The distribution of turbulence driven wind speed extremes; a closed form asymptotic formulation

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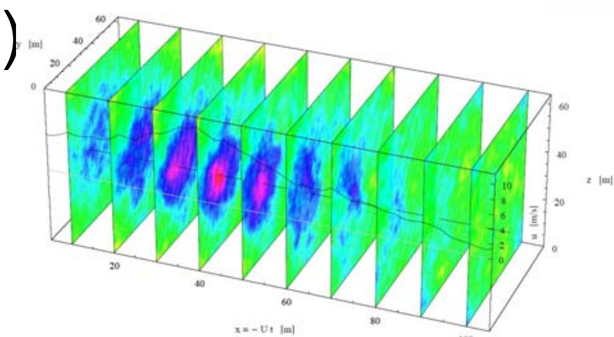
Outline

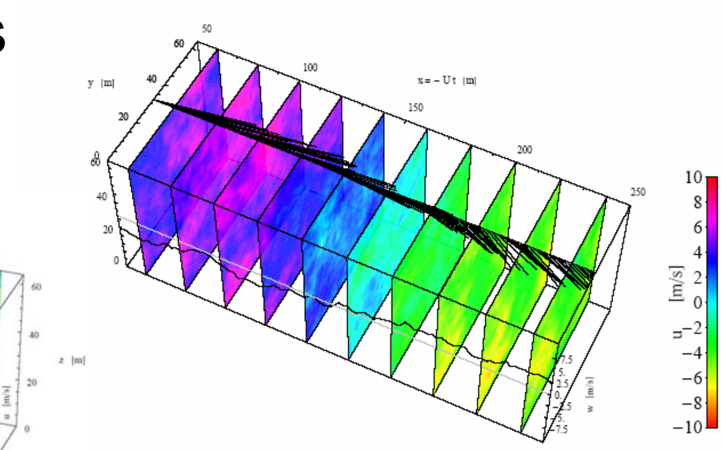
- Introduction
- Modeling
  - The classical approach - Cartwright /Longuet-Higgins [1]
  - An approach based on a non-Gaussian tail behavior
    - … with an empirical distribution parameter [3]
    - … with the requested asymptotic tail behavior derived from a subclass of the GH distribution
- Conclusion
- Outlook
- References

Introduction

- Wind sensitive structures … in particular wind turbines
- Extreme wind events … driven by turbulence
- “Gust-generator” for generation of stochastic turbulence fields with specified gust events consistently embedded … magnitudes of gust events

  (e.g. in an optimization context)





- … Relevant for aeroelastic design computations of wind turbines as well as structural reliability considerations

Introduction

- Focus on the simplest possible class of gust events … characterized by *wind speed increase* (coherent analogy: IEC 64100-1; extreme load case EOG)
- Aim: Asymptotic closed form solution for the distribution of the *largest* turbulence driven wind speed excursion within a specified span of time … both turbulence generated excursions and recurrence period are assumed to be large (but otherwise arbitrary)

Cartwright /Longuet-Higgins

- Based on pioneering work of Rice [2]
- Basic assumptions
  - Stationary process with Gaussian “parent distribution”
  - Independent local extremes
  - Large magnitudes … in terms of process standard deviations
  - Large number of local extremes contribution to the global extreme
- Approach
  - Distribution of local extremes
  - Distribution of the global extreme

Cartwright /Longuet-Higgins

- Result (normalised with process root mean square)
  - Distribution $$f_{\max}(\eta_m) = \eta_m \, Exp\left\{ -e^{-\frac{1}{2}\eta_m^2 + \ln(T\upsilon)} \right\} e^{-\frac{1}{2}\eta_m^2 + \ln(T\upsilon)} .$$
  - Mean $$E(\eta_m) = \sqrt{2\ln(\upsilon T)} + \frac{\gamma}{\sqrt{2\ln(\upsilon T)}} ,$$
  - Root mean square $$\sigma(\eta_m) = \frac{\pi}{\sqrt{12\ln(\upsilon T)}} ,$$
  - Mode $$m(\eta_m) = \sqrt{2\ln(\upsilon T)} .$$
  - … with $$\upsilon = \sqrt{\frac{m_2}{m_0}} \qquad \gamma \approx 0.5772$$

$$m_i = \int_0^\infty S(f) f^i df \, ,$$

Cartwright /Longuet-Higgins

- Characteristics:
  - Distribution resemble (some of) the functional characteristics of the EV1 distribution
  - Mean increases with increasing time span T
  - Mode increases with increasing time span T
  - Root mean square decreases with increasing time span T
- Performance … comparing with data
  - Good for small/moderate recurrence periods
  - May underestimate substantially for large recurrence periods

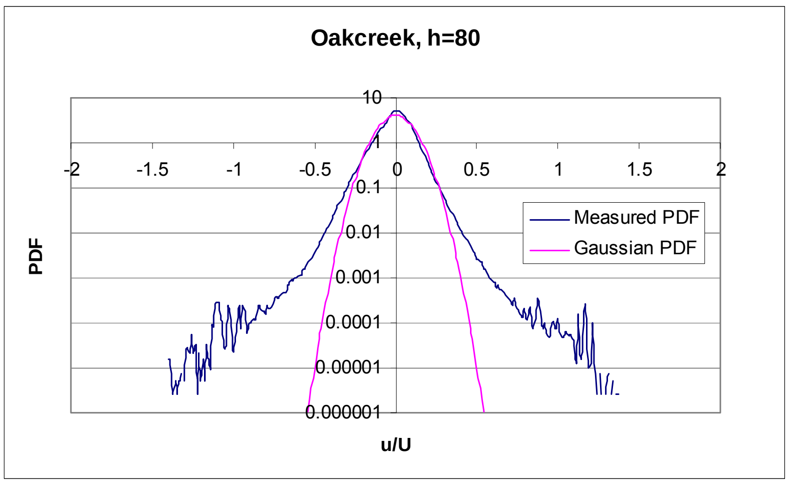
Cartwright /Longuet-Higgins

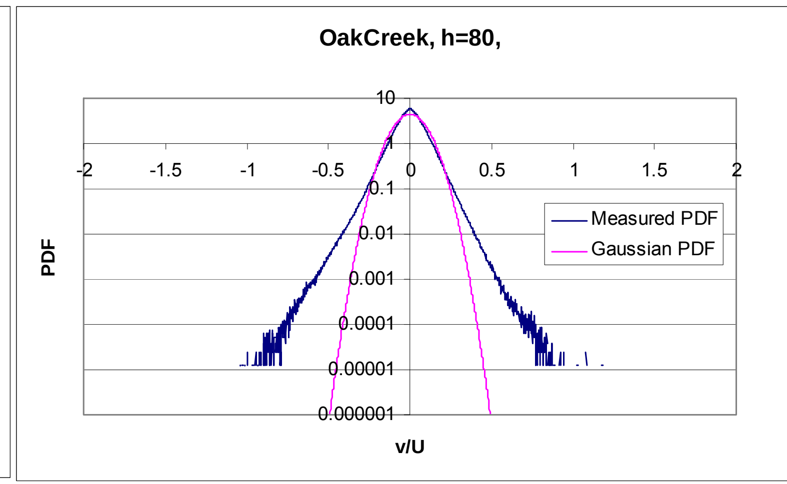
- Performance … an example

Site	Cartwright/ Longuet-Higgens	Extreme value analysis of measurements
Skipheia; 101m; 1 month	4.9 m/s	7.5 ± 0.1 m/s
Skipheia; 101m; 1 year	5.4 m/s	9.1 ± 0.2 m/s
Skipheia; 101m; 50 year	6.1 m/s	11.7 ± 0.2 m/s
Näsudden; 78m; 1 month	5.0 m/s	7.7 ± 0.2 m/s
Näsudden; 78m; 1 year	5.4 m/s	9.3 ± 0.3 m/s
Näsudden; 78m; 50 year	6.1 m/s	11.9 ± 0.4 m/s
Oak Creek; 79m; 1 month	7.9 m/s	12.4 ± 0.2 m/s
Oak Creek; 79m; 1 year	8.6 m/s	15.2 ± 0.2 m/s
Oak Creek; 79m; 50 year	9.6 m/s	19.6 ± 0.3 m/s

Prelude to non-Gaussian tail behavior approach

- Two observations:
  1. Conventional Gaussian assumption is inadequate for description of events associated with large excursions from the mean





  2. Extremes, associated with turbulence driven full-scale events in the atmospheric boundary layer, usually seems to be well described by a Gumbel EV1 distribution

- … the suggested model aims at providing the link between these observations

Model

Key elements:

- Assumptions
- Monotonic transformation
- Distribution of local extremes in transformed domain
- Distribution of the global extreme in transformed domain
- Number of local extremes as function of recurrence period
- Synthesis
- Resulting distribution expressed in the physical domain
- Parameter estimation

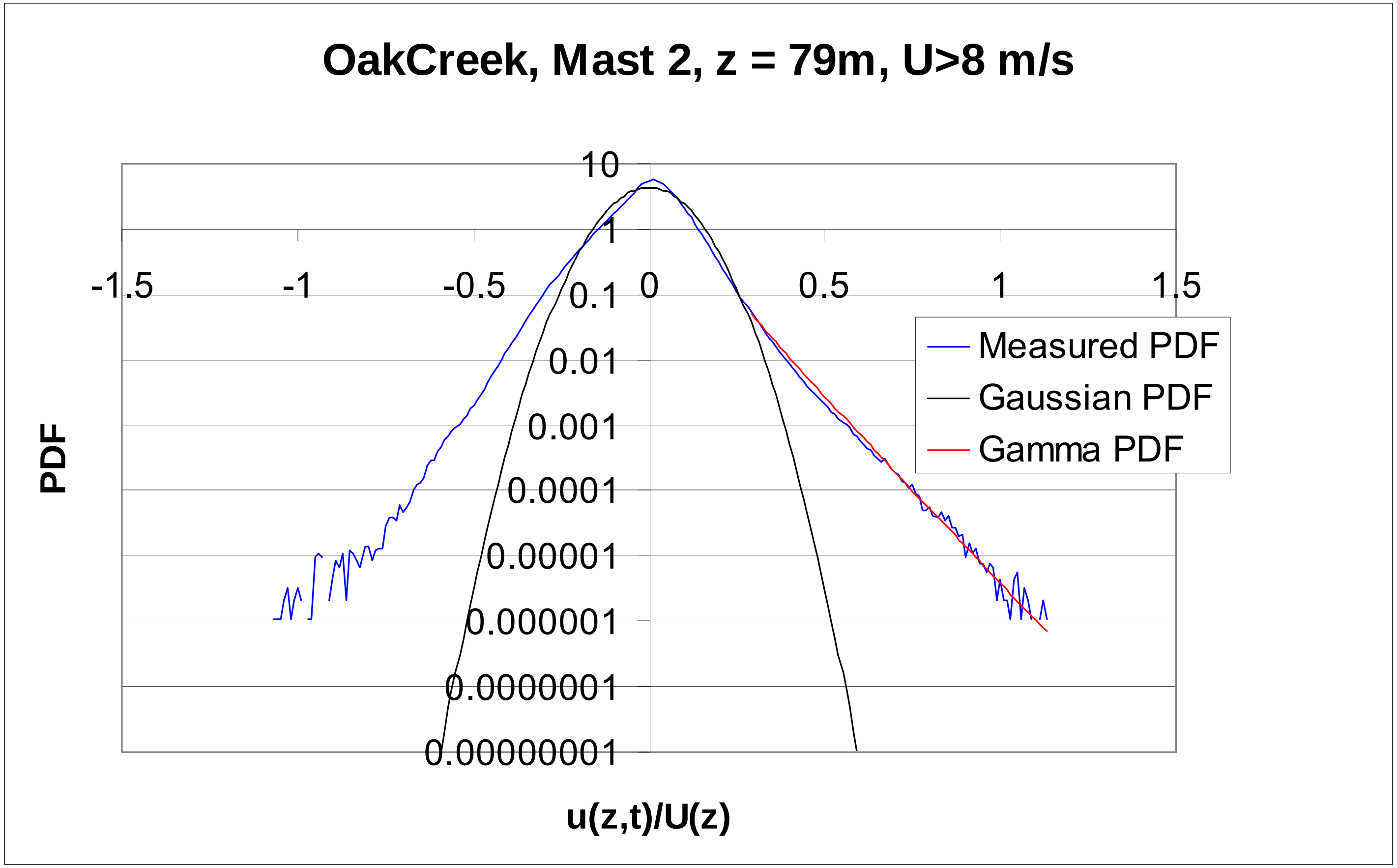
Model - Assumptions

- We postulate the following distribution of turbulence driven *large* excursions from mean (double sided Gamma dist.; shape par. =1/2):

$$f_{u_e}\left(u_e(z);\sigma(z),C(z)\right)=\frac{1}{2\sqrt{2\pi C(z)\sigma(z)}\sqrt{|u_e(z)|}}Exp\left(-\frac{|u_e(z)|}{2C(z)\sigma(z)}\right),$$

- *σ(z)* is the standard deviations of the *total* data population measured at altitude z
- *C(z)* is a dimensionless, but site- and height-dependant, *positive* constant

Model – ex. distribution fit in the asymptotic regime



Model - Monotonic transformation

- *We* introduce the *monotonic* transformation:

$$v_e = g(u_e) = sign(u_e)\sqrt{\frac{\sigma}{C(z)}}\sqrt{|u_e|}$$

- The (standard) "trick" is:
  - A monotonic transformation will transform local extremes in the physical domain into local extremes in the transformed domain
  - Thus, the number of local extremes (and their position on the time-axis) is invariant with respect to (strictly) monotone transformations
  - Therefore, *global* extremes may be analyzed in the transformed domain and subsequently transformed back to the physical domain

Model - Monotonic transformations

- In the transformed domain we obtain the following Gaussian PDF

$$f_{Gauss}(v_e;\sigma)=\frac{1}{\sqrt{2\pi}\sigma}Exp\left(-\frac{{v_e}^2}{2\sigma^2}\right).$$

… and the analysis of the extremes in this domain can take advantage of a Gaussian variable having a *tractable joint Gaussian distribution* of the variable and its associated *first* and *second* order derivatives (required for formulation of conditions for an extreme occurrence)

Model - Distribution of local extremes

- Rice [2] has established the statistics of local extremes, η_e, for a Gaussian process (normalized with σ):

$$f_\eta(\eta_e;\delta)=\frac{1}{\sqrt{2\pi}}\left[\delta\, e^{-\frac{\eta_e^2}{2\delta^2}}+\eta_e\sqrt{1-\delta^2}\, e^{-\frac{\eta_e^2}{2}}\, g(\eta_e,\delta)\right],$$

$$g(\eta_e,\delta)=\sqrt{\frac{\pi}{2}}\left(1+sign(\eta_e)\, Erf\left(\frac{|\eta_e|\sqrt{1-\delta^2}}{\sqrt{2}\,\delta}\right)\right),$$

… the statistics depends only on the *band width parameter*, which may be expressed in terms of process spectral moments as

$$\delta=\sqrt{\frac{m_0 m_4-m_2^2}{m_0 m_4}}\ ,$$

Model - Distribution of the global extreme

- We assume the *local extremes to be statistical independent*
- D.E. Cartwright and M. S. Longuet-Higgins derived the following *asymptotic* expression (i.e. *large* excursions) for the largest among N independent local maxima:

$$f_{\max\eta}(\eta_{em};N,\delta)=N\sqrt{1-\delta^2}\,\eta_{em}\,Exp\left[-N\sqrt{1-\delta^2}\,e^{-\frac{1}{2}\eta_{em}^2}\right]e^{-\frac{1}{2}\eta_{em}^2}\quad,$$

… which for *large* N can be approximated as

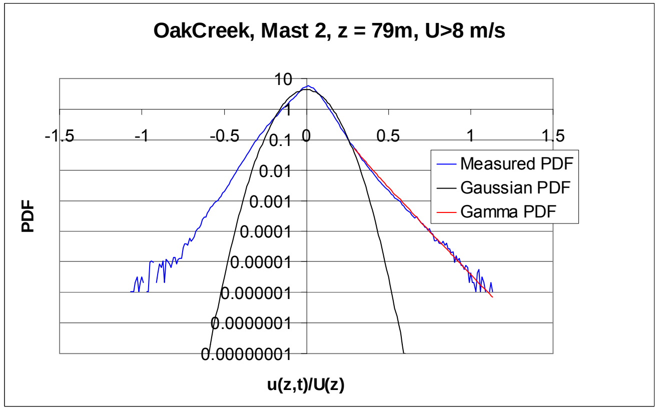
$$f_{\max\eta}(\eta_{em};N,\delta)=\eta_{em}\,Exp\left[-e^{-\frac{1}{2}\eta_{em}^2+\ln\left(N\sqrt{1-\delta^2}\right)}\right]e^{-\frac{1}{2}\eta_{em}^2+\ln\left(N\sqrt{1-\delta^2}\right)}\quad.$$

Model - Number of local extremes

- In the pure Gaussian case, N was obtained from Rice's estimate for the expected number of maxima [2]

$$N = \sqrt{\frac{m_4}{m_2}}\, T \quad .$$

- *Not* consistent within this approach



- The *expected number* of extremes of the process should include only contributions from large extremes (i.e. extremes exceeding ~2σ in the physical domain)

Model - Number of local extremes

- A large extreme in the transformed (Gaussian) domain is accordingly

$$V_0 = \sqrt{\frac{2}{C}}\sigma$$

- Closed form (asymptotic) expression for the expected number of maxima exceeding V_0 obtained using Rice's asymptotic result for expected number of excursions above a pre-defined threshold

$$N = \kappa T; \quad \kappa \equiv Exp\left(-\frac{1}{C}\right)\sqrt{\frac{m_2^3}{m_0^2 m_4}} \ .$$

Model - Synthesis

- Combine expressions for extreme PDF, bandwidth parameter, and rate of local (large) maxima to obtain

$$f_{max\,\eta}(\eta_{em};T,\kappa)=\eta_{em}\,Exp\left[-e^{-\frac{1}{2}\eta_{em}^{2}+ln(\kappa T)}\right]e^{-\frac{1}{2}\eta_{em}^{2}+ln(\kappa T)}.$$

- Transformation to the normalized *physical* domain

$$f_{max\,\mu}(\mu_m;T,\kappa,C)=\frac{1}{2C}\,Exp\left(-e^{-\frac{1}{2C}|\mu_m|+ln(\kappa T)}\right)e^{-\frac{1}{2C}|\mu_m|+ln(\kappa T)}.$$

Model – Characteristics

- Gumbel EV1 type of distribution … as “requested”

- Mean $$E(\mu_m) = 2C(\gamma + ln(\kappa T)) ,$$

- Root mean square $$\sigma(\mu_m) = \pi C\sqrt{\frac{2}{3}} ,$$

- Mode $$m(\mu_m) = 2C \; ln(\kappa T) \; .$$

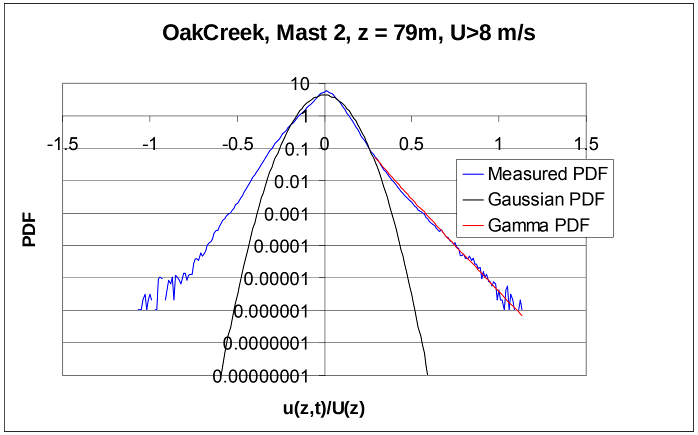
- Comparison with C/LH: We predict faster increase in mean and mode with T, and our root mean square is *independent* of T

Model – Required parameters

- Required parameters:
  - Standard deviation of the driving process σ
  - Spectral moments (m_2 and m_4): from measurements *or* closed form expressions based on generic wind spectra as specified in codes (including length scale specifications) – e.g. Kaimal spectrum
  - *C(z)* … requires a huge number of fast sampled data (which is seldom available), or an empirical “pre-calibration”

Model – Calibration of *C(z)*

- The "constant" *C(z)* is calibrated using a huge fast sampled data material representing three different terrain categories
  - offshore/coastal
  - flat homogeneous terrain, and
  - hilly scrub terrain
- …by minimizing the functional

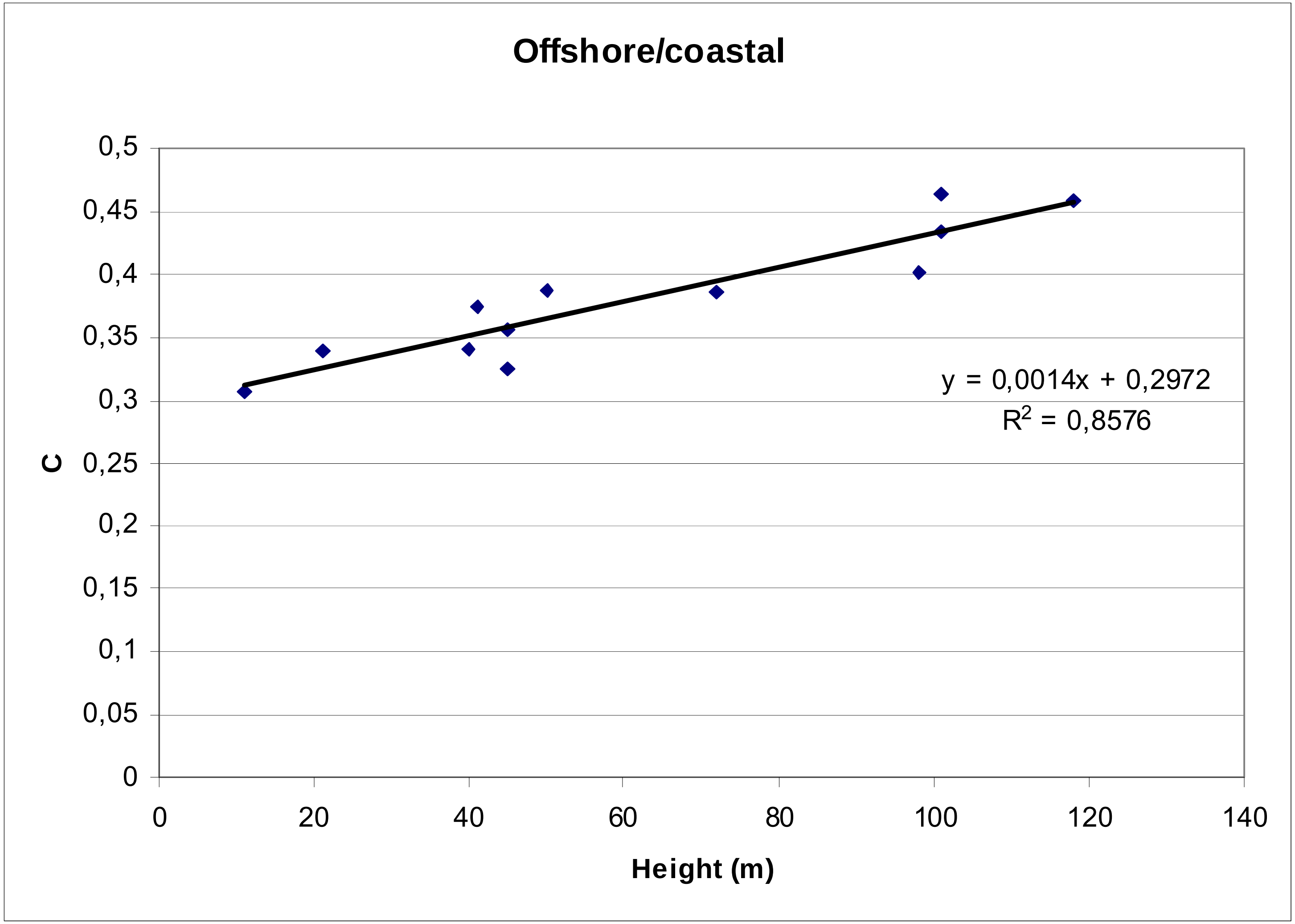


$$\Pi(C(z)) = \int_{2\sigma}^{+\infty} du(z)\left(f_{u_e}(u_e(z);\sigma(z),C(z)) - f_m(u_e(z))\right)^2 \ .$$

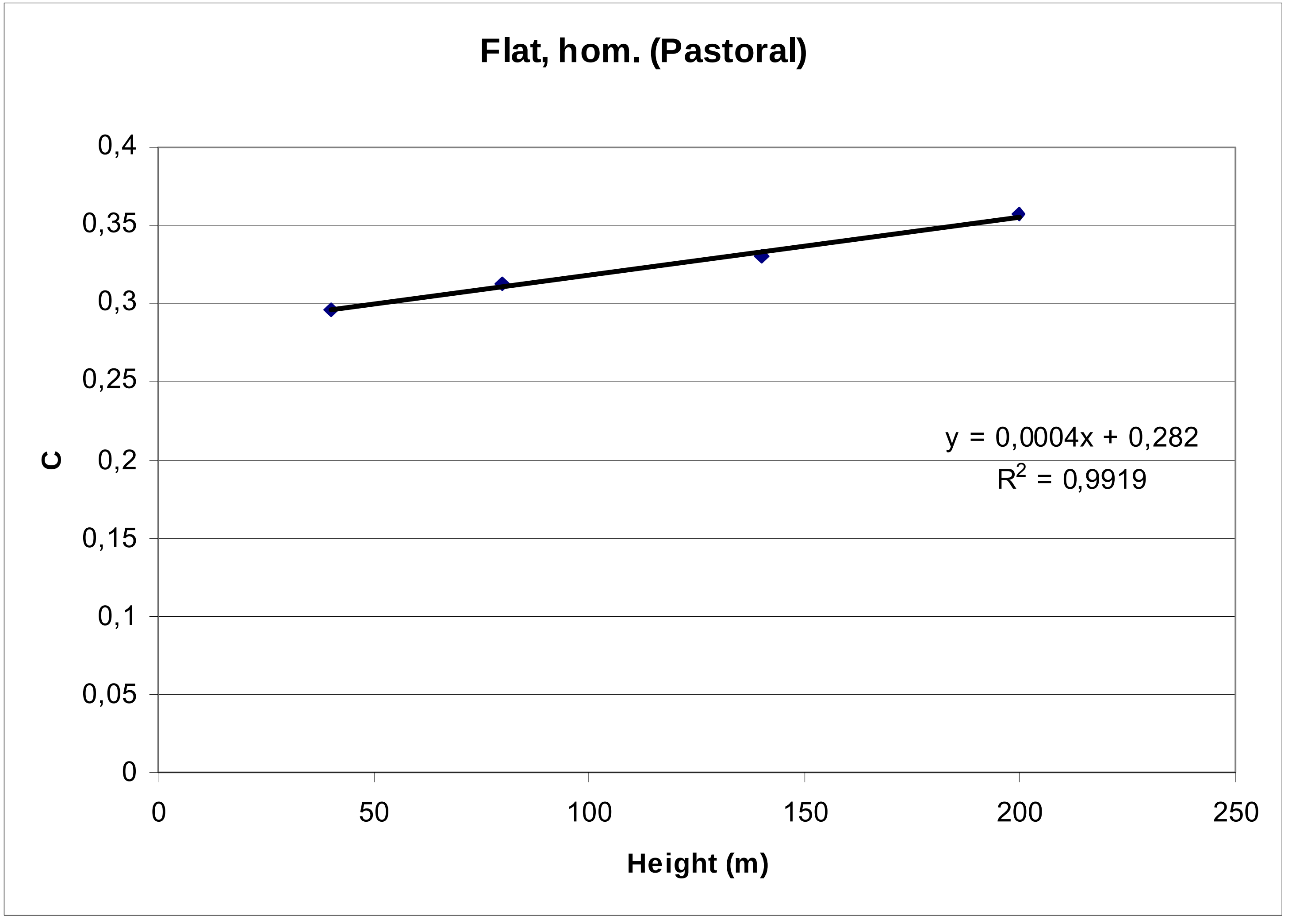
Model – Calibration of *C(z)*

Site	Type of site	Obs. height [m]	No. hours	Scan freq. [Hz]	C
Gedser rev	Offshore	45	385	5	0.357
Rødsand	Offshore	45	390	5	0.325
Horns Rev	Offshore	50	629	20	0.387
Nasudden	Coastal; flat	40	1122	1	0.340
Nasudden	Coastal; flat	98	1548	1	0.401
Nasudden	Coastal; flat	118	1589	1	0.459
Skipheya	Coastal; roling hills	11	5200	0.85	0.307
Skipheya	Coastal; roling hills	21	5737	0.85	0.339
Skipheya	Coastal; roling hills	41	6408	0.85	0.373
Skipheya	Coastal; roling hills	72	4446	0.85	0.386
Skipheya	Coastal; roling hills	101	3904	0.85	0.434
Skipheya	Coastal; roling hills	101	3550	0.85	0.463
Cabauw	Flat, hom. (Pastoral)	40	377	2	0.297
Cabauw	Flat, hom. (Pastoral)	80	421	2	0.313
Cabauw	Flat, hom. (Pastoral)	140	440	2	0.331
Cabauw	Flat, hom. (Pastoral)	200	404	2	0.358
Oak Creek (M1)	Hill, scrub	79	1671	8	0.437
Oak Creek (M2)	Hill, scrub	10	2593	8	0.366
Oak Creek (M2)	Hill, scrub	50	1916	8	0.404
Oak Creek (M2)	Hill, scrub	79	3210	8	0.426

Model – Calibration of *C(z)*



Model – Calibration of *C(z)*



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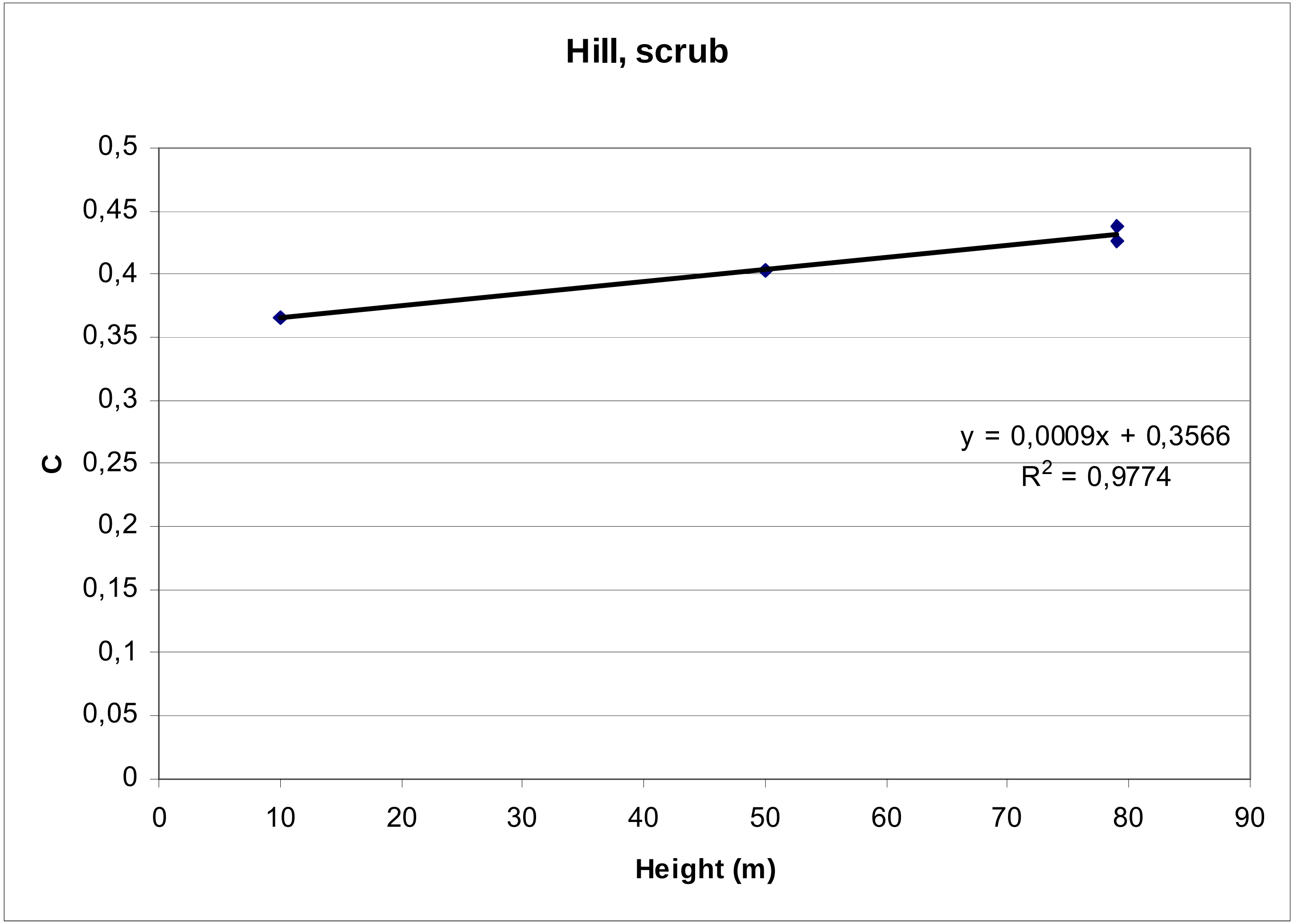
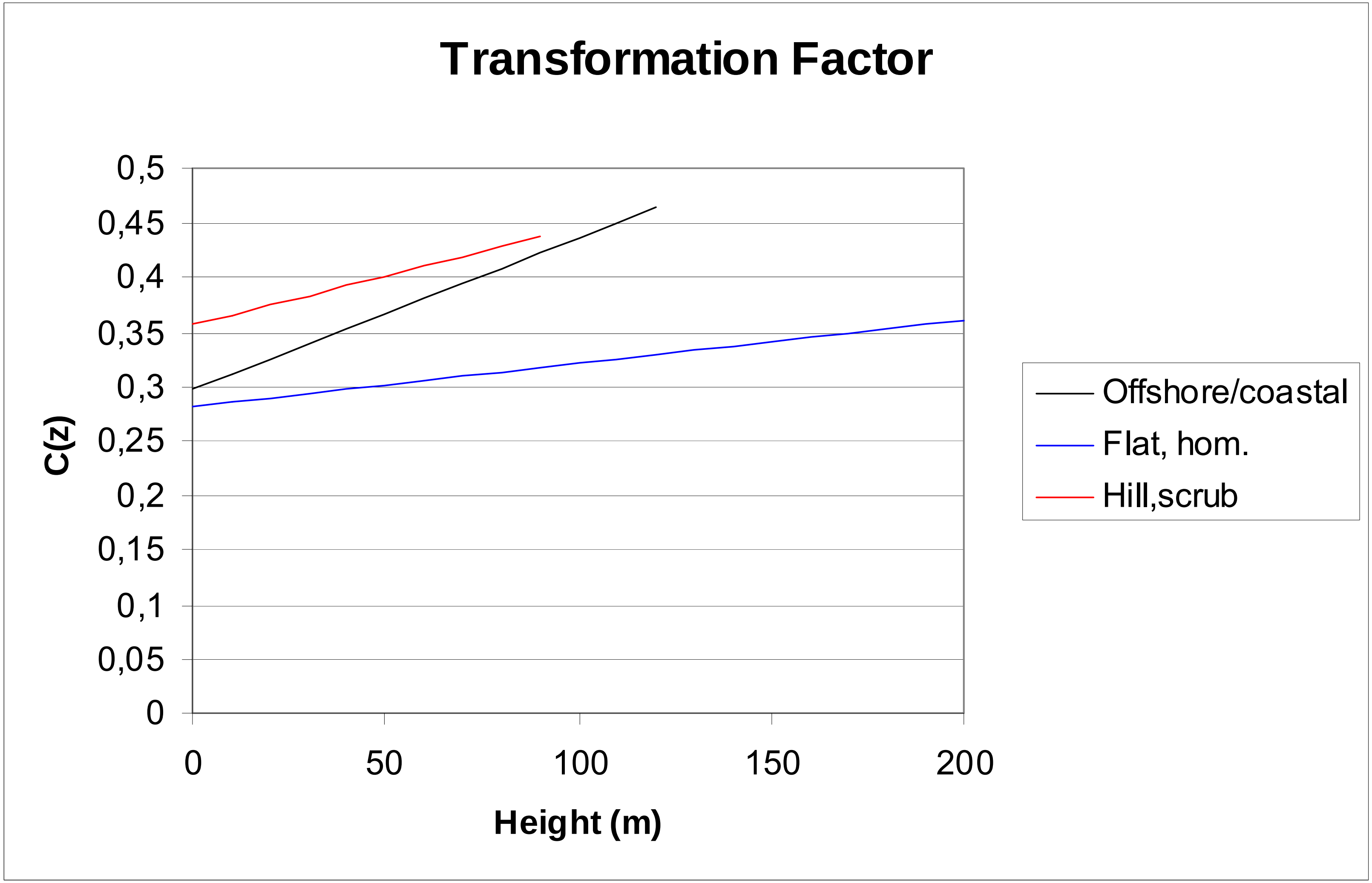


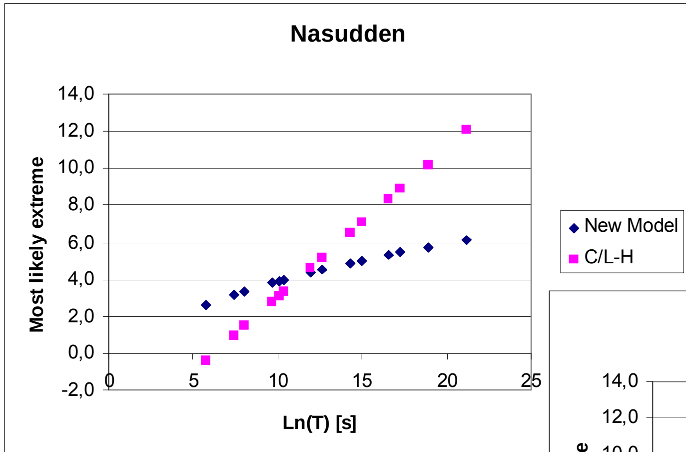
Model – Calibration of *C(z)*

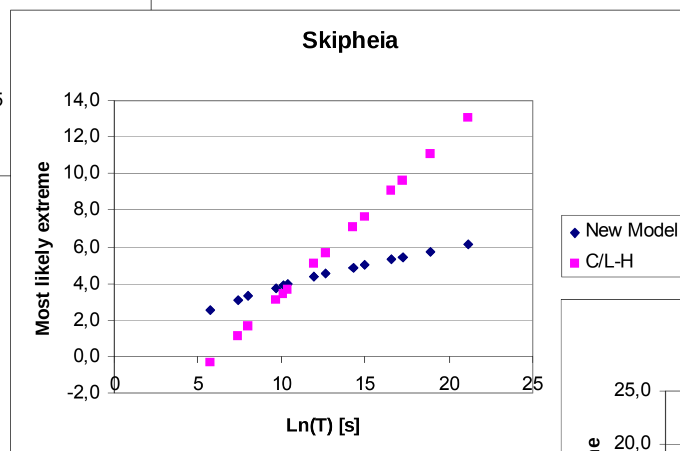


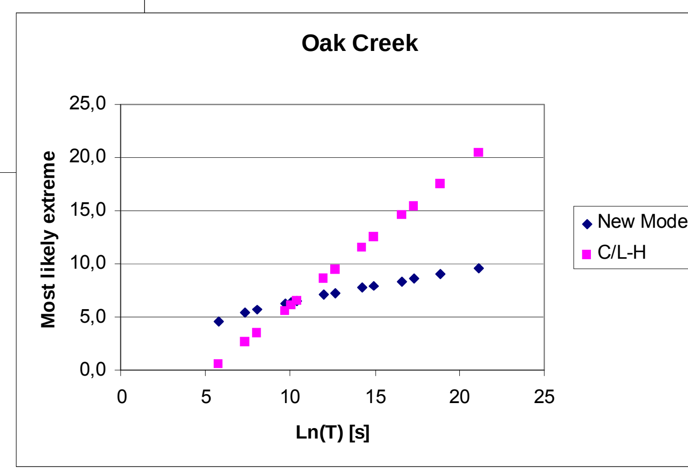
Model – Performance

	Cartwright/ Longuet-Higgens	Proposed model	Extreme value analysis of measurements
Skipheia; 101m; 1 month	4.9 m/s	7.5 m/s	7.5 ± 0.1 m/s
Skipheia; 101m; 1 year	5.4 m/s	9.6 m/s	9.1 ± 0.2 m/s
Skipheia; 101m; 50 year	6.1 m/s	13.0 m/s	11.7 ± 0.2 m/s
Näsudden; 78m; 1 month	5.0 m/s	6.9 m/s	7.7 ± 0.2 m/s
Näsudden; 78m; 1 year	5.4 m/s	8.9 m/s	9.3 ± 0.3 m/s
Näsudden; 78m; 50 year	6.1 m/s	12.1 m/s	11.9 ± 0.4 m/s
Oak Creek; 79m; 1 month	7.9 m/s	12.2 m/s	12.4 ± 0.2 m/s
Oak Creek; 79m; 1 year	8.6 m/s	15.4 m/s	15.2 ± 0.2 m/s
Oak Creek; 79m; 50 year	9.6 m/s	20.5 m/s	19.6 ± 0.3 m/s

Model – The asymptotic constraint







Model – *C* based on GH distribution approach

- Strategy:
  - Assume turbulence excursions generalized hyperbolic (GH) distributed (fatter than Gaussian tails)
  - The distribution of the largest extreme is preferred evaluated in a Gaussian domain as GH distribution is not particularly analytically tractable (joint GH(u,ú,ü) needed for extreme assessment)
  - When resulting EV1 is required constraints are imposed on the GH asymptotic behavior → specific GH subclass follows …

Model – GH distribution

- Distribution of turbulent excursions

$$f_{GH}(u;\alpha,\beta,\lambda,\mu,\delta)=\frac{\left(\alpha^2-\beta^2\right)^{\lambda/2}K_{\lambda-1/2}\left(\alpha\sqrt{\delta^2+(u-\mu)^2}\right)Exp(\beta(u-\mu))}{\sqrt{2\pi}\,\alpha^{\lambda-1/2}\,\delta^{\lambda}\,K_{\lambda}\left(\delta\sqrt{\alpha^2-\beta^2}\right)\left(\sqrt{\delta^2+(u-\mu)^2}\right)^{1/2-\lambda}}\ .$$

$$\delta\geq 0\wedge|\beta|<\alpha \text{ for } \lambda>0$$

$$\delta>0\wedge|\beta|<\alpha \text{ for } \lambda=0$$

$$\delta>0\wedge|\beta|\leq\alpha \text{ for } \lambda<0$$

… with the requirement imposed that the asymptotic behavior resembles the characteristics of the Gamma distribution with shape parameter 1/2

Model – GH asymptotes

- GH subclass defined by subclass parameter λ= ½
- GG defined by

$$f_{GG}(u;\alpha,\beta,\mu,\delta) \equiv \frac{\left(\alpha^2-\beta^2\right)^{1/4} K_0\left(\alpha\sqrt{\delta^2+(u-\mu)^2}\right) Exp\left(\beta(u-\mu)\right)}{\sqrt{2\pi\delta}\; K_{1/2}\left(\delta\sqrt{\alpha^2-\beta^2}\right)} \; ,$$

$$\delta \geq 0 \wedge |\beta| < \alpha$$

- GG asymptotics

$$f_{GG}(u;\alpha,\beta,\mu,\delta) \propto \frac{Exp\left(\delta\sqrt{\alpha^2-\beta^2}+\mu(\alpha-\beta)\right)\sqrt{\alpha^2-\beta^2}}{\sqrt{2\pi\alpha}\sqrt{|u|}} Exp\left(-\alpha|u|+\beta u\right) \quad \text{for u} \to \pm\infty \, .$$

Model – GG symmetric

- First attempt … assume symmetry of distribution of excursions
- Consequence β = 0
- Turbulent excursions have zero mean

$$E_{GG}[U] = \mu + \frac{\beta\delta}{\gamma}\frac{K_{3/2}(\delta\gamma)}{K_{1/2}(\delta\gamma)} = \mu = 0 \ , \quad \gamma \equiv \sqrt{\alpha^2 - \beta^2} \ .$$

- With β = 0 and μ = 0 GG asymptotes simplifies to

$$f_{GG,asymp}(u;\alpha,0,0,\delta) = \frac{\sqrt{\alpha}\,Exp(\delta\alpha)}{\sqrt{2\pi}\sqrt{|u|}} Exp(-\alpha|u|) \ .$$

Model – Parameter match

$$f_{GG,asymp}(u;\alpha,0,0,\delta) = \frac{\sqrt{\alpha}\, Exp(\delta\alpha)}{\sqrt{2\pi}\sqrt{|u|}} Exp(-\alpha|u|) \ .$$

$$f_G(u;\sigma) = \frac{1}{2\sqrt{\pi}\sqrt{2C\sigma|u|}} Exp\left(-\frac{|u|}{2C\sigma}\right),$$

- $C = \dfrac{1}{2\sigma\alpha}$
- and $\sqrt{2}Exp(\delta\alpha) = 1$, or $\delta\alpha = -\dfrac{Ln(2)}{2}$.
- … but $\delta \geq 0 \wedge |\beta| < \alpha$

DTU

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Model – GG asymmetric

- *Symmetric* GG gives too fat tails compared to the requested Γ-behavior … but $\beta \neq 0$ potentially opens for the needed affinity/scaling of the tail behavior
- Second (and last) attempt … require asymmetry of the GG parent distribution by assuming $\beta \neq 0$ … *even* in case a symmetric empirical distribution (engineering approach!)
- GG fit based on
  - Statistical moments (even order)
  - An additional parameter constraint arising from the requested type of asymptotic distribution behavior.

Model – GG fit

- Mean [4]

$$0 = \mu + \frac{\beta\delta}{\gamma}\frac{K_{3/2}(\delta\gamma)}{K_{1/2}(\delta\gamma)} , \qquad \gamma \equiv \sqrt{\alpha^2 - \beta^2} .$$

- Variance [4]

$$\sigma^2 = \delta^2\left(\frac{K_{3/2}(\delta\gamma)}{\delta\gamma\, K_{1/2}(\delta\gamma)} + \frac{\beta^2}{\gamma^2}\left(\frac{K_{5/2}(\delta\gamma)}{K_{1/2}(\delta\gamma)} - \left(\frac{K_{3/2}(\delta\gamma)}{K_{1/2}(\delta\gamma)}\right)^2\right)\right) ,$$

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Model – GG fit

- 4th order central moment

$$\mu_4 = \frac{\partial^4}{\partial\theta^4} C(\theta)\bigg|_{\theta=0} + 3\left[\delta^2\left(\frac{K_{3/2}(\delta\gamma)}{\delta\gamma K_{1/2}(\delta\gamma)} + \frac{\beta^2}{\gamma^2}\left(\frac{K_{5/2}(\delta\gamma)}{K_{1/2}(\delta\gamma)} - \left(\frac{K_{3/2}(\delta\gamma)}{K_{1/2}(\delta\gamma)}\right)^2\right)\right)\right]^2 .$$

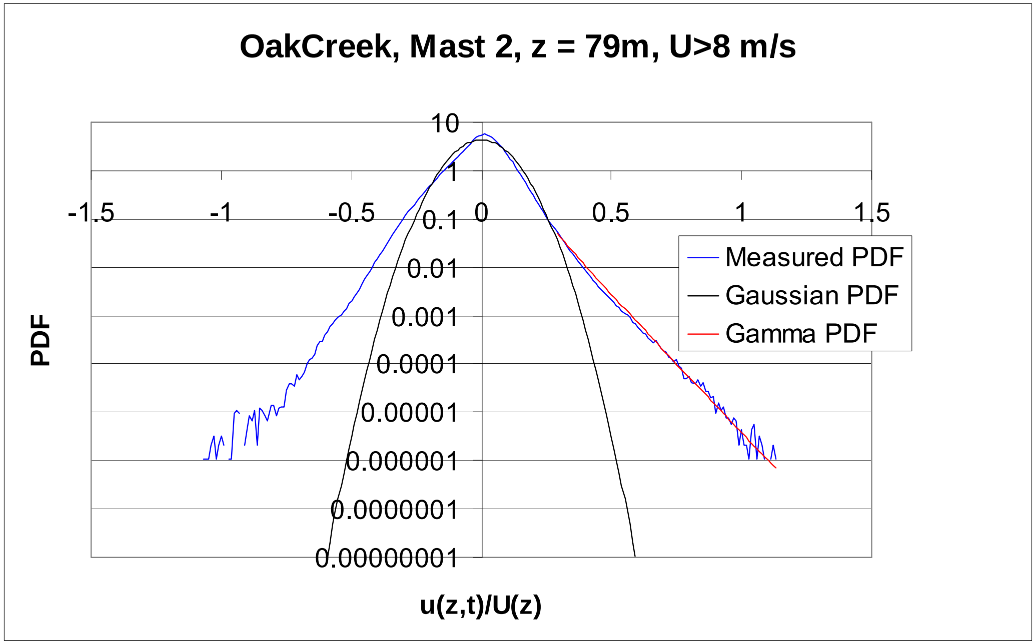
with the GG cumulant function *C*(*Θ*) given by [4]

$$C(\theta) = \frac{1}{4} Ln\left(\frac{\gamma}{\alpha^2 - (\beta+\theta)^2}\right) + Ln\left(\frac{K_{1/2}\left(\delta\sqrt{\alpha^2 - (\beta+\theta)^2}\right)}{K_{1/2}\left(\delta\sqrt{\alpha^2 - \beta^2}\right)}\right) + \frac{\theta}{2} ,$$

- Requested asymptotic match

$$\frac{Exp\left(\delta\sqrt{\alpha^2 - \beta^2} + \mu(\alpha - \beta)\right)\sqrt{2(\alpha+\beta)}}{\sqrt{\alpha}} = 1 .$$

Model – Definition of asymptotic regime



- Crossing between Gauss PDF and continuation of GG asymptote

$$\frac{u_0^{\ 2}}{2\sigma^2} = \frac{u_0}{2C\sigma} + \frac{Ln(u_0)}{2} - Ln\left(\frac{\sqrt{\sigma}}{2\sqrt{C}}\right); \qquad C = (2\sigma)^{-1}(\alpha - \beta)^{-1}$$

Model – Implication for local extremes to be counted

- Rate of extremes in the asymptotic regime (i.e. rate of extremes exceeding u_0)

$$\kappa \equiv Exp\left(-\frac{k}{2C}\right)\sqrt{\frac{m_2^3}{m_0^2 m_4}}\ ; \qquad k=\frac{u_0}{\sigma} \qquad C=\left(2\sigma\right)^{-1}\left(\alpha-\beta\right)^{-1}$$

Model – Syntheses

- Assume the existence of a monotonic memoryless (time independent) variable transformation that transforms the GG distribution onto a Gaussian distribution
- This transformation does not have to be known, except for its asymptotic properties

$$v = g(u) \propto g_{Asymp}(u) = \sqrt{\frac{\sigma}{C}}\sqrt{u} \quad for\ u \to +\infty ,$$

- The steps from here is analogue to the previous model with the empirical determination of C …

Conclusions

- An *asymptotic* model for the PDF of the *largest* wind speed excursion is derived
- The model is based on a “mother” distribution that reflects the Exponential-like distribution behaviour of *large* wind speed excursions … and is shown to be of the Gumbel EV1 type
- The recurrence period is assumed *large*, but may otherwise be arbitrary
- The model requires only a few, easy accessible, input parameters … these are basic parameters characterizing the stochastic wind speed processes in the atmopheric boundary layer together with the recurrence period

Conclusions

- The model parameter, C, have been calibrated against a large number of full-scale time series wind speed measurements for application in three common terrain categories
- Model predictions have been successfully compared to results derived from full-scale measurements of wind speeds extracted from “Database on Wind Characteristics”
- A fit of the C parameter has been attempted by assuming a parent distribution as a subclass (GG) of the GH distribution family

Conclusions

- This approach in addition opens for a consistent definition of the asymptotic regime
- The *symmetric* version of the GG distribution inevitable results in too fat tails compared to the requested asymptotic behaviour
- This has lead to the proposal of a GG fit with the skewness parameter β *required* different from zero! … but up to now it has not been investigated if this approach leads to parameter estimates within the allowable regime $\delta \geq 0 \wedge |\beta| < \alpha$

Outlook

- Analyze the monotony of the transformation
g: GG → Gauss
- Analyze if the fitting system of equations can be solved within the allowable parameter regime

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