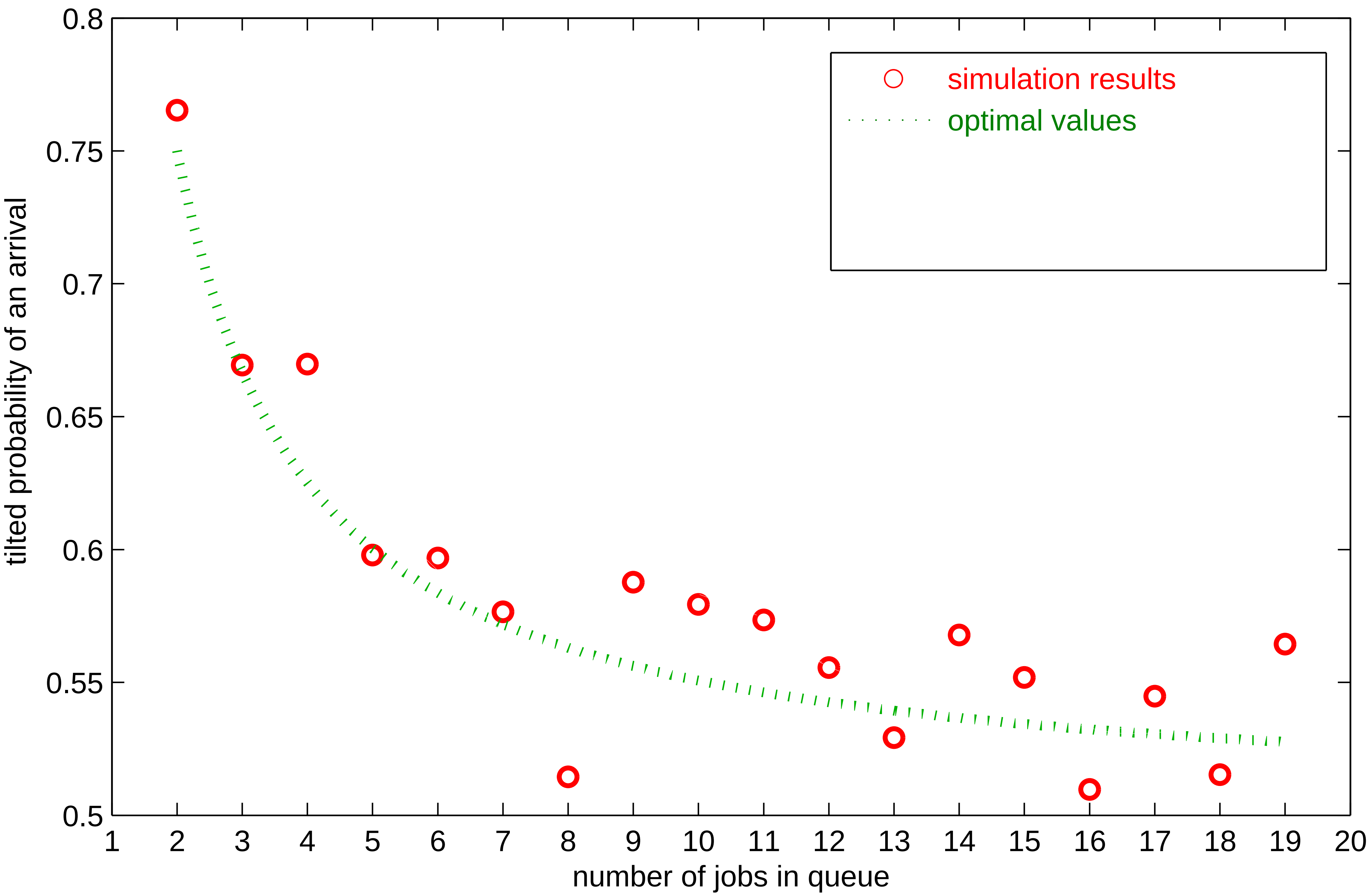

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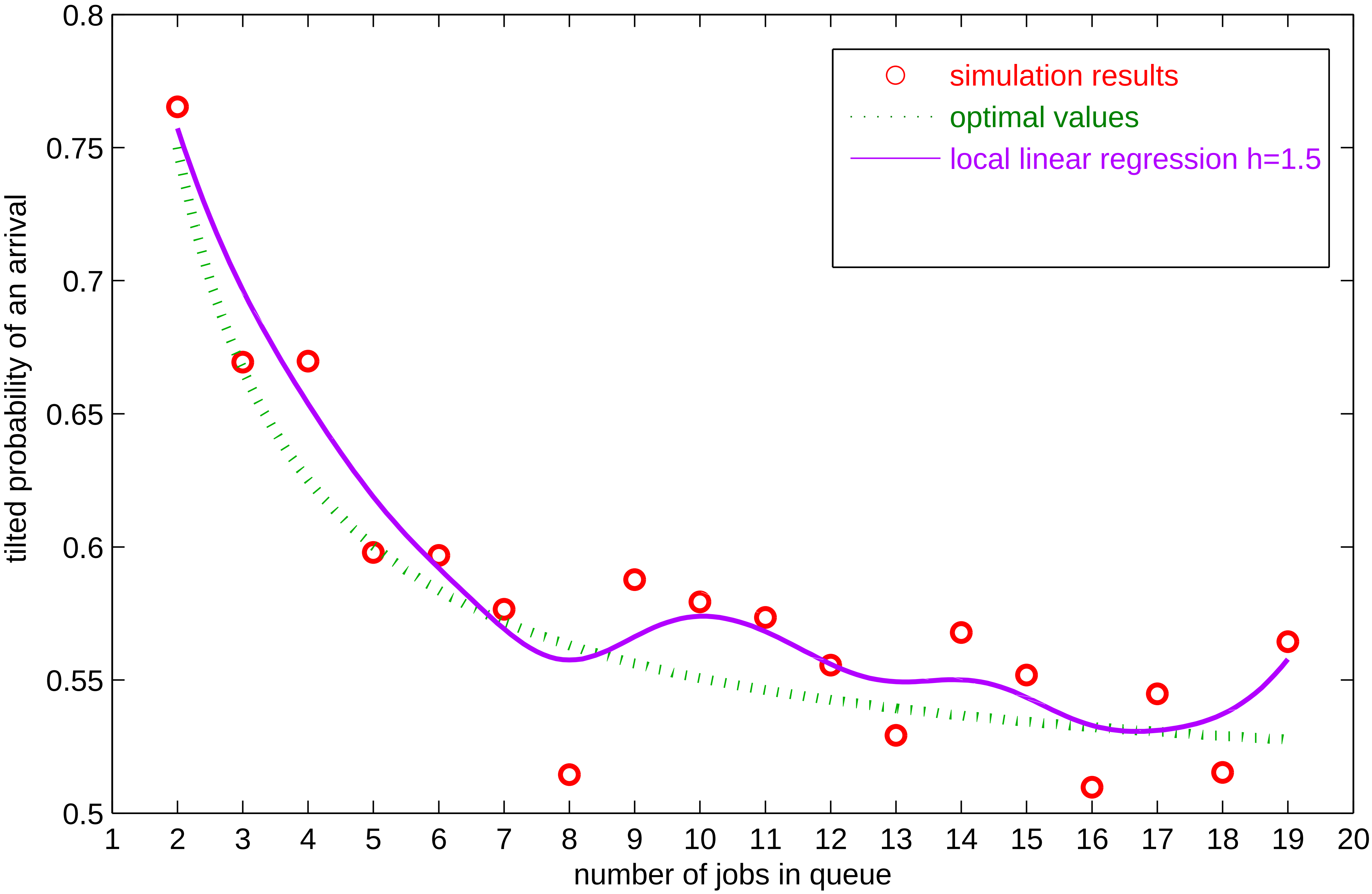
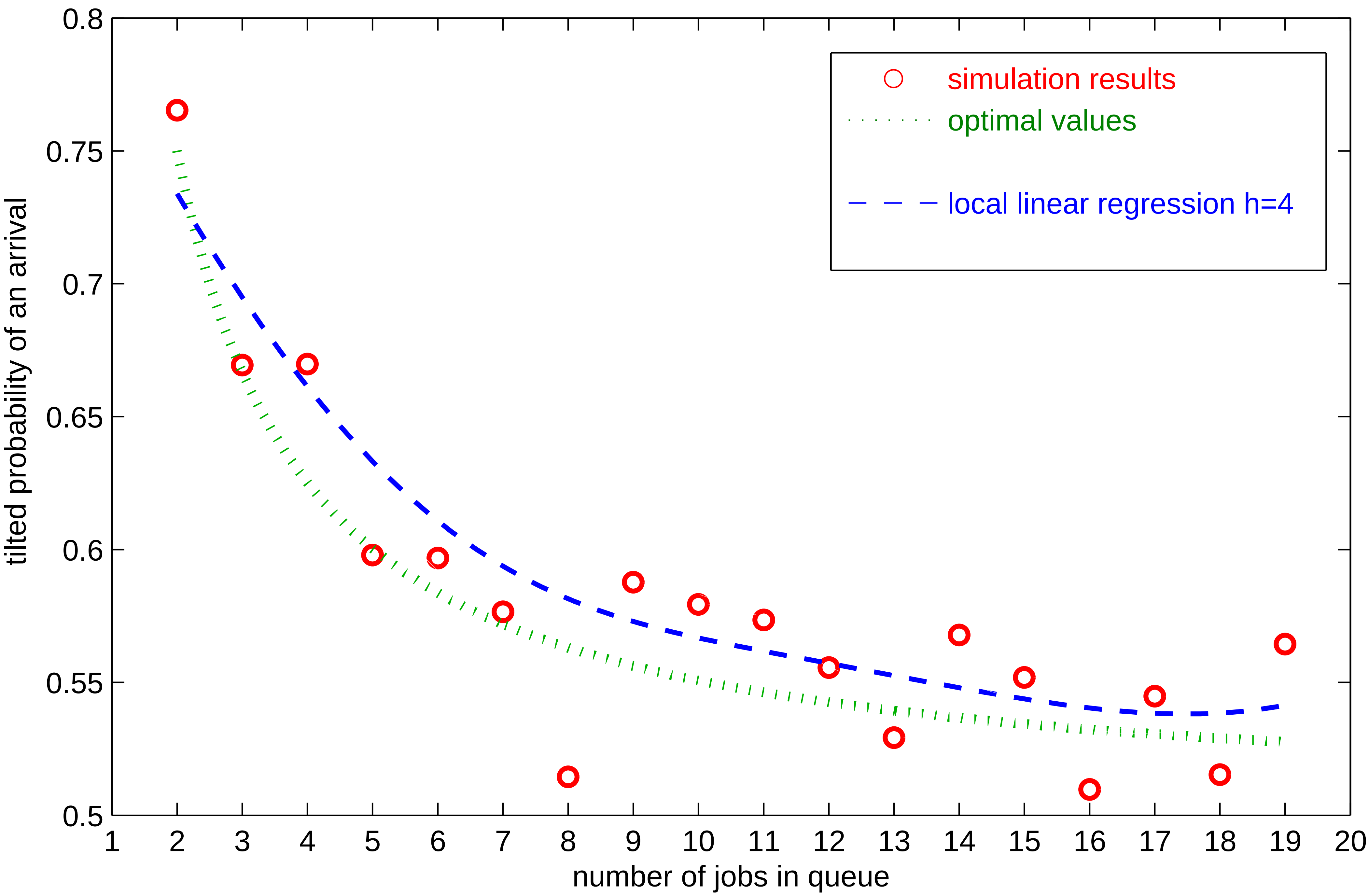


M/M/1/20 example



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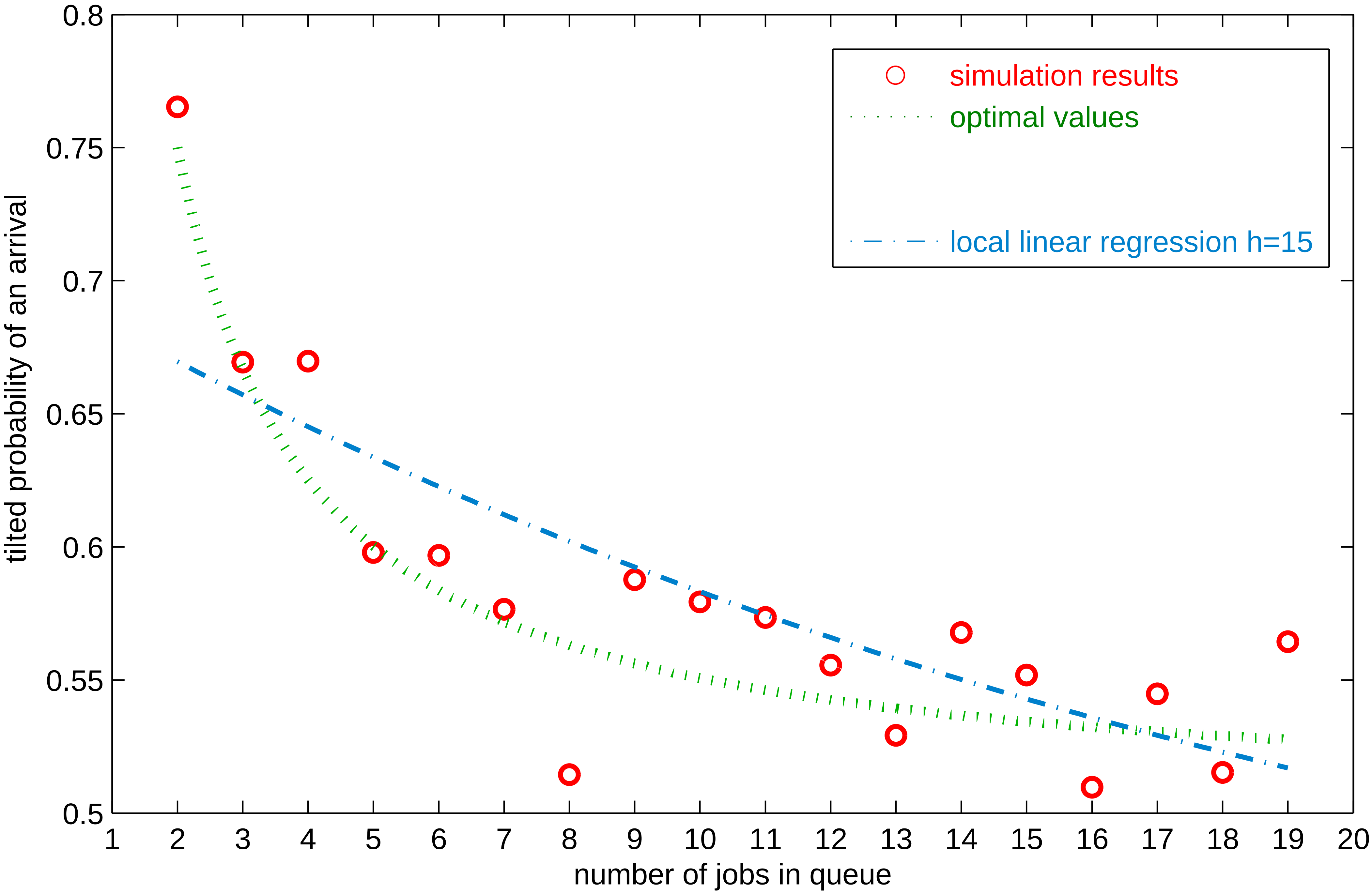
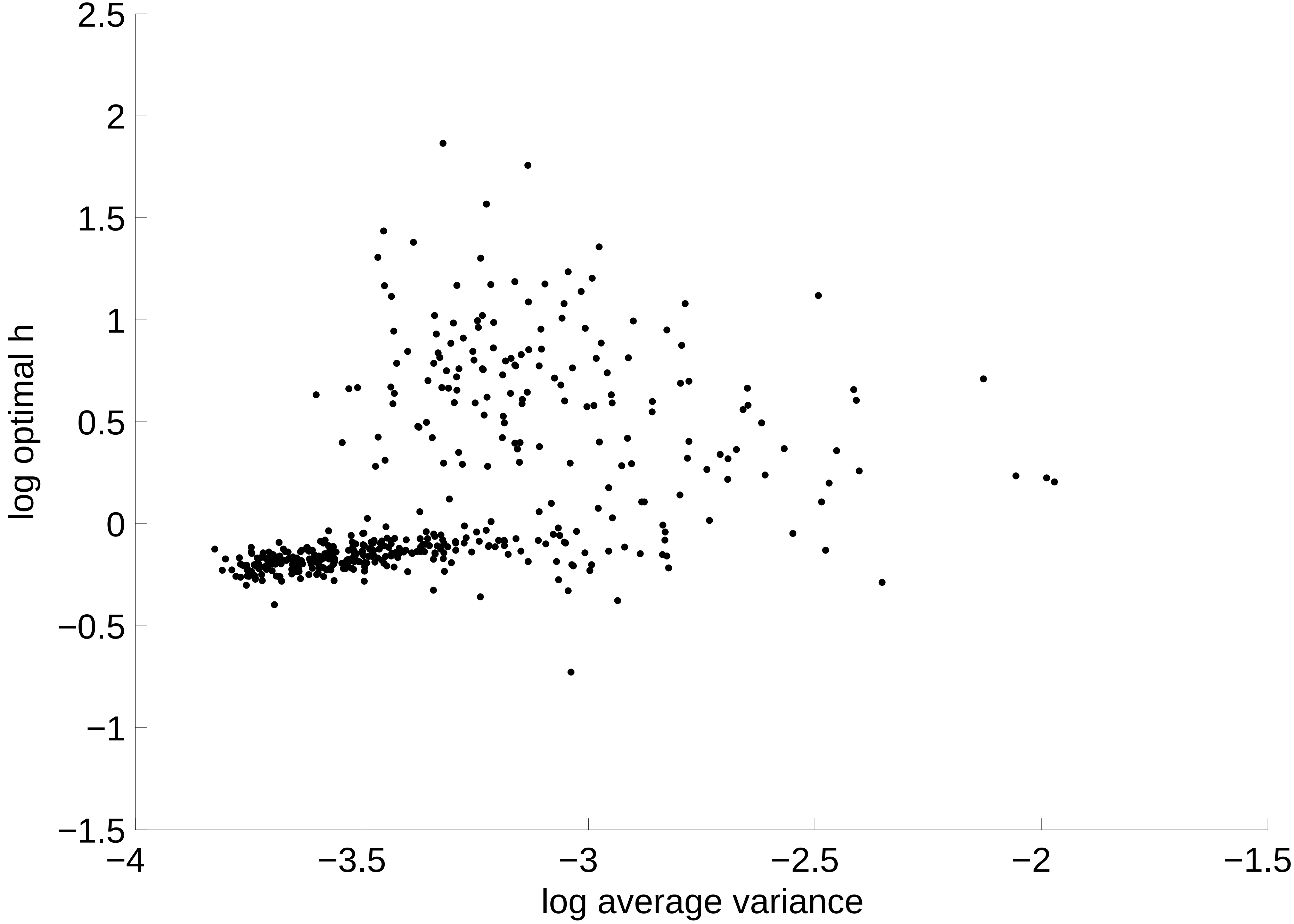


Choosing the kernel width h



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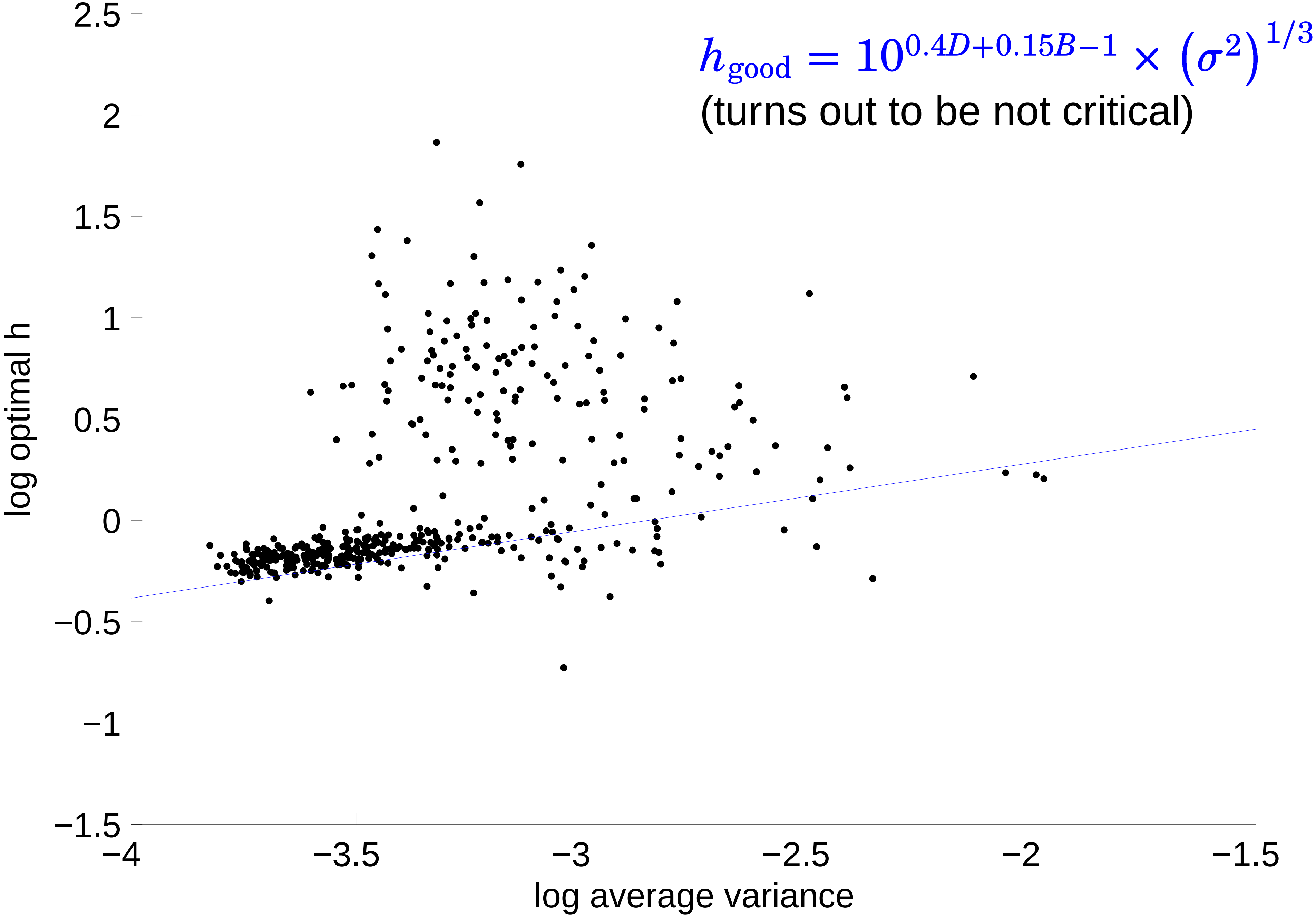
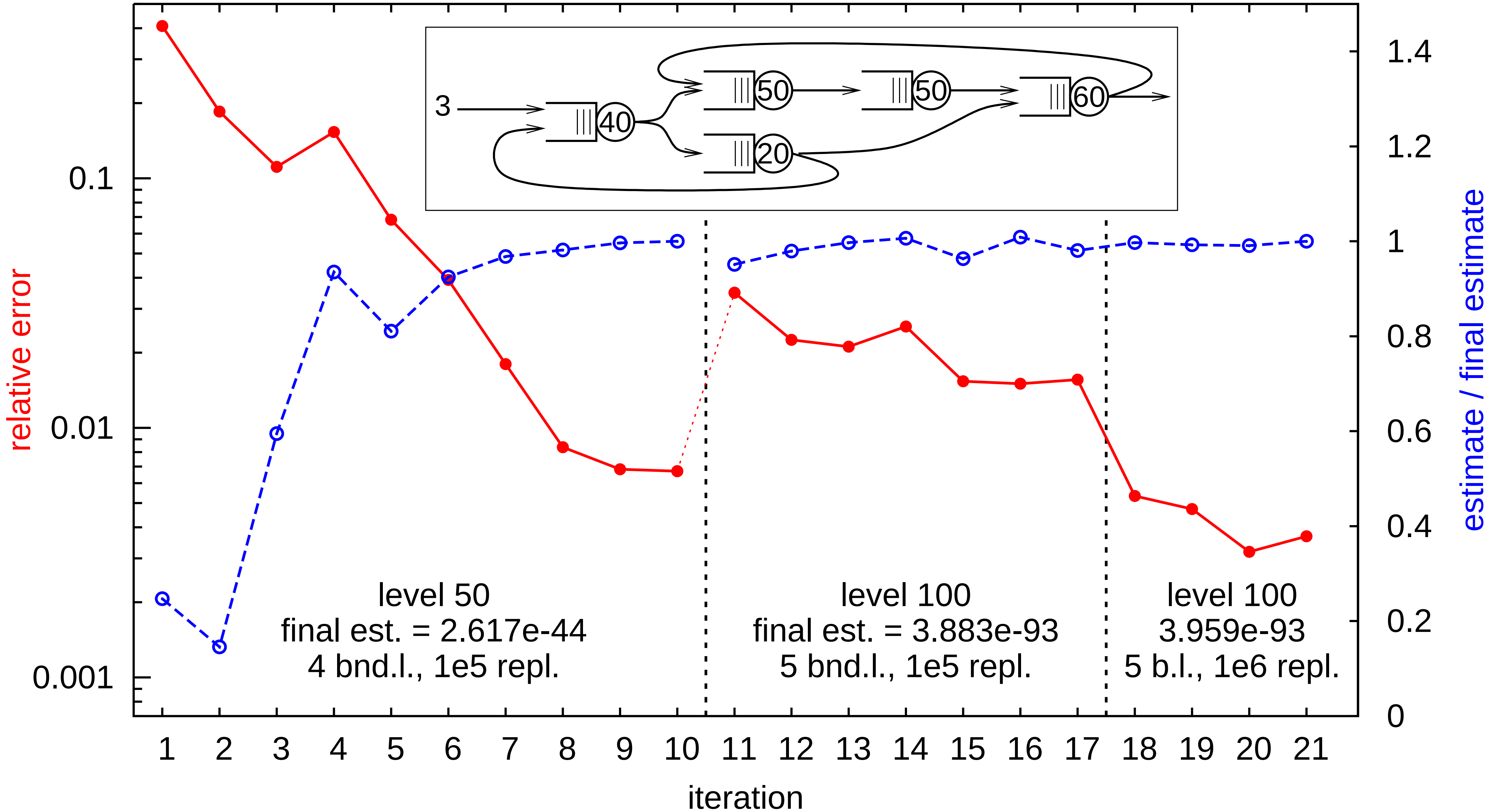


Example results



Concluding remarks

- Non-parametric smoothing works better than the old spline smoothing.
- Non-parametric smoothing is computationally feasible.
- Perhaps non-parametric smoothing is also useful in other CE problems with many parameters?