Response Surface Methodology for Simulating Hedging and Trading Strategies

R. Evren Baysal

Barry L. Nelson

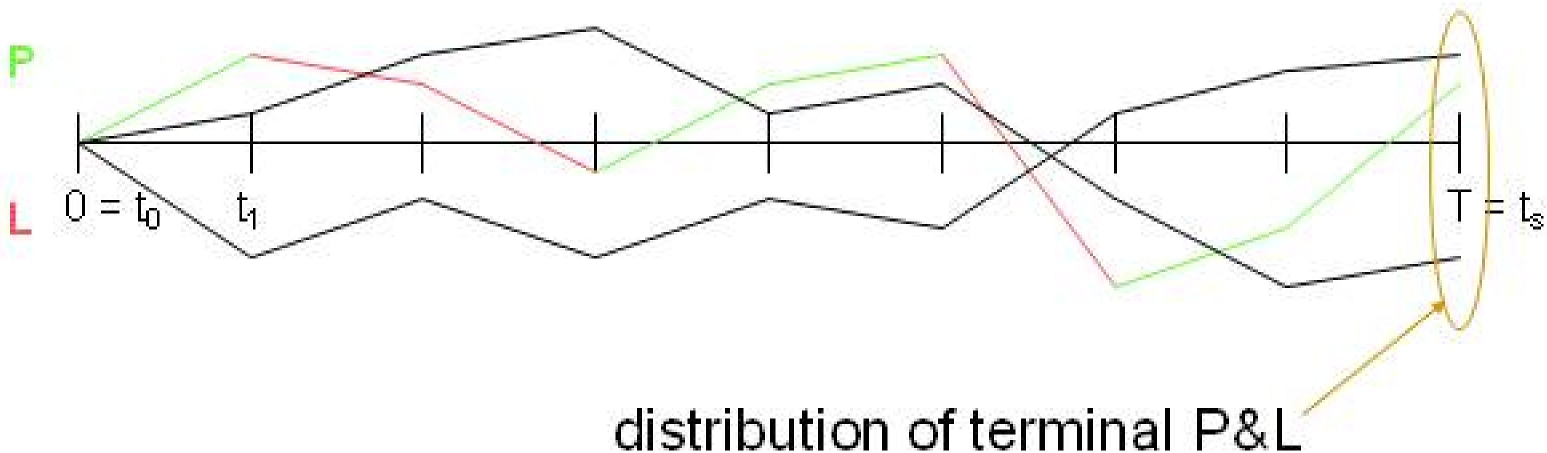
Jeremy Staum

Dept. of Industrial Engineering and Management Sciences

Northwestern University

Evaluating Portfolio Strategies

- Goal: approximate the distribution of P&L
  - especially at time horizon T
  - especially mean and <u>variance</u>
- Further goal: optimize over strategies.



Simulating Portfolio Strategies

- stochastic model of financial market
  - simulate $S_1, \ldots, S_s$ using probability measure **P**
  - What will happen? distribution of S_{i+1} given S_i
- rules of our behavior: What will we do?
  - θ_i = step-i decision given S_i (Markov)
- P&L depends on events and decisions:
  - P&L incurred at step i: $\theta_{i-1}(V_i - V_{i-1})$
    - security value vector $V_i = f(t_i, S_i)$
  - total P&L by step i: $\Pi_i = \theta_i V_i - \theta_0 V_0$ (self-financing)

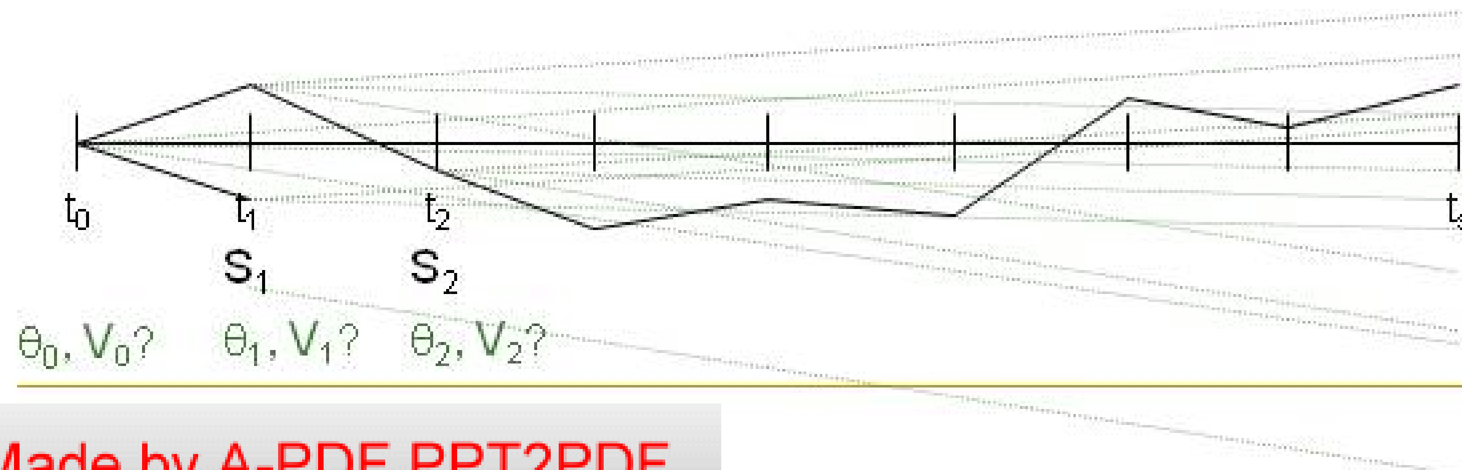
Example: Hedging a Put Option

- stochastic model of stock price S
- securities: put option, stock, bank account
  - bank account $V_{i1} = \exp(r\, t_i)$
  - stock $V_{i2} = S_i$
  - put option $V_{i0} = f_0(t_i, S_i)$
- portfolio weights ($i = 0, 1, \ldots, s-1$)
  - put option $\theta_{i0} = 1$ (no choice)
  - stock $\theta_{i2} = -\partial V_{i0}/\partial S_i$ (delta-hedge to reduce risk)
  - bank account $\theta_{i1} V_{i1} = \theta_{i-1,1} V_{i-1,1} + (\theta_{i-1,2} - \theta_{i2}) V_{i2}$

(self-financing) *old bank* *value of stock sold*

Nested? Simulation of Hedging

- Black-Scholes model (formulae known):
  - option value $V_{i0} = f_0(t_i, S_i)$ = Black-Scholes formula
  - hedge ratio $\theta_{i2} = g_2(t_i, S_i) = -\partial f_0(t_i, S_i)/\partial S_i$ = delta
  - simulate S_i, update V_i, Π_i, and θ_i, repeat
- No formulae: for each path, estimate $V_{00}, \ldots, V_{s-1,0}$, and $\theta_{00}, \ldots, \theta_{s-1,0}$ by Monte Carlo.



Review of Nested Simulation

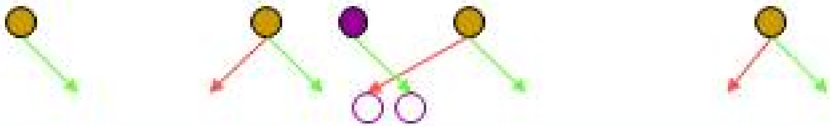
- outer level: sample *k* risk factors / scenarios
  - scenario Z = path of length *s*
- inner level: estimate value of path
  - sample *m* replications to estimate each mean
  - for security values and hedge ratios at *s* steps
- simulation at *ks* points (t_i, S^j_i)
  - total effort ksm = 1,000 × 60 × 1,000 = 60 million
- portfolio strategies: path value $\Pi_s(Z)$ is a function of many means $V_{00}, \ldots, V_{s-1,0}$ and $\theta_{00}, \ldots, \theta_{s-1,0}$ estimated by simulation

Nested Simulation Approaches

- quantile of conditional expectation (VaR)
  - learn distribution of scenario value E[X|Z]
  - Henderson-Steckley: learn density
- Glynn-Lee-Szechtman and Gordy-Juneja
  - analyze MSE, reduce bias
  - allocate *k* vs. *m*, variable inner-level sample size
- Lan-Nelson-Staum
  - confidence intervals using ranking-and-selection
  - choose *k*, variable inner-level sample sizes

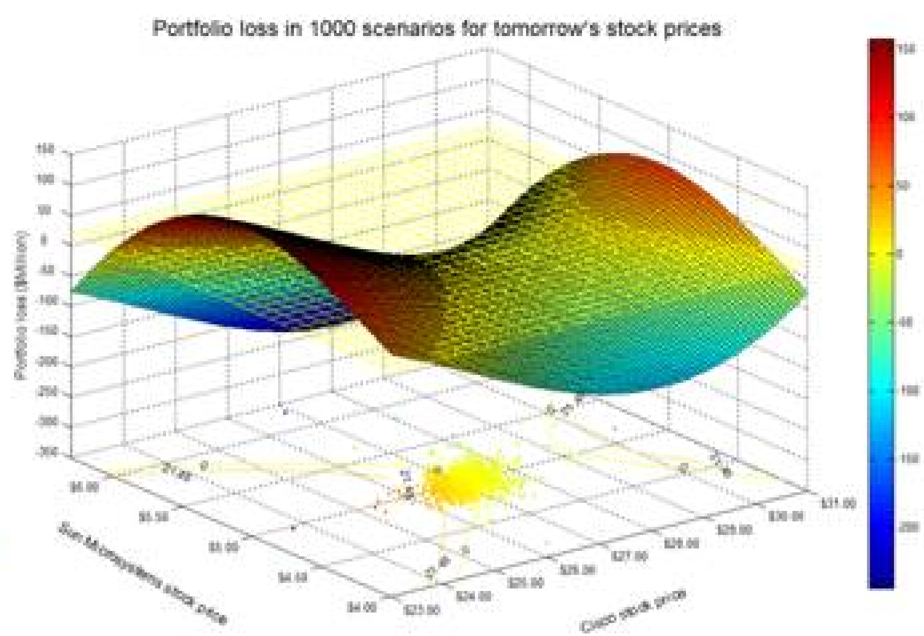
How Hard is Nested Simulation?

- say value = E[X|Z], sample X given Z
  - risk management, service level agreements
  - Bayesian approach to input uncertainty
- What functional of the distribution of E[X|Z]?
  - mean: no nesting! E[E[X|Z]] = E[X], pick m=1
  - quantile: inner-level noise can cancel
    - common random numbers: inner- and outer-level add
  - variance: getting Var[E[X|Z]] + E[Var[X|Z]]/m
    - attempt bias correction?



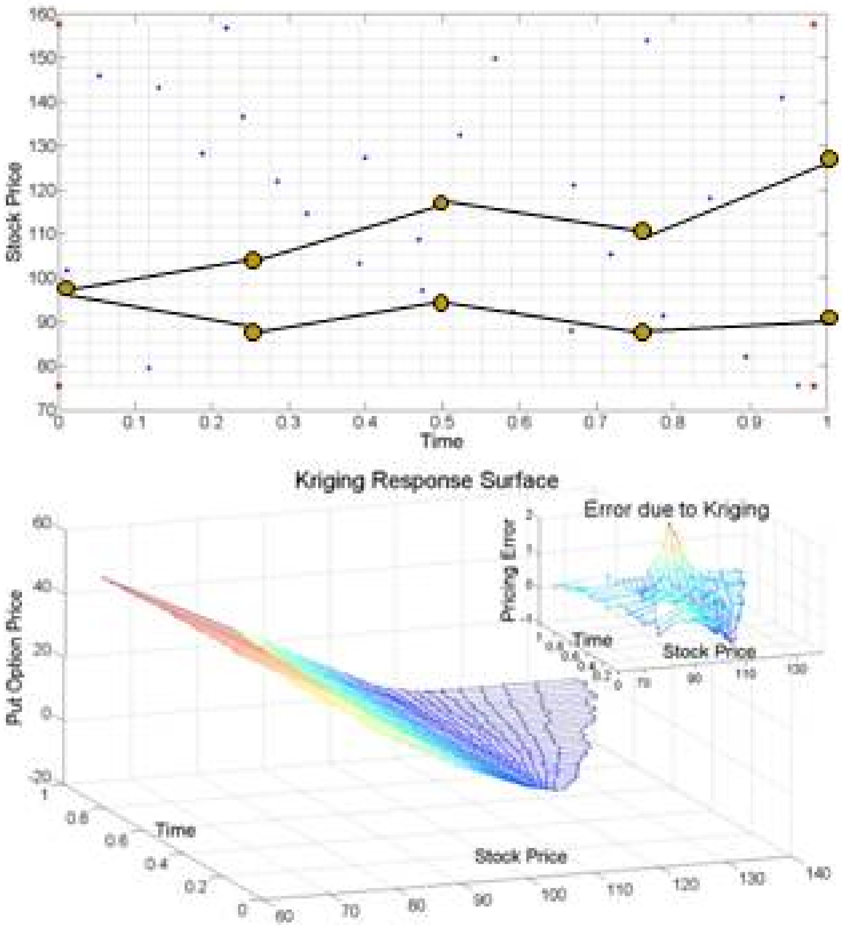
Response Surface Modeling

- Idea: exploit "spatial" structure among scenarios to aid in estimating their value
  - V=f(t,S) and θ=g(t,S) are nice functions
  - simulation at (t',S') can yield information about V and θ at (t,S)
- Frye, Shaw: VaR
  - surface = loss vs. scenario
  - basic response surfaces



Response Surface Modeling Procedure

- sample *k* paths with *s* steps (*ks* points)
- choose *n* << *ks* design points
  - (*n* = 404, *ks* = 60,000)
- sample *m* payoffs at design points
- build surface by <u>kriging</u>
- estimate V, θ at all points using surface



Kriging

stationary Gaussian random field having spatial correlation

- Response surface model:

$$Y(x) = \underline{\beta_0 + \beta h(x)} + M(x)$$

value at point x

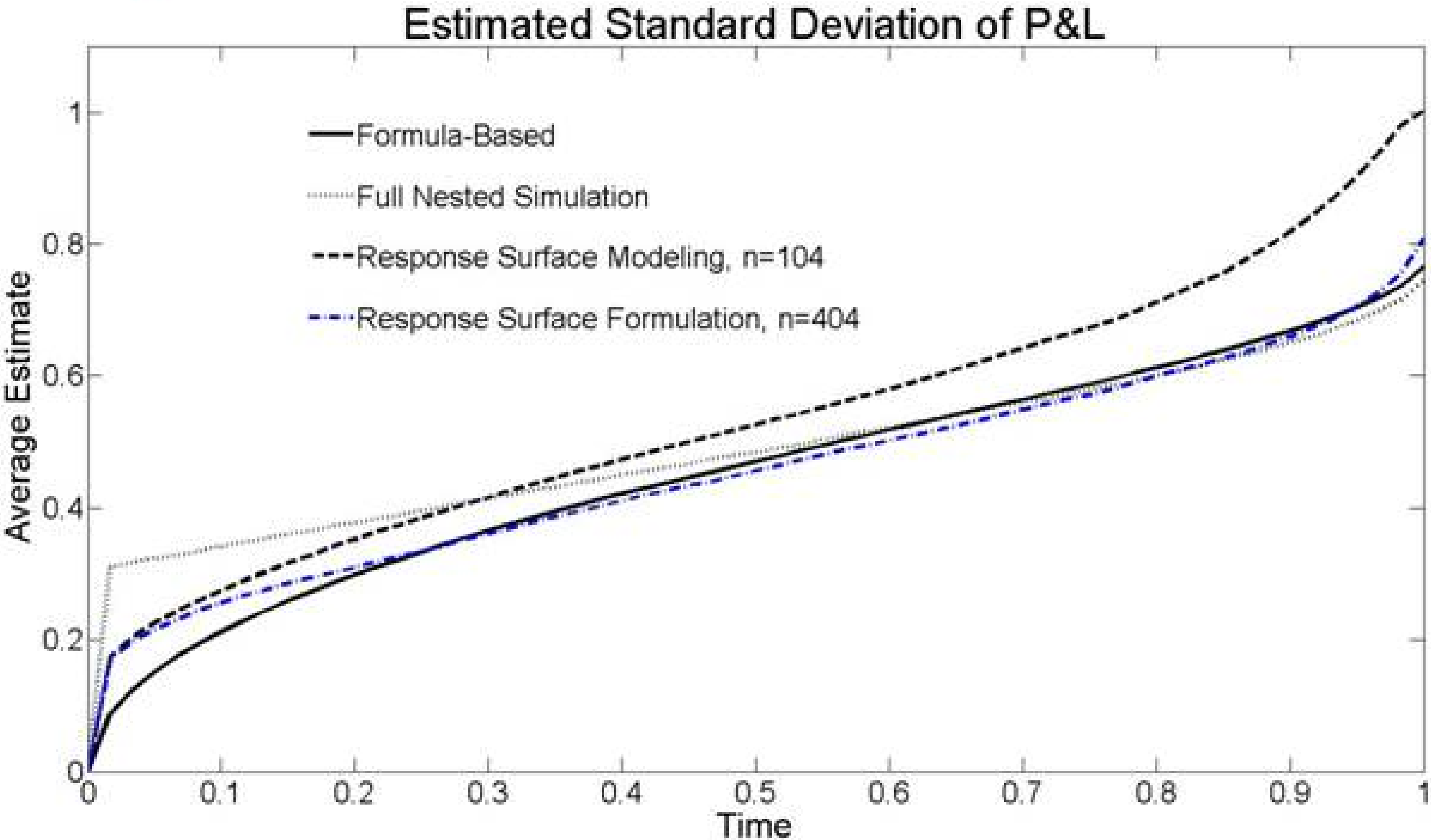
linear combination of basis functions ("trend")

- Simulate $Y(x_1), \ldots, Y(x_n)$ at design points.
- Estimate coefficients of M and β.
- Predict $Y(x_0)$ to be

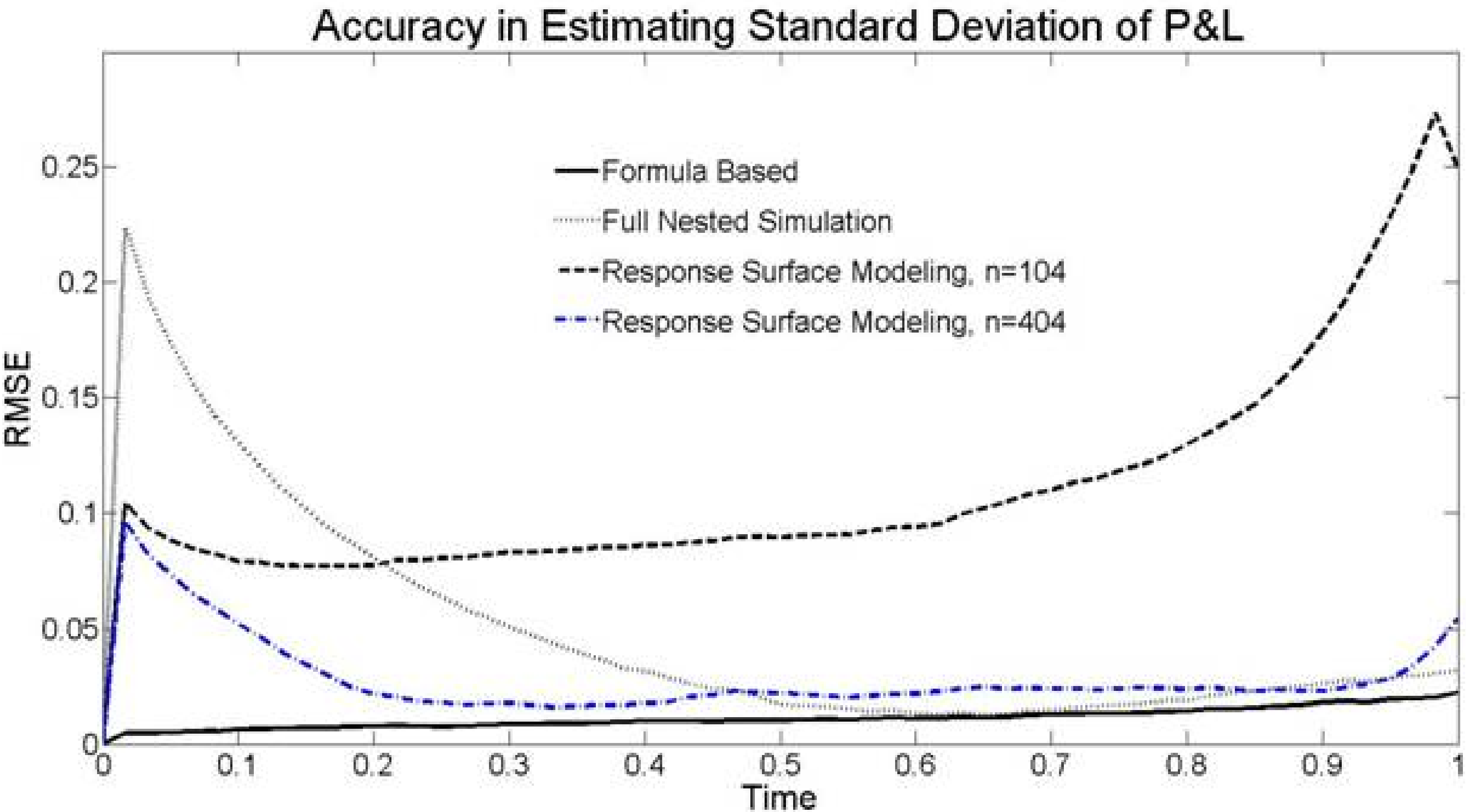
$$\underline{\beta_0 + \beta h(x_0)} + \sum w_i(x_0)(\underline{Y(x_i) - \beta_0 + \beta h(x_i)})$$

trend at x_0 weight = influence of x_i at x_0 residual at x_i

Experimental Results



Experimental Results



Ongoing Research: Portfolio Strategies

- higher-dimensional examples
  - 3D: time, stock price, level of stochastic volatility
- regression ("trend") in kriging
  - use related option formulae; improves fit
- experimental design
- stochastic kriging
  - kriging is for deterministic responses
  - incorporate uncertainty about true value after stochastic inner-level simulation

Conclusions

- Response surface modeling speeds up nested simulations:
  - simulation within simulation
  - simulation within optimization
- Nested simulation:
  - can attain high accuracy with great speed
  - it helps to leverage problem structure