Modelling the electricity markets

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Plan of the talk

1. Nord Pool – example of an electricity market
2. Multi-factor arithmetic spot price modelling
3. Forward pricing
4. Cross commodity modelling

The NordPool Market

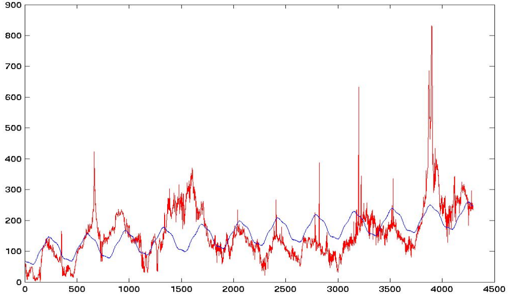
- The NordPool market organizes trade in
  - Hourly spot electricity, next-day delivery
  - Financial forward contracts
    - In reality mostly futures, but we make no distinction here
    - Frequently called *swaps*
  - European options on forwards
- Difference from "classical" forwards:
  - Delivery over a period rather than at a fixed point in time
- Crucial point in modeling

Elspot: the spot market

- A (non-mandatory) hourly market with physical delivery of electricity
- Participants hand in bids before noon *the day ahead*
  - Volume and price for each of the 24 hours next day
  - Maximum of 64 bids within technical volume and price limits
- NordPool creates demand and production curves for the next day before 1.30 pm

- The *system price* is the equilibrium
- Reference price for the forward market
- Due to congestion (non-perfect transmission lines), *area prices* are derived
  - Sweden and Finland separate areas
  - Denmark split into two
  - Norway may be split into several areas
- The area prices are the actual prices for the consumers/producers in the area in question

- Historical system price from the beginning in 1992



The forward market

- Forward with delivery over a period
- Financial market
- Settlement with respect to system price in the delivery period
- Delivery periods
  - Next day, week or month
  - Quarterly (earlier seasons)
  - Yearly
- Overlapping settlement periods (!)
- Contracts also called *swaps*: Fixed for floating price

The forward curve March 25, 2004

The option market

- European call and put options on electricity forwards
  - Quarterly and yearly electricity forwards
- Low activity on the exchange
- OTC market for electricity derivatives huge
  - Average-type (Asian) options, swing options ....

Multi-factor arithmetic models

A stochastic spot price model

- Desirable features of a stochastic electricity spot model are

1. Honours the statistical properties of the observed price data
   - Seasonality
   - Mean reversion (multi-scale)
   - Price spikes
2. Analytically tractable
   - Possible to price electricity forwards (swaps) analytically
   - Option pricing feasible

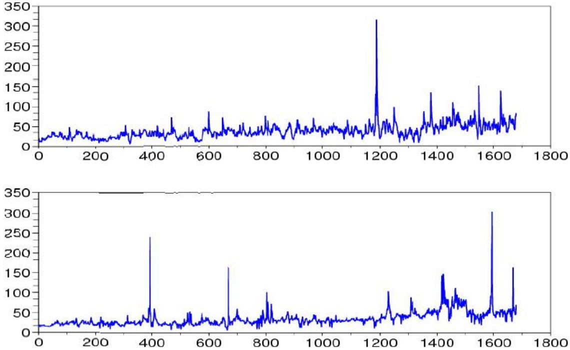
The model and properties

- The spot price as a *sum of non-Gaussian OU-processes*
  - BNS stochastic volatility model

$$S(t) = \Lambda(t) \times \sum_{i=1}^{n} Y_i(t)$$

$$dY_i(t) = -\alpha_i Y_i(t)\, dt + dL_i(t)$$

- $\Lambda(t)$ deterministic seasonality function
- $L_i(t)$ are independent *increasing* time-inhomogeneous pure jump Lévy processes
  - Called *independent increment processes*



- A simulation of $S(t)$ fitted to EEX electricity data
  - Calibration will come later....
  - Top: simulated, bottom: EEX prices

- Dynamics of $S(t)$

$$dS(t) = \left\{X(t) - \left(\alpha_n - \frac{\Lambda'(t)}{\Lambda(t)}\right) S(t)\right\} dt + \Lambda(t)\, d\bar{L}(t)$$

- AR(1)-process, with stochastic mean and seasonality
  - Mean-reversion to stochastic base level

$$X(t) = \Lambda(t) \times \sum_{i=1}^{n-1} (\alpha_n - \alpha_i) Y_i(t)$$

  - Seasonal speed of mean-reversion $\alpha_n - \Lambda'(t)/\Lambda(t)$
  - Seasonal jumps, where $d\bar{L}(t) = \sum_{i=1}^{n} dL_i(t)$, dependent on the stochastic mean

- Autocorrelation function for $\widetilde{S}(t) := S(t)/\Lambda(t)$

$$\rho(t,\tau) = \mathrm{corr}[\widetilde{S}(t), \widetilde{S}(t+\tau)] = \sum_{i=1}^{n} \omega_i(t,\tau) \mathrm{e}^{-\alpha_i \tau}$$

- If Y_i are stationary, $\omega_i(t,\tau) = \omega_i$
  - The weights ω_i sum to 1
- The theoretical ACF can be used in practice as follows:
  1. Find the number of factors n required
  2. Find the speeds of mean-reversion by calibration to empirical ACF

- $L_i(t)$ jumps only upwards
  - Jump size is a *positive* random variable
  - Called a subordinator process
- Y_i will mean-revert to zero
  - However, Y_i is always *positive*
- Ensures that $S(t)$ is positive
- NO Brownian motion component in the factors
  - Probability for $S(t)$ becoming negative
- In practice, one may use a Brownian motion component
  - Very small probability for negative prices
  - Calibration may become simpler?

Calibration to the EEX spot price

- Report here a calibration study by Thilo Meyer-Brandis (CMA & TU Munich)
  - We only give basic ideas here....
- 1652 daily Phelix Base electriity spot prices, starting from medio June, 2000
- Assume 3-factor model
  - First factor accounts for spikes (fast reversion)
  - Two remaining the “normal” variations in the market (medium and slow reversion)

$$S(t) = \Lambda(t)\left\{Y_1(t) + Y_2(t) + Y_3(t)\right\}$$

Steps in the estimation procedure

1. Fit a seasonal function to $S(t)$
   - Using a linear trend and trigonomewtric functions with 6 and 12 months periods
   - De-seaonalize data; $X(t) = S(t)/\Lambda(t)$
2. Separation of data into a spike component and a base component
3. Fitting the spike component to Y_1
4. Fitting $Y_2 + Y_3$ to the base component

Step 2: Spike component

- Estimate the mean-reversion of spikes as

$$\alpha_1 = -\log\left(\min_t \frac{X(t)}{X(t-1)}\right) = 1.3$$

- $\alpha_1 = 1.3$ corresponds to a half-life of 0.5 days for a spike
  - A spike is halfed over 0.5 days on average
- Transform the data into reversion-adjusted differences

$$\begin{aligned}\Delta X(t) := X(t) - e^{-\alpha_1} X(t-1) &= (Y_2(t) + Y_3(t)) \\ &\quad - e^{-\alpha_1}(Y_2(t-1) + Y_3(t-1)) + \epsilon(t)\end{aligned}$$

- $\epsilon(t) \approx L_1(t) - L_1(t-1)$ is the size of the spikes (iid)

Step 3: Fitting the spike component to data

- Estimation of $\epsilon(t)$ goes in two steps
  1. Estimating a threshold u which identifies spikes
  2. Estimating the spikes distribution
- Use techniques from Extreme Value Theory to fit a generalized Pareto distribution

$$P(\Delta X(t) - u \leq x | \Delta X(t) > u) = G_{\xi,\beta}(x) = 1 - (1 + \xi x/\beta)^{-1/\xi}$$

- Following estimates are found:

$$u = 1.6\,, \qquad \xi = 0.384\,, \qquad \beta = 0.472$$

- Based on 38 exceedances
  - Gives a jump frequency of 0.023
- Hence, $L_1(t) = ZdN(t)$
  - Z jump size: generalized Pareto distributed
  - N Poisson process, with frequency 0.023

- Next step is to filter out the spike component from the data
- This is simply done by subtracting $X_1(t)$ from the data $X(t)$

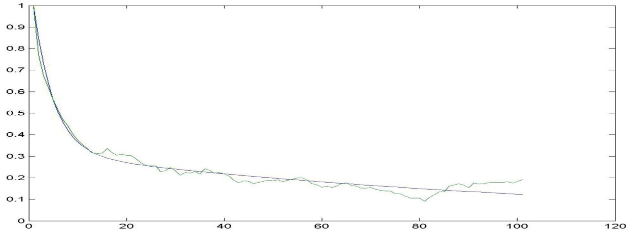
$$X(t) - X_1(t)\,, \qquad X_1(t) = \mathrm{e}^{-\alpha_1} X_1(t-1) + \widehat{\epsilon}(t)$$

with

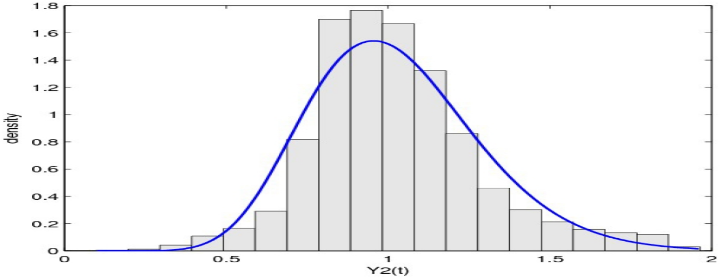
$$\widehat{\epsilon}(t) = (\Delta X(t) - u)1(\Delta X(t) > u)$$

- This leaves us with data cleaned of spikes
- Modelled using $Y_2(t) + Y_3(t)$

Step 4: Fitting the base component



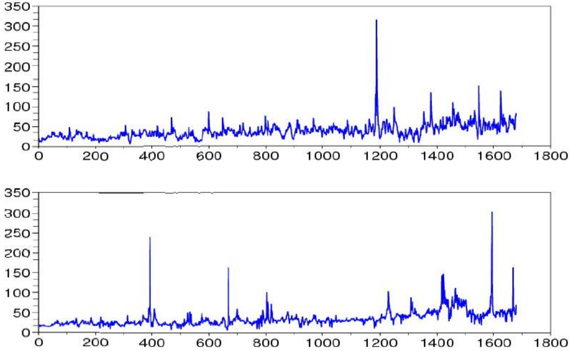
- Calibration of mean-reversion using empirical ACF
  - Estimates: $\alpha_2 = 0.243$ and $\alpha_3 = 0.009$



- Stationary distribution of $Y_2 + Y_3$ described by $\Gamma(14.8, 14.4)$
  - Both Y_2 and Y_3 are mean-reversion models
  - A stationary distribution for both exists
  - The sum must be stationary as well

- We assume $Y_2 \sim \Gamma(10.2, 14.4)$ and $Y_3 \sim \Gamma(4.6, 14.4)$
  - Then, $Y_2 + Y_3 \sim \Gamma(14.8, 14.4)$
- Choice based on that the medium mean-reversion process (Y_2) should have bigger jumps than the slow one (Y_3)
- BDLP of Y_2 and Y_3 known
  - Compound Poisson process with exponential jump distribution
  - Fast simulation algorithms exist
- We have a full specification of the model

- A simulation of $S(t)$ fitted to EEX electricity data



Forward pricing

The spot and electricity forward relation

- Let $S(t)$ be the spot price
  - Not necessarily a semimartingale
- Consider a forward contract delivering (financially) electricity over a period $[T_1, T_2]$
- Payoff from a long forward position entered at time $t \leq T_1$

$$\int_{T_1}^{T_2} S(t)\,dt - (T_2 - T_1)F(t, T_1, T_2)$$

- The forward price $F(t, T_1, T_2)$ denoted in Euro/MWh

- From general theory:
  - Price of any derivative is given as the present expected value with respect to a risk-neutral measure Q
- The spot $S(t)$ not storable
  - Any $Q \sim P$ risk-neutral
- Cost of entering the contract should be zero
- Price of a forward with constant interest rate
  - Assuming financial settlement at maturity T_2
  - Using adaptedness of $F(t, T_1, T_2)$

$$F(t, T_1, T_2) = \mathbb{E}_Q \left[\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(u)\, du \,|\mathcal{F}_t \right]$$

- Interchanging expectation and integration leads to

$$F(t, T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f(t, u)\, du$$

- Here, $f(t, u)$ is the price of a forward with fixed-delivery time at u,

$$f(t, u) = \mathbb{E}_Q \left[S(u) \,|\mathcal{F}_t \right]$$

- Question: What Q to use?
  - No hedging argument possible (buy-and-hold)
  - No storage or convenience yield arguments can be used
  - Possible approaches
    1. Condition on future information (B., Meyer-Brandis)
    2. Utility indifference (B, Cartea and Kiesel)

- Choose a simple approach here
- Restrict to a subclass of measures Q
  - Usual choice: Esscher transform
  - Structure preserving
- Essentially, a measure change introduces an modification in the spot drift
  - Coined the *market price of risk*
- Jump measure under Q

$$\ell_i^Q(dz, dt) = e^{\theta_i(t)z}\ell_i(dz, dt)$$

- Radon-Nikodym derivative for measure change:

$$\frac{dQ}{dP}|_{\mathcal{F}_t} = \prod_{i=1}^{n} Z_i(t)$$

- Z_i martingales defined as

$$Z_i(t) = \exp\left(\int_0^t \theta_i(s)\, dL_i(s) - \psi_i(0, t, -\mathrm{i}\theta_i(\cdot))\right)$$

- ψ_i is the cumulant function of L_i

Derivation of the forward price

- Calculate $f(t,u)$

$$
\begin{aligned}
f(t,u) &= \Lambda(u) \times \mathbb{E}_Q[Y(u) \,|\, \mathcal{F}_t] \\
&= \Lambda(u) \times \sum_{i=1}^{n} Y_i(t) e^{-\alpha_i(u-t)} + \int_t^u e^{-\alpha_i(u-s)}\, d\gamma_i(s) \\
&\quad + \Lambda(u) \sum_{i=1}^{n} \int_t^u \int_{\mathbb{R}_+} e^{-\alpha_i(u-s)} z \{ e^{\theta_i(s) z} - 1_{|z|<1} \}\, \ell_i(dz, ds)
\end{aligned}
$$

- Integrating over the delivery period $[T_1, T_2]$ yields the electricity forward price

- In conclusion:

$$F(t,T_1,T_2)=\Theta(t,T_1,T_2)+\sum_{i=1}^n \overline{\alpha}_i(t,T_1,T_2)Y_i(t)$$

where Θ is a risk-adjustment function, defined as

$$(T_2-T_1)\Theta(t,T_1,T_2)=\sum_{i=1}^n\int_t^{T_2}\int_{\max(v,T_1)}^{T_2}\Lambda(u)\mathrm{e}^{-\alpha_i(u-v)}\,du\,d\gamma_i(v)$$
$$+\sum_{i=1}^n\int_t^{T_2}\int_{\mathbb{R}_+}\int_{\max(v,T_1)}^{T_2}\Lambda(u)\mathrm{e}^{-\alpha_i(u-v)}\,du\,z\{\mathrm{e}^{\theta_i(v)z}-1_{z<1}\}\,\ell_i(dz,dv)$$

- $\overline{\alpha}_i$ is the seasonally weighted average of $\exp(-\alpha_i(u-t))$ for $u \in [T_1, T_2)$

$$\overline{\alpha}_i(t, T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(u) e^{-\alpha_i(u-t)} \, du$$

- Seasonally weighted average Samuelson effect
  - $\exp(-\alpha_i(u-t))$ increasing when time to maturity $u-t$ goes to zero
  - "Volatility" goes up as we approaches delivery at time u
  - Delivery over a period, so we average using a seasonal weighting!

Dynamics of the forward price

$$dF(t, T_1, T_2) = \sum_{i=1}^{n} \overline{\alpha}_i(t, T_1, T_2)\, d\widetilde{L}_i(t)$$

- $\widetilde{L}_i$ is the compensated L_i
- $F(t, T_1, T_2)$ is a martingale (under Q)

Pricing of options on forwards

- Let g be the payoff of an option
  - E.g, a put option $g(x) = \max(K - x, 0)$
  - Call options require a damping factor in what follows (or one can use the put-call parity)
- Option price is

$$p(t, T; T_1, T_2) = e^{-r(T-t)} \mathbb{E}_Q \left[\max\left(K - F(T, T_1, T_2), 0\right) \mid \mathcal{F}_t \right]$$

- Calculate this using Fourier transformation
  - Pricing expression suitable for FFT

- Using the inverse Fourier transform:

$$g(x) = \frac{1}{2\pi} \int \widehat{g}(y) \exp(ixy)\, dy$$

- By the independent increment property (using $n = 1$)

$$\mathbb{E}_Q\left[g(F(T, T_1, T_2)) \,|\, \mathcal{F}_t\right] = \frac{1}{2\pi} \int \widehat{g}(y) \mathbb{E}_Q\left[e^{iyF(T,T_1,T_2)} \,|\, \mathcal{F}_t\right] dy$$

$$= \frac{1}{2\pi} \int \widehat{g}(y) e^{iyF(t,T_1,T_2)} \mathbb{E}_Q\left[e^{iy \int_t^T \overline{\alpha}(s,T_1,T_2)\, d\widetilde{L}(s)} \,|\, \mathcal{F}_t\right] dy$$

- Introducing a cumulant $\widetilde{\psi}$

$$\widetilde{\psi}(t, T, \theta) = \int_t^T \int_0^\infty \left\{ e^{i\theta(s)z} - 1 \right\} \ell^Q(dz, ds)$$

- Fourier expression for option price ($\star$ the convolution product)

$$p(t, T; T_1, T_2) = e^{-r(T-t)} \left(g \star \Phi_{t,T} \right) \left(F(t, T_1, T_2) \right)$$

where

$$\widehat{\Phi}_{t,T}(y) = \exp\left(\widetilde{\psi}(t, T, y\overline{\alpha}(\cdot, T_1, T_2)) \right)$$

Cross-commodity multi-factor models

- Generalization of the arithmetic model for several commodities
- Applications to spread options and area prices
- Example: Options on the spark spread:
  - Option written on the spread between an electricity and gas forward
- Spark spread forward, supposing the same delivery period $[T_1, T_2]$,

$$F_s(t, T_1, T_2) = \mathbb{E}\left[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} E(s) - cG(s)\,ds|\mathcal{F}_t\right]$$

- $E(t)$ and $G(t)$ are the spot electricity and gas, resp.
- c is the heat rate (conversion of gas units into electricity)

- Model electricity and gas spot using the multi-factor arithmetic model

$$E(t) = \Lambda_E(t) \times \sum_{i=1}^{m} X_i(t)$$
$$G(t) = \Lambda_G(t) \times \sum_{j=1}^{n} Y_j(t)$$

- X_i and Y_j are non-Gaussian mean-reversion processes (as defined above)
- Spark spread forward price F_s computable in terms of $X_i(t)$ and $Y_j(t)$, as we have seen
- Expression suitable for transform-based pricing of options
  - Use of FFT or numerical Laplace transform

- Modelling idea: separate into common and unique factors
  - Let the jump components in the first k factors be equal
  - That is, X_i and Y_i are different *only* in the mean-reversion speeds α_i^E and α_i^G
  - Similar shock, but the two markets dampen them differently
  - Left with $m-k$ and $n-k$ unique factors
- Assuming stationary common factors

$$\mathrm{Cov}\left(\widetilde{E}(t), \widetilde{G}(t)\right) = \sum_{i=1}^{k} \frac{w_i}{\alpha_i^E + \alpha_i^G}$$

Conclusions

- Proposed a multi-factor OU model for electricity spot prices
- Analytical forward prices feasible
  - Forwards delivering the power over a period
- Option prices available using transform-based methods
- Extensions to cross-commodity modelling discussed
  - Spark spread modelling

Coordinates

- fredb@math.uio.no
- http://folk.uio.no/fredb
- www.cma.uio.no

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