Asymptotics of conditional moments of the summand in Poisson compound

Tomasz Rolski (joint work with Agata Tomanek)

Conference in Honour of Søren Asmussen

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- $X, X_1, X_2, \ldots$ a sequence of i.i.d. $\mathbb{Z}_+$ r.v.s independent of N.
- We are interested in

$$N_k =_{\mathrm{d}} \left(N \middle| \sum_{j=1}^{N} X_j = k \right).$$

In particular we want to know the conditional mean $\mathbb{E} N_k$ or the conditional variance $\mathbb{V}\mathrm{ar}\, N_k$ and their asymptotics for $k \to \infty$.
In this talk N is Poisson with mean a.

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Suppose X is Poisson with mean b. We will call this case as (Poi(a),Poi(b)). Compute

$$\mathbb{E}N_k = \frac{\sum_{m=0}^{\infty} m \frac{a^m}{m!} e^{-a} \frac{(mb)^k}{k!} e^{-bm}}{\sum_{m=0}^{\infty} \frac{a^m}{m!} e^{-a} \frac{(mb)^k}{k!} e^{-bm}} = \frac{B^c(k+1)}{B^c(k)},$$

where

$$B^c(k) = \sum_{m=1}^{\infty} m^k \frac{c^m}{m!} e^{-c}$$

is the k-th moment of the Poisson distribution and $c = ae^{-b}$.

More generally,

$$\mathbb{E}(N_k)^l = \frac{B^c(k+l)}{B^c(k)},$$

$$\mathbb{V}\mathrm{ar}\, N_k = \frac{B^c(k+1)}{B^c(k)} \left(\frac{B^c(k+2)}{B^c(k+1)} - \frac{B^c(k+1)}{B^c(k)} \right).$$

Therefore of particular interest is ratio $J^c(k) = B^c(k+1)/B^c(k)$.

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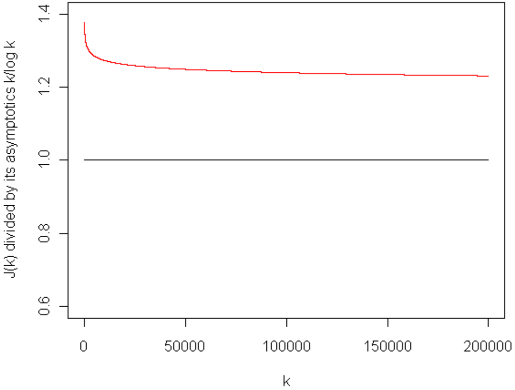
- For $c = 1$, the asymptotics of $J^1(k) = J(k)$ was earlier written in Harper (66), however with redundant e in the denominator.

Unfortunately, this asymptotics is extremely slow:



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- Dobinski (1877): $B(k)$ is equal to the k-th Bell number.
- De Bruijn (1981) gave

$$\frac{\log B(n)}{n} = \log n - \log\log n - 1 + o\left(\frac{\log\log n}{\log n}\right).$$

Lovász (93)(who quotes Moser and Wyman)

$$B(k) \sim k^{-1/2}[\Lambda(k)]^{k+1/2} e^{\Lambda(k)-k-1},$$

where $\Lambda(x)$ is the function defined by $\Lambda(x) \log \Lambda(x) = x$.

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where $\Lambda(x)$ is the function defined by $\Lambda(x)\log\Lambda(x) = x$. The function Λ is related to the Lambert W-function by $W(x) = x/\Lambda(x)$.

- From de Bruijn (1981)

$$W(x) = \log x - \log\log x + O\left(\frac{\log\log x}{\log x}\right),$$

and hence

$$\Lambda(x) \sim \frac{x}{\log x}\left(1 + \frac{\log\log x}{\log x} + O(\left(\frac{\log\log x}{\log x}\right)^2)\right).$$

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- We also refer to Pitman (97) for interesting connections between Bell numbers and Poisson distributions.

- Jessen *et al*(2010)

$$B^c(k) = (1 + o(1)) \sum_{m \in \left[\frac{k(1-\epsilon)}{\log k}, \frac{k(1+\epsilon)}{\log k}\right]} m^k e^{-c} \frac{c^m}{m!},$$

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- from which they concluded $J^c(k) = B^c(k+1)/B^c(k) \sim k/\log k$. We will use their ideas of proof for other cases.

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Natural estimator seems to be expected value conditioned on $N_0, \ldots, N_j$:

$$\hat{D}_{j+l} = \mathbb{E}\left[D_{j+l} \middle| D_0, \ldots, D_j\right] \qquad \text{dla} \qquad l = 0, 1, \ldots.$$

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See e.g. Mack (1993, 1994)

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$$p_j = \mathrm{P}(Y_{11} = j), \qquad j = 0, 1, \ldots;$$

- M, (K_m), (Y_{mk}) are independent.
- D_j–number of payments of claims from year 0 paid in year j:

$$D_j = \sum_{m=1}^{M} \sum_{k=1}^{K_m} 1_{\{Y_{mk}=j\}}, \qquad j = 0, 1, \ldots;$$

Jessen et al (2010)

- Denote by X_{mk} the value of the k-th payment in the m-th claim ((X_{mk}) are iid and independent of M, (K_m) and (Y_{mk})), then

$$S_j = \sum_{m=1}^{M} \sum_{k=1}^{K_m} X_{mk} 1_{\{Y_{mk}=j\}}, \qquad j = 0, 1, \ldots$$

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- Then

$$\hat{S}_{j+l} = \mathbb{E}(X_{11}) \hat{D}_{j+l}.$$

Theorem

Theorem (Jessen et al (2010))

If $EM < \infty$, then

$$\hat{D}_{j+l} = \mu \; p_{j+l} \; \mathbb{E}\left[M \middle| D_0 + \cdots + D_j = n_0 + \cdots + n_j \right].$$

Thus asymptotics is of interest:

$$R_{k,j} = \mathbb{E}\left[M \middle| D_0 + \cdots + D_j = k \right] \qquad \text{przy } k \to \infty.$$

Asymptotics - $M \sim Poi(\lambda)$

In this case

$$R_{k,j} = \frac{\mathrm{E}\left(\widetilde{M}\right)^{k+1}}{\mathrm{E}\left(\widetilde{M}\right)^{k}},$$

where $\widetilde{M}$ is Poisson with parameter

$$c = \lambda e^{-\theta_j}$$

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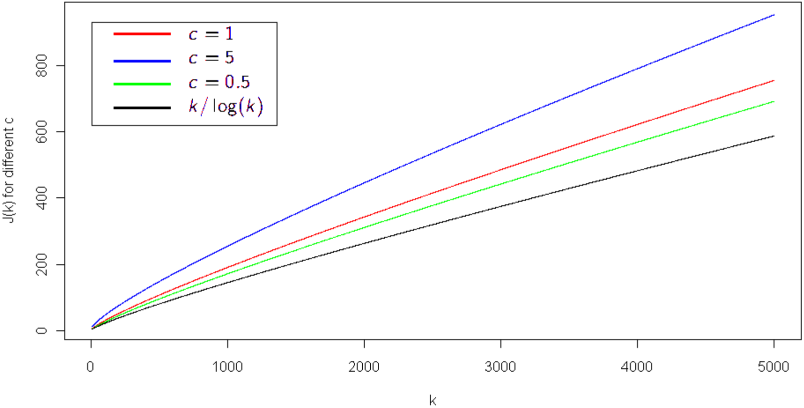
Lemma (Jessen eta al (2010)

$M \sim \text{Poisson}(\lambda)$. Then

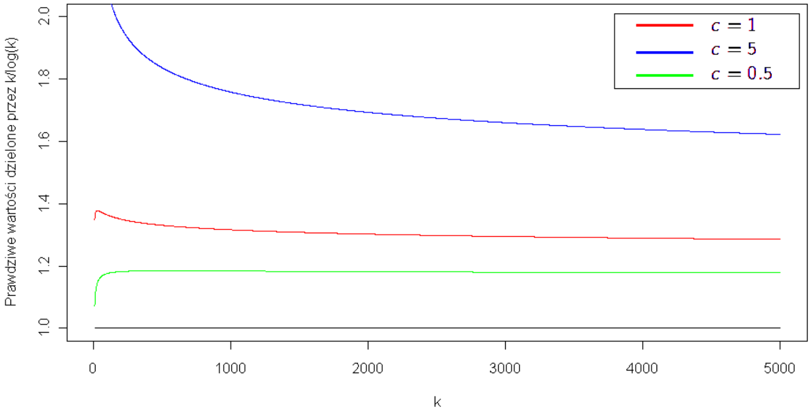
$$J^c(k) = \frac{\mathrm{E}(M)^{k+1}}{\mathrm{E}(M)^k} \sim \frac{k}{\log k},$$

Remarks

Comparison of $J^c(k)$ i $k/\log k$.


c = 1
c = 5
c = 0.5
k / log(k)
J(k) for different c
0
200
400
600
800
0
1000
2000
3000
4000
5000
k

Motivations: reserves in nonlife insurance



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- $(L_k(t), t \geq 0)$, $k = 1, 2, \ldots$ i.i.d. Levy processes.
- Poisson cluster of Levy processes:

$$S(t) = \sum_{k=1}^{N(1)} L_k(t - T_{(k)}), \qquad t \geq 1.$$

(Matsui and Mikosch (2010))

Probabilistically equivalent to:

$$S(t) = \sum_{k=1}^{N(1)} L_k(t - T_k),$$

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- $(L_k), N(t), (T_i)$ are independent.

Rolski and Tomanek (2011))

For simplicity assume

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$$S(t, t+s] = S(t+s) - S(t), \qquad t \geq 1, s > 0.$$

Proposed estimator:

$$\widehat{S}_k(t, t+s] = \mathbb{E}[S(t, t+s] | S(t) = k]$$

Lemma

dla $t \geq 1$

$$\widehat{S}_k(t, t+s] = bs\mathbb{E}(M(1)|\sum_{k=1}^{N(1)} L_k(t-T_k) = k)$$

$$= bs\mathbb{E}(N(1)|\sum_{j=1}^{N(1)} X_j = k),$$

gdzie $X_1, X_2, \ldots,$ are iid.

Question

How to compute!!!

$$\mathbb{E}N_k = \mathbb{E}(N(1)|\sum_{j=1}^{M(1)} X_j = k).$$

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- where $F \sim \mathrm{U}(t-1, t)$.

Conditioned moments

$$N_k =_{\mathrm{d}} \left(N \Big| \sum_{j=1}^{N} X_j = k \right),$$

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Then

$$\mathbb{E}N_k = \frac{B^c(k+1)}{B^c(k)},$$

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In general

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and

$$\begin{aligned} \mathbb{V}\mathrm{ar}\, N_k &= \frac{B^c(k+2)}{B^c(k)} - \left(\frac{B^c(k+1)}{B^c(k)}\right)^2 && (1) \\ &= \frac{B^c(k+1)}{B^c(k)} \left(\frac{B^c(k+2)}{B^c(k+1)} - \frac{B^c(k+1)}{B^c(k)}\right). && (2) \end{aligned}$$

Asymptotics

Proposition

For $c > 0$

$$\mathbb{E}N_k \sim \Lambda^c(k+1)$$

and

$$\mathbb{V}\text{ar}\, N_k \sim \frac{\Lambda^c(k+1)^2}{\Lambda^c(k+1)+k},$$

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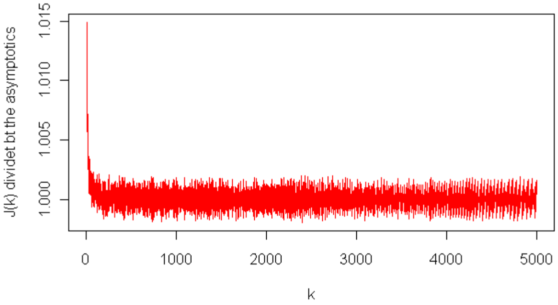
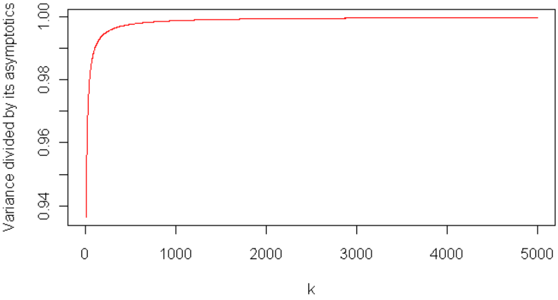
$$\Lambda^c(k)\log(\Lambda^c(n)/c) = n.$$

Asymptotics



Rysunek: Comparison of $J^c(k)$ and Λ^c.

Asymptotics



Rysunek: Comparison for variances.

General scheme

- Consider
$$G(k) = \sum_{m\geq 1} f_k(m), \qquad F(k) = \sum_{m\geq 1} m f_k(m),$$
and quotient $R(k) = F(k)/G(k)$.

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For

$$q_k(m) = \frac{f_k(m+1)}{f_k(m)}$$

$\lambda(k)$ is the solution of the so called λ-equation $q_k(\lambda) = 1$.

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$$l^* = l^*(k) = \lfloor (1-\epsilon)\lambda(k) \rfloor, \; r^* = r^*(k) = \lceil (1+\epsilon)\lambda(k) \rceil$$

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A.3.

$$\rho_k = \sup_{m \geq r^*} q_k(r^*), \qquad \rho_k^{'} = \sup_{m \leq r^*} (1/q_k(r^*))$$

i

$$\limsup_k \rho_k < 1, \qquad \limsup_k \rho_k^{'} < 1.$$

General scheme

Proposition

If A.1–A.3 hold, then $R(k) \sim \lambda(k)$ for $k \to \infty$.

Consider (Poi(a),Poi(b)).

$$f_k(l) = l^k \frac{c^l}{l!}, \qquad c = ae^{-b}.$$

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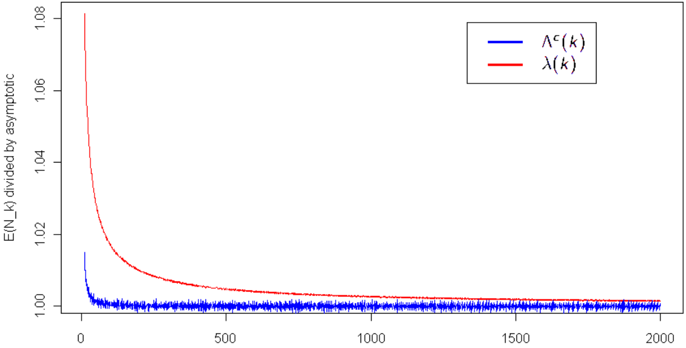
and hence λ-equation:

$$\frac{c}{\lambda+1}\left(\frac{\lambda+1}{\lambda}\right)^k = 1.$$

(Poi(a),Poi(b)) – case

Different results for (Poi(a),Poi(b)).



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If $\xi \sim F$, then mixPoi(F) is mixed Poisson with mixing distr. F.

Then $X \sim$mixPoi(F) i.e.

$$\mathbb{P}(X = k) = \mathbb{E}\left[\frac{\xi^k}{k!}e^{-\xi}\right].$$

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We have

$$f_k(m) = \frac{m^k}{m!} C_k(m).$$

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Denote $S_m = \xi_1 + \ldots + \xi_m$.

Specjalny przypadek; (Poi(a)),mixPoi(b))

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Then

$$\mathbb{E}(\xi_1 + \ldots + \xi_m)^k e^{-(\xi_1 + \ldots + \xi_m)} = \phi^m(1)\tilde{\mathbb{E}}(\xi_1 + \ldots + \xi_m)^k.$$

Special case – (Poi(a)),mixPoi(b))

For this case the λ-equation is:

$$\frac{c}{l+1}\left(\frac{l+1}{l}\right)^{k}\frac{\tilde{\mathbb{E}}\bar{S}_{l+1}^{k}}{\tilde{\mathbb{E}}\bar{S}_{l}^{k}}=1.$$

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We know that for $k \to \infty$:

$$\frac{l}{l+1}\frac{\tilde{\mathbb{E}}\bar{S}_{l+1}^k}{\tilde{\mathbb{E}}\bar{S}_l^k} \sim \tilde{f}(r)(1+o(1)).$$

Unfortunately this is not uniform with respect l.

Thus only conjecture:

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Conjecture

Suppose $0 < \tilde{f}(r-) < \infty$.

$$\mathbb{E}M_k \sim \lambda(k) \sim \Lambda^{c'}(k),$$

where $c' = c\tilde{f}(r-)$ and λ is the solution of λ-equation

$$\frac{c'}{l+1}\left(\frac{l+1}{l}\right)^{k+2} = 1.$$

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- $$C_k(m) = \tilde{\mathbb{E}}(\xi_1 + \ldots + \xi_m)^k = \frac{(m+k-1)!}{(b+1)^k(m-1)!}.$$

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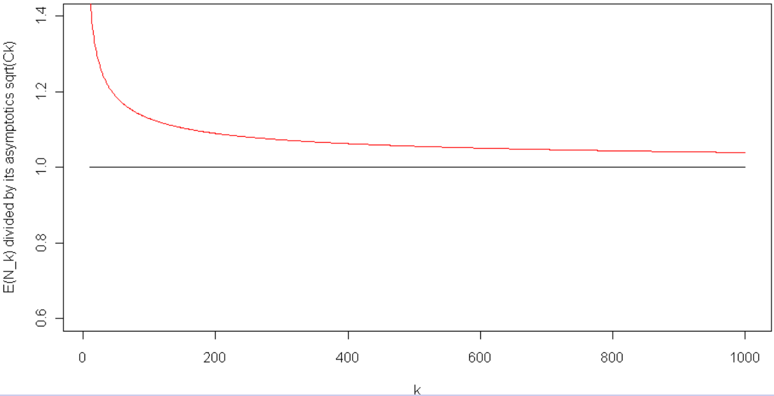
- Let ξ ~Exp(b)
- $$C_k(m) = \tilde{\mathbb{E}}(\xi_1 + \ldots + \xi_m)^k = \frac{(m+k-1)!}{(b+1)^k(m-1)!}.$$
- The solution of λ-equation

$$\lambda(k) \sim \sqrt{Ck} \qquad \text{as} \qquad k \to \infty,$$

where $C = ab/(b+1)$.

(Poi(a)),mixPoi(F)); exponential ξ.



(Poi(a)),mixPoi(F)); wykładnicze ξ.

Proposition

$$\mathbb{E}M_k \sim \lambda(k).$$

Saddlepoint approximations - see Asmussen and Albrecher (2010)

$$\mathbb{E}[N|A=k] = \frac{\mathbb{E}^{\theta}[N; \sum_{j=1}^{N} X_j = k]}{\mathbb{P}^{\theta}(\sum_{j=1}^{N} X_j = k)}$$

and θ is the solution

$$\mathbb{E}^{\theta} \sum_{j=1}^{N} X_j = k$$

Continuous-time models for claims reserving.

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Continuous-time models for claims reserving.

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- $S(t) = \sum_{j=1}^{N(1)} L_i(t - T_i)$, where $L_1, L_2, \ldots$ are iid copies.

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