Ruin excursions, the G/G/∞ queue and tax payments in renewal risk models

Hansjörg Albrecher (Lausanne)

Relations between queueing models and collective risk models are a classical topic in applied probability, to which Soren Asmussen has contributed significantly over the last decades. In this talk some further links between queueing and risk theory will be discussed and it will be illustrated how these can be used to obtain simple identities for ruin probabilities and related quantities in risk models with dividend and tax payments.

Volatility and Variation

Ole E. Barndorff-Nielsen (Aarhus)

A review is given of recent and ongoing work on modelling and measurement of volatility, with particular reference to applications in turbulence and finance.

Moment distributions of phase–type

Mogens Bladt (National University of Mexico) and Bo Friis Nielsen (DTU Copenhagen)

Phase–type distributions are a computationally and analytically convenient tool in applied probability where explicit and/or exact solutions (through numerical evaluation) frequently may be obtained even in rather complex stochastic models. Examples from queueing theory and risk analysis are numerous. Phase–type distributions are contained in the class of matrix–exponential distributions, which are distributions on the positive reals which have a rational Laplace transform. Most results that are valid for phase–type distributions may be calculated by similar means for the wider class of matrix–exponential distributions.

In this talk we show that phase–type and matrix–exponential distributions are closed under the formation of moment distributions. If X is a positive random variable and μ_i is its i'th moment, then the function $f_i(x) = x^i f(x)/\mu_i$ is a density function, and the corresponding distribution is called the i'th order moment distribution.

This closure property of phase–type and matrix–exponential distributions can be added to a number of already known closure properties for such distributions, and in particular phase–type representations of the moment distributions are of interest and non–trivial to establish.

First order moment distribution appear naturally in a number contexts, the classical example being the income distribution in a population and which is related to the so–called Lorenz curve and Gini index. For the first order distribution we present an explicit formula for the related Lorenz curve and Gini index when the underlying distributions is of phase–type or matrix–exponential.

Moment distributions of orders one, two and three have been extensively used in areas such as economy, physics, demography and civil engineering.

On perfect sampling and conditional large deviations

Jose Blanchet (Columbia)

This talk touches upon various areas in applied probability that have been largely influenced by the work of Soren Asmussen over the years. The talk centers around a theme that, as we shall see, and maybe surprisingly, is common to both the design of efficient perfect sampling and rare-event simulation algorithms: conditional distributions given rare events. We will revisit one of the earliest papers in the area of perfect simulation (i.e. sampling without bias from steady-state distributions), due to Asmussen, Glynn and Thorisson (1992) and show that the design of efficient perfect simulation algorithms relates to questions such as good asymptotic descriptions of conditional processes given suitable rare events. We illustrate how recent simulation techniques whose aim is to capture such good descriptions allow to design perfect samplers for models for which no previous samplers are known (such as heavy-tailed queues).

A Markov chain Monte Carlo method for rare-event probability estimation

Zdravko Botev and Pierre L'Ecuyer (Montreal)

Importance sampling and Markov chain Monte Carlo are typically considered two distinct simulation methods, where importance sampling is predominantly used for rare-event estimation problems, and Markov chain Monte Carlo is typically used for Bayesian computations. In this talk we propose a generic approach to rare-event probability estimation that combines both methods within a single framework.

The BRAVO effect and other problems involving 'biological' models

Daryl J. Daley (Univ. of Melbourne and ANU Canberra)

Many simple models for biological problems are formulated stochastically and studied deterministically with an underlying limit operation largely suppressed. A recent example involving stationary birth–death processes has both first and second moment asymptotics accessible: a curious discontinuity occurs in its variance asymptotics. Nazarathy and Weiss (QUESTA, 2008) demonstrated its existence (the BRAVO effect) in the linear asymptotics of the variance function of the output of M/M/1/K queues (large K), and a modified version holds for the asymptotics of M/M/c/K systems: are such phenomena related in any way to change-point behaviour in e.g. pressure–temperature phase-change diagrams for some physical systems? I

shall refer to problems for other systems involving models for epidemic phenomena.

Risk aggregation

Paul Embrechts (ETH Zurich)

Motivated by applications to risk aggregation, I discuss numerical algorithms (AEP and GAEP) for the calculation of the distribution function of possibly dependent random variables. These algorithms are based on a geometric decomposition of the domain of integration. A comparison with (quasi) Monte Carlo methods is made. The results for (G)AEP are to be found in:

[1] Arbenz, P., Embrechts, P., Puccetti, G. (2011): The AEP algorithm for the fast computation of the distribution of the sum of dependent random variables. Bernoulli 17(2), 562-591

[2] Arbenz, P., Embrechts, P., Puccetti, G. (2011): The GAEP algorithm for the fast computation of the distribution of a function of dependent random variables. To appear in Stochastics,

[3] Embrechts, P., Puccetti, G. (2010) Risk Aggregation. Copula Theory and its Applications, P. Jaworski, F. Durante, W. Haerdle, and T. Rychlik (Eds.) Lecture Notes in Statistics - Proceedings 198, Springer Berlin/Heidelberg, pp. 111-126.

Regenerative structure of processes that depends on infinite future: examples and elements of theory

Sergey Foss (Heriot-Watt Edinburgh and Novosibirsk)

First, I will discuss several examples in the same spirit: the dynamics of a stochastic process is highly dependent, but there exist random events that depend on entirely infinite future that make the process regenerative.

Second, I will describe the dynamics of such a process by a Markovian stochastic recursion and introduce some elements of theory which seems to be complementary to the classical theory of Harris Markov chains.

This is a part of an ongoing joint work with Stan Zachary.

A Martingale Fixed-Point Problem Arising From Financial Regulation

Paul Glasserman (Columbia University)

Some proposals for new approaches to financial regulation include having regulators use market signals (in particular, a bank's stock price) in deciding when to intervene. The motivation for this approach is the idea that market prices incorporate expectations of future performance and the likelihood of distress. Sundaresan and Wang (2010) have pointed out that this raises a consistency issue: the regulator is influenced by the stock price, but the stock price is influenced by the anticipated action of the regulator. If we

require the discounted stock price to be a martingale (reflecting the notion that the price reflects market expectations), this issue can be formulated as a question of existence and uniqueness of a martingale solution to a fixed-point problem. We analyze a specific instance of the general problem, contrasting what happens in discrete and continuous time. This is based on on-going work with Behzad Nouri, Suresh Sundaresan, and Zhenyu Wang.

Limit Theorems for Random Search
Peter W. Glynn (Stanford)

We will discuss several simulation-based algorithms for minimizing a smooth function defined as an expectation. Each algorithm involves randomly generating points within the feasible region, after which one conducts a series of simulations at each such point. In the simplest such algorithm, the simulations are averaged to obtain an estimator for the function value at each of the points; the minimizer is then estimated via the minimizing sample average. The convergence characteristics of such algorithms are heavily affected by the particular trade-off made between the amount of exploration (i.e. the number of random points) and the simulation accuracy demanded (i.e. the number of simulation samples per point). We discuss consistency results in the presence of both light and heavy tails, as well as a limit theorem that describes the optimal rate of convergence (when the trade-off is made optimally). We then go on to discuss large deviations for the simulation-based minimizer, and discuss also a variant for minimizing convex functions in which one assumes that both function values and gradients are estimated via simulation at each randomly chosen point, after which supporting hyperplanes are determined and an approximating convex function constructed. We further discuss optimal rates of convergence in this latter setting.

Because of their ease of use, such algorithms are sometimes used by practitioners. We also view these results as providing theoretical benchmarks against which to compare more sophisticated algorithms that more intelligently search within the feasible region. This involves joint work with Yen Lin Chia, Abhay Subramanian, and Wei Wu.

Exact Gerber-Shiu formulas for random walks and compound Poisson processes.
Martin Jacobsen (Copenhagen)

For a random walk in discrete time started at a strictly positive level, consider the time where the walk first becomes strictly negative. Assuming that the distribution of the downward jumps of the walk is a linear combination of exponentials, an expression is derived for the joint moment generating function of the time to exit, the level reached at exit (deficit at ruin) and the level just before exit (surplus before ruin). Analogous results are available for some compound Poisson processes in continuous time with

two-sided jumps and a positive drift. The results form a natural continuation of those presented in the author's contribution to the Asmussen festschrift and for random walks are shown by finding partial eigenfunctions for the transition operator of the Markov chain consisting of the sequence of pairs of consecutive values of the walk.

Population size dependent, age structured branching processes linger around their carrying capacity

Peter Jagers (Chalmers) and Fima C. Klebaner (Monash University)

Dependence of individual reproduction upon the size of the whole population is studied in a general branching process context. The particular feature under scrutiny is that of reproduction changing from supercritical in small populations to subcritical in large ones. The transition occurs when population size passes a critical threshold, known in ecology as the carrying capacity. We show that populations either die out directly, never coming close to the carrying capacity, or else they grow quickly towards the latter, subsequently lingering around it for a time that is expected to be exponentially long in terms of a carrying capacity tending to infinity.

Asymptotic normality of M-estimators in nonhomogeneous hidden Markov models

Jens Ledet Jensen (Aarhus University)

Results on asymptotic normality for the maximum likelihood estimate in hidden Markov models are extended in two directions. The stationarity assumption is relaxed, which allows for a covariate process influencing the hidden Markov process. Furthermore a class of estimating equations is considered instead of the maximum likelihood estimate. The basic ingredients are mixing properties of the process and a general central limit theorem for weakly dependent variables.

On some tractable growth collapse processes with renewal collapse epochs

Offer Kella (The Hebrew University of Jerusalem)

In this paper we generalize existing results for the steady state distribution of growth collapse processes with independent exponential inter-collapse times to the case where they have a general distribution on the positive real line having a finite mean. In order to compute the moments of the stationary distribution, no further assumptions are needed. However, in order to compute the stationary distribution, the price that we are required to pay is the restriction of the collapse ratio distribution from a general one concentrated on the unit interval to minus-log-phase-type distributions. A random variable has such a distribution if the negative of its natural logarithm has a phase type distribution. Thus, this family of distributions is dense in the

family of all distributions concentrated on the unit interval. The approach is to first study a certain Markov modulated shot-noise process from which the steady state distribution for the related growth collapse model can be inferred via level crossing arguments.

Characterization of tails through hazard rate and convolution closure properties

Dimitrios G. Konstantinides and A. G. Bardoutsos (University of the Aegean)

We use properties of the Matuszewska indices for density functions to show asymptotic inequalities for hazard rates. Two statements about the relation between membership in the classes of dominatedly or extended rapidly varying tail distributions and the corresponding hazard rate conditions are provided. These results lead to the further study of the relation between subexponential subclasses. Convolution closure is established for the class of distributions with extended rapidly varying tails. A revision of Pitman's criterion of subexponentiality is given.

Identifying separated time scales in stochastic models of reaction networks

Tom Kurtz (University of Wisconsin)

This is joint work with Hye-Won Kang. For chemical reaction networks in biological cells, reaction rates and chemical species numbers may vary over several orders of magnitude. Combined, these large variations can lead to subnetworks operating on very different time scales. Separation of time scales has been exploited in many contexts as a basis for reducing the complexity of dynamic models, but the interaction of the rate constants and the species numbers makes identifying the appropriate time scales tricky at best. Some systematic approaches to this identification will be discussed and illustrated by application to one or more complex reaction network models.

Checkpointing For The RESTART Problem in Markov Networks

Lester Lipsky, Derek Doran and Swapna Gokhale (University of Connecticut)

We apply the known formulas of the RESTART problem, and derive new equations, to Markov models of software (and many other) systems. We show how checkpoints might be included, with their resultant performance under RESTART. The result is a complete procedure for finding the mean, variance, and tail behavior of the job completion time as a function of the failure rate. We also provide a detailed example.

Threshold strategies for risk processes and their relation to queueing theory

Andreas Löpker, Onno Boxma (TU Eindhoven) and David Perry (Haifa)

We consider a risk model with threshold strategy, where the insurance company pays off a certain percentage of the income as dividend whenever the current surplus is larger than a given threshold. We investigate the ruin time, ruin probability and the total dividend, using known results from queueing theory.

Precise large deviations probabilities for a heavy-tailed random walk

Thomas Mikosch (University of Copenhagen)

In this talk we will consider the tail probabilities of partial sum processes for stationary processes whose marginal distribution has power law tails. These results generalize the classical results by A.V. and S.V. Nagaev who showed that the "heavy-tail heuristics" applies in this case: the power law tails of the partial sums are essentially due to the maximum term in the sum.

The situation changes in the case of dependent sequences. Then extremal clusters shape the form of the tails of the partial sums. But in contrast to the tails of the maxima, the extremal index does not appear in these quantities. In contrast to the tail behavior of partial maxima there are only very few particular cases where we can determine the tail behavior of partial sums for stationary sequences. We will consider some known cases, compare them with the iid case and indicate how these large deviation results can be used to proved results about ruin probablities.

Distributions with rational moment generating functions

Bo Friis Nielsen (DTU Copenhagen)

Numerous definitions of multivariate exponential and gamma distributions can be retrieved from the literature. These distribtuions belong to the class of Multivariate Matrix–Exponetial Distributions (MVME) whenever their joint Laplace transform is a rational function. The majority of these distributions further belong to an important subclass of MVME distributions where the multivariate random vector can be interpreted as a number of simultaneously collected rewards during sojourns in a the states of a Markov chain with one absorbing state, the rest of the states being transient. We use this observation to unify the description of the majority of previously defined distributions with rational moment generating function. We also discuss the extension of these distributions to distributions for vectors that take values in a real space with rational moment generating function.

Loss rate for a general Lévy process with downward periodic barrier

Zbygniew Palmowski and P. Świątek (Wroclaw University)

In this paper we consider a general Lévy process X reflected at downward periodic barrier A_t and constant upper barrier K giving a process $V_t^K = X_t + L_t^A - L_t^K$. We find the expression for a loss rate defined by $l^K = \mathbb{E}L_1^K$ and identify its asymptotics as $K \to \infty$ when X has light-tailed jumps and $\mathbb{E}X_1 < 0$.

Duality Results for Markov Modulated Fluid Flow Models

V. Ramaswami (AT&T Labs-Research) and S. Ahn (University of Seoul)

We establish some interesting duality results for Markov modulated fluid flow models. Though fluid flow models are continuous state analogues of quasi birth and death processes, some duality results do differ by the inclusion of a scaling factor.

Laplace transforms of Lognormal distributions

Leonardo Rojas-Nandayapa (University of Queensland), Søren Asmussen and Jens Ledet Jensen (Aarhus)

The lognormal distribution is extremely popular in several areas such as insurance, finance and telecommunication. In particular, many applications require the Laplace transform of a lognormal random variable. We use a saddlepoint argument to obtain a sharp approximation for the Laplace transform. In our calculations we have used the Lambert function (also known as Omega function) which allows to make extensive simplifications to our results. We complement our work by showing some interesting applications where diverse methodologies using Laplace transforms are employed.

Asymptotics of conditional moments of the summand in Poisson compounds

Tomasz Rolski and A. Tomanek

Suppose that N is a $\mathbb{Z}_+$-valued random variable and $X, X_1, X_2, \ldots$ a sequence of i.i.d. $\mathbb{Z}_+$ random variables independent of N. In this paper we are interested in properties of the conditional variable $N_k =_{\mathrm{d}} \left(N \middle| \sum_{j=1}^N X_j = k\right)$. In particular we want to know the asymptotic behavior of the conditional mean EN_k or the conditional variance $Var(N_k)$ as $k \to \infty$. We consider the cases when X are Poisson or mixed Poisson. The problem is motivated by modelling loss reserves in non-life insurance.

Reuven Rubinstein (Technion)

TBA

Exact Simulation of the Stationary Distribution of M/G/c Queues

Karl Sigman (Columbia University)

We present two exact simulation algorithms for the stationary distribution of customer delay D for first-in-first-out (FIFO) multi-server queues in which the arrival process is Poisson at rate λ, and the service times $\{S_n\}$ are general iid (with finite mean $0 < E(S) = 1/\mu < \infty$); the M/G/c model. We assume that the service-time distribution $G(x) = P(S \leq x),\ x \geq 0$, and its corresponding equilibrium distribution $G_e(x) = \mu \int_0^x P(S > y)dy$ are such that samples of them can be simulated.

Our first algorithm is in continuous time and is only for the special case when $\rho = \lambda/\mu < 1$ (super stable case). This algorithm involves the general method of dominated coupling from the past (DCFTP) and we use the single-server queue operating under the *processor sharing (PS)* discipline as an upper bound. The algorithm is shown to have finite expected termination time if and only if service times have finite second moment.

Our second algorithm is for the general case of $\rho < c$. Here we use discrete-time processes and basic regenerative simulation, in which as regeneration points, we use return visits to state 0 of a corresponding random assignment (RA) model which serves as a sample-path upper bound.

Both algorithms yield, as output, a stationary copy of the entire Kiefer-Wolfowitz workload vector.

Ruin probabilities in a diffusion environment

Hanspeter Schmidli (Cologne) and Jan Grandell (KTH Stockholm)

We consider an insurance model, where the underlying point process is a Cox process. Using a martingale approach applied to diffusion processes, finite-time Lundberg inequalities are obtained. By change of measure techniques Cramér–Lundberg approximations are derived.

Decay rates for Quasi-Birth-Death Processes with Infinitely-Many Phases

Peter Taylor (University of Melbourne)

Over the last few years, there has been considerable interest in the calculation of decay rates for models that can be viewed as quasi-birth-and-death (QBD) processes with infinitely-many phases. The convergence of the decay rates of finite-phase truncations has also been studied.

In this talk, I shall first present some background, and then move on to discuss a contribution to this endeavour that involves some classes of models in which the transition function is not homogeneous in the phase direction. For these classes of models, I shall characterise the range of decay rates that are compatible with the dynamics of the process away from the boundary.

Our approach relies on the use of orthogonal polynomials to determine when certain equations have a positive solution.

What is Typical ?

Hermann Thorisson (Reykjavik) and Günther Last (Karlsruhe)

Let ξ be a random measure on a locally compact second countable topological group and let X be a random element in a measurable space on which the group acts. In the compact case, we give a natural definition of the concept that the origin is a typical location for X in the mass of ξ, and prove that when this holds the same is true on sets placed uniformly at random around the origin. This result motivates an extension of the concept of typicality to the locally compact case where it coincides with the concept of mass-stationarity. We describe recent developments in Palm theory where these ideas play a central role. This is joint work with Guenter Last.