Transport, mixing and agglomeration of particles in turbulent flows

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Symposium Particle Transport, University of Aarhus 6-7 Nov, 2014

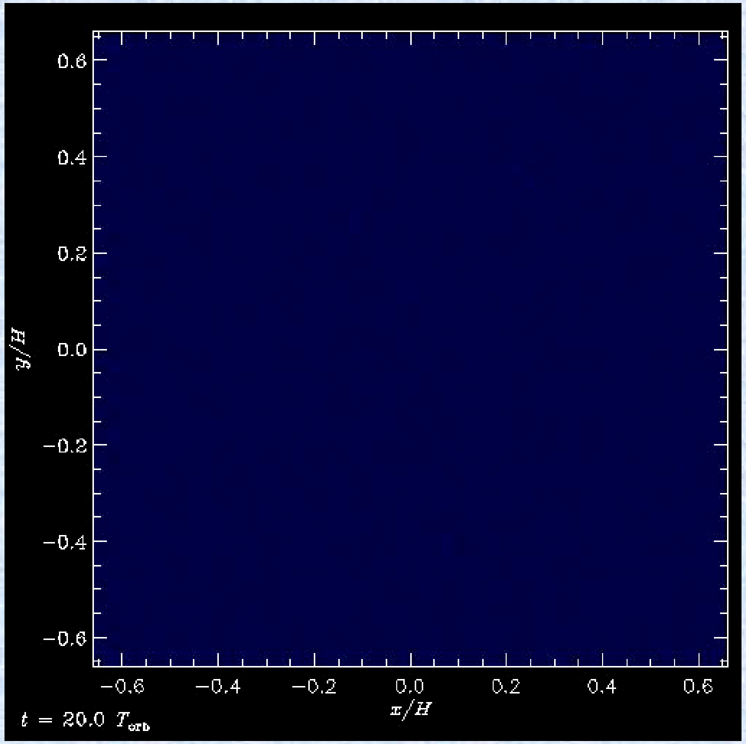
Gas - particle / droplet flows

Dilute flows

- Pollutant dispersion : PM10s
- Aerosol formation
  - Smoke - radioactive releases
- Weather - rain, mist and fog

Dense flows

- Volcanic eruptions
- Fluidized beds
- Mixing & combustion
- Pneumatic conveying
- Fouling / deposition
- Spraying
- Planet formation from inter-stellar dust



Outline

- Background
  - Scope of the physics in gas particle droplet flows
    - Role of inertia / Stokes number
- One-particle transport/dispersion
  - PDF approach
  - Dispersion in simple / complex flows
  - Transport in a turbulent boundary layer
    - Deposition and concentration
- Two-particle transport
  - Methods and approaches
    - Segregation
    - collisions

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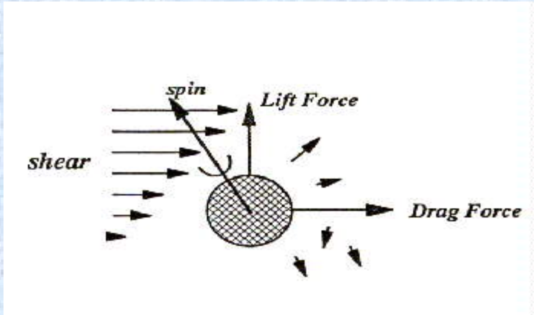
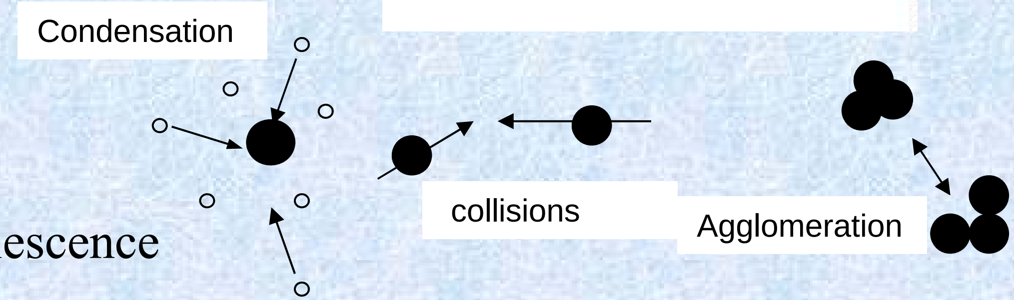
Gas-droplet / particle flows: scope of the physics

- Transport / dispersion
  - –Equation of motion
  - –Aerodynamic forces
  - –Stokes number $St=\tau_p/\tau_f$

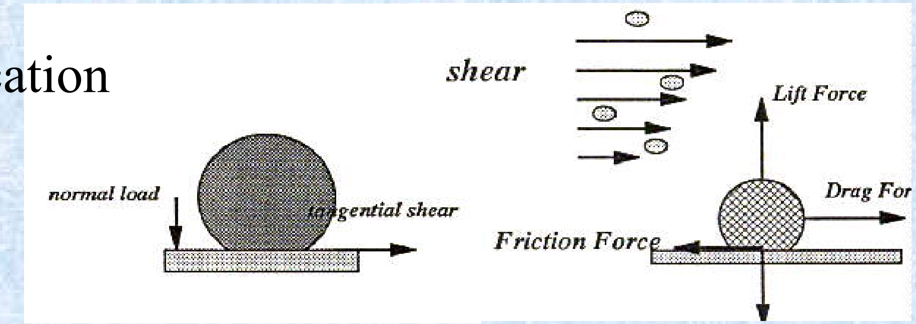


- Formation and growth
  - Condensation / evaporation
  - Collisions / agglomeration/coalescence

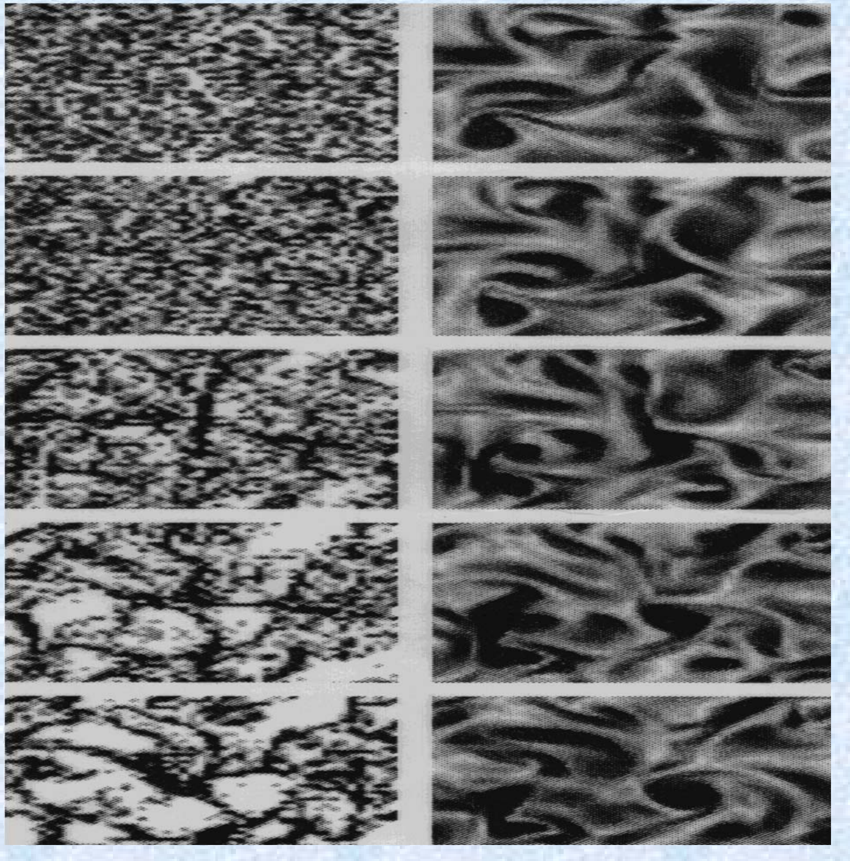


- Two way coupling
  - interfacial drag/turbulence modification

- Boundary conditions
  - Bounce / resuspension



Unmixing by turbulent flows



Wang & Maxey JFM 1993

Need for a PDF approach

- Two-fluid approach
  - Representing the dispersed phase as a fluid in same way as carrier flow
    - What are the continuum equations-constitutive relations?
      - Does it behave as a simple Newtonian fluid?
    - Boundary conditions (near wall behaviour)
- Particle segregation
  - Interaction with turbulent structures
  - Pair dispersion
  - Collision processes
- Need for a statistical approach c.f. Kinetic theory of gases

Slide 6

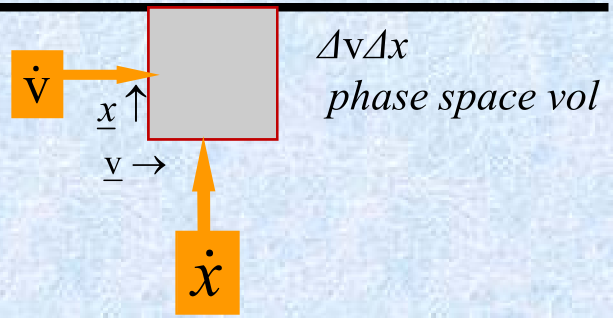
A5 Administrator; 24-06-2010

PDF Transport Equations

phase space vector $\underline{X} = (\underline{x}, \underline{\mathrm{v}}, m, T)$

phase space density $W(\underline{X}, t)$

mass conservation $\dfrac{\partial W}{\partial t} + \dfrac{\partial}{\partial X} \cdot (\underline{\dot{X}} W) = 0$



Eqns of Motion : $\underline{\dot{x}} = \underline{\mathrm{v}}$; $\underline{\dot{\mathrm{v}}} = \beta(\underline{u} - \underline{\mathrm{v}})$; $\dot{m} = f\left(\dfrac{\alpha_v}{\alpha_{sv}}\right) Sh(\mathrm{Re}_p, Sc)$; $\dot{T} = \dot{T}(\mathrm{Re}_p, \mathrm{Pr})$

$\underline{F}_a = \frac{1}{2} \rho_f C_D (\mathrm{Re}_p) |\underline{u} - \underline{\mathrm{v}}| (\underline{u} - \underline{\mathrm{v}})$,

$\beta^{-1} = \tau_p(|\underline{u} - \underline{\mathrm{v}}|, \mathrm{Re}_p)$ - particle relaxation time

$$P = \langle W \rangle ; \underline{\dot{X}} = \langle \underline{\dot{X}} \rangle + \underline{X}'$$

Averaged (PDF) Eqn; $\dfrac{\partial P}{\partial t} + \dfrac{\partial}{\partial \underline{\mathrm{X}}} \cdot (\langle \underline{\dot{\mathrm{X}}} \rangle P) = -\dfrac{\partial}{\partial \underline{\mathrm{X}}} \cdot (\langle \underline{\dot{\mathrm{X}}}' W \rangle) + \left(\dfrac{\partial P}{\partial t} \right)_{coll}$

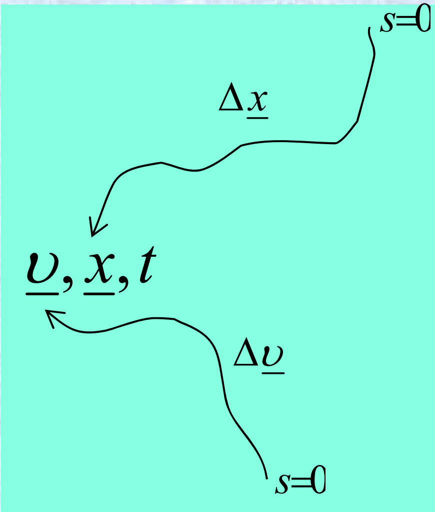
$d_p{}^2 \iint \Delta p_2(\mathrm{v}_1, x, x + d_p \hat{k}, t) \mathrm{v}_{21} \cdot \hat{k} d\hat{k} d\mathrm{v}_2$

Simonin Zaichik

Gas-particle flow $\dfrac{\partial P}{\partial t} + \dfrac{\partial}{\partial \underline{x}} \cdot \underline{\mathrm{v}} P + \dfrac{\partial}{\partial \underline{\mathrm{v}}} \cdot \beta(\langle \underline{u} \rangle - \underline{\mathrm{v}}) P = -\dfrac{\partial}{\partial \underline{\mathrm{v}}} \cdot \boxed{\beta \langle \underline{u}' W \rangle}$ net force due to turbulence

Closure approximations for $\langle \beta \underline{u}' W \rangle$

$$\left\langle \beta \underline{u}'(\underline{x},t) W(\underline{\mathrm{v}},\underline{x},t) \right\rangle = -\left(\frac{\partial}{\partial \underline{\mathrm{v}}} \cdot \underline{\mu} + \frac{\partial}{\partial \underline{x}} \cdot \underline{\lambda} + \underline{\kappa} \right) \langle W \rangle$$



$$\underline{\mu} = \overbrace{\beta \left\langle \Delta \underline{\upsilon}(\underline{\upsilon},\underline{x},t \,|\, 0)\; \underline{u}'\langle \underline{x},t \rangle \right\rangle}^{\text{diffusion in velocity}} \qquad \underline{\lambda} = \overbrace{\beta \left\langle \Delta \underline{x}(\underline{\upsilon},\underline{x},t \,|\, 0)\; \underline{u}\langle \underline{x},t \rangle \right\rangle}^{\text{diffusion in x-space}}$$

$$\underline{\kappa} = \underbrace{-\left\langle \Delta \underline{x}(\underline{\upsilon},\underline{x},t|0) \cdot \frac{\partial}{\partial \underline{x}} \underline{u}'(\underline{x},t) \right\rangle}_{\text{body force}}$$

LHDI Approximation

Reeks,

$$\begin{pmatrix} \mu \\ \lambda \end{pmatrix} = \beta \int_0^t \begin{pmatrix} \dot{g}(t-s) \\ g(t-s) \end{pmatrix} \left\langle \left\langle \underline{u}'(\underline{x}_p(s),s) \underline{u}'(\underline{x},t) \right\rangle_{\underline{x}_p} \right\rangle ds$$

Furutsu Novikov

Zaichek, Swailes, Minier

Moments of PDF Eqns

spatial density $\langle\rho\rangle = \int d\,\underline{\mathrm{v}}\,P(\underline{\mathrm{v}},\underline{x},t)$; mean velocity $\underline{\overline{\mathrm{v}}} = \langle\rho\rangle^{-1}\int d\,\underline{\mathrm{v}}\,P(\underline{\mathrm{v}},\underline{x},t)\;\underline{\mathrm{v}}$;

Velocity fluctuations $\underline{\mathrm{v}}' = \underline{\mathrm{v}} - \underline{\overline{\mathrm{v}}}$ $\quad \overline{\mathrm{v}_i'^{\,l}\,\mathrm{v}_m'^{\,m}\,\mathrm{v}_n'^{\,n}} = \langle\rho\rangle^{-1}\int d\,\underline{\mathrm{v}}\,P(\underline{\mathrm{v}},\underline{x},t)\;\mathrm{v}_i'^{\,l}\,\mathrm{v}_m'^{\,m}\,\mathrm{v}_n'^{\,n}$

mass $\quad \dfrac{\partial\langle\rho\rangle}{\partial t} + \dfrac{\partial}{\partial \underline{x}}\cdot\langle\rho\rangle\underline{\overline{\mathrm{v}}} = 0$

momentum $\quad \langle\rho\rangle\dfrac{D}{Dt}\overline{\mathrm{v}}_i = -\dfrac{\partial}{\partial x_j}\langle\rho\rangle\overline{\mathrm{v}_i'\mathrm{v}_j'} + \beta\big(\langle u_i\rangle - \overline{\mathrm{v}}_i\big)\langle\rho\rangle + \beta\langle\rho\rangle\overline{u_i'}$

Reynolds /kinetic stresses $\qquad$ Net force due turbulence

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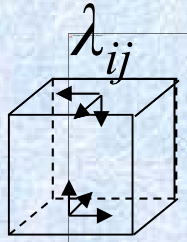
Momentum Eqn from PDF Eqn.



pressure tensor

$$\beta\langle\rho\rangle\overline{u_i'} = -\frac{\partial}{\partial\underline{x}}\cdot\overline{\underline{\lambda}}\langle\rho\rangle + \overline{\underline{\kappa}}\langle\rho\rangle$$

net turbulent driving force

body force

$$\langle\rho\rangle\frac{D\overline{\upsilon_i}}{Dt} = -\frac{\partial}{\partial\underline{x}}\cdot\underline{p} + \beta\left(\langle u_i\rangle - \overline{\upsilon_i}\right)\langle\rho\rangle + \overline{\kappa}_i\langle\rho\rangle$$

total pressure *drag force*

Eqn of state $\dfrac{p_{ij}}{\langle\rho\rangle} = \overline{\upsilon_i'\upsilon_j'} + \lambda_{ij}$

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Momentum equation as a diffusion equation

$$\langle\rho\rangle\overline{\underline{\upsilon}} = \langle\rho\rangle\underline{\upsilon}_C - \underline{\varepsilon}\cdot\frac{\partial\langle\rho\rangle}{\partial\underline{x}} - \beta^{-1}\langle\rho\rangle\frac{D\overline{\underline{\upsilon}}}{Dt}$$

Momentum eqn.

$s=0$

$Y(\underline{x},t|s)$

$$-\int_0^t ds\left\langle \underline{u}(\underline{x},t)\left[\nabla\cdot\underline{\upsilon}_p(\underline{y},s)\right]_{y=Y(\underline{x},t|s)}\right\rangle$$

$\underline{x},t$

$$\underline{\upsilon}_C = \langle\underline{u}\rangle + \underline{\upsilon}_d \qquad \underline{\upsilon}_d = -\beta^{-1}\frac{\partial}{\partial\underline{x}}\cdot\overline{\underline{\upsilon}'\underline{\upsilon}'} - \beta^{-1}\underline{\kappa}$$

Drift due spatial inhomogeneity

turbophoresis

Homogeneous stationary turbulence

$$\underline{\varepsilon} = \beta^{-1}\left(\overline{\underline{\upsilon}'\underline{\upsilon}'} + \underline{\lambda}\right) = \int_0^\infty \underbrace{\left\langle \underline{u}_p(0)\underline{u}_p(t)\right\rangle}_{\text{Lagrangian}} dt$$

Long time particle diffusion coefficient

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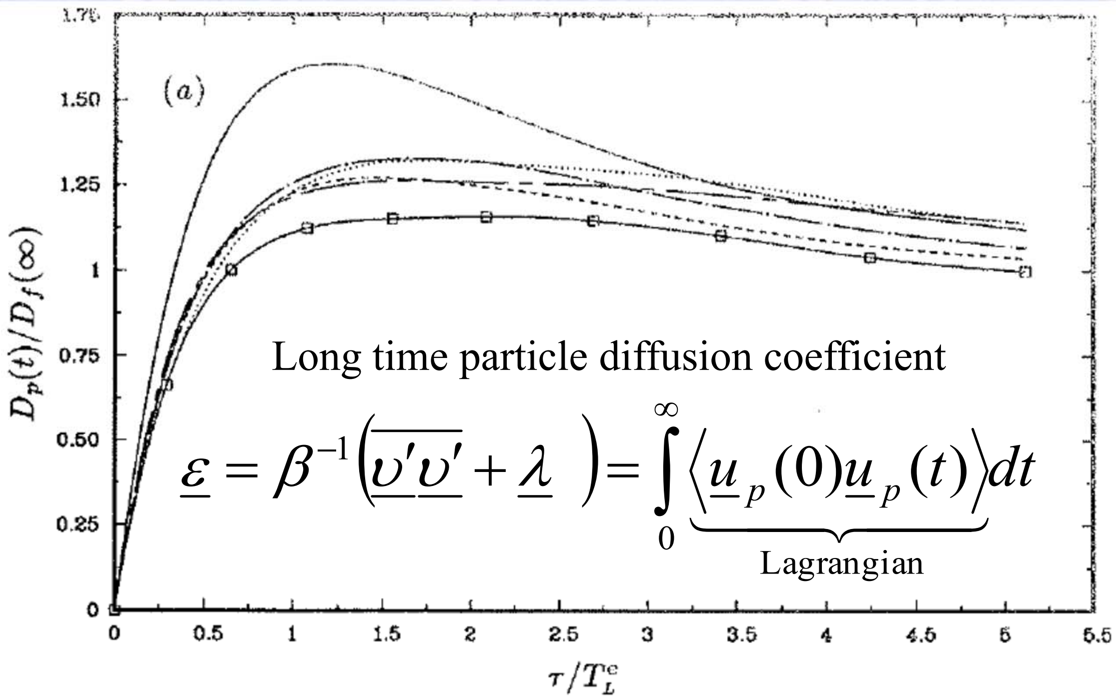
Diffusion coefficient versus particle inertia

Homogeneous stationary turbulence



Long time particle diffusion coefficient

$$\underline{\varepsilon} = \beta^{-1}\left(\overline{\underline{\upsilon}'\underline{\upsilon}'} + \underline{\lambda}\right) = \int_0^\infty \underbrace{\left\langle \underline{u}_p(0)\underline{u}_p(t)\right\rangle}_{\text{Lagrangian}} dt$$

FIGURE 4.6A. Effect of inertia on particle eddy diffusivity, $Re_\lambda = 30.8$. ⊟——⊟, fluid (Case DI1); ----, $\tau_p/T_f = 0.05$; —·—, $\tau_p/T_f = 0.11$; ········, $\tau_p/T_f = 0.33$; —--—, $\tau_p/T_f = 0.65$; ——, $\tau_p/T_f = 1.09$.

Particle kinetic stress transport equation

Kinetic stress flux

$$\langle\rho\rangle\frac{D}{Dt}\overline{\upsilon'_m\upsilon'_n}=-\frac{\partial}{\partial x_i}\langle\rho\upsilon'_i\upsilon'_m\upsilon'_n\rangle$$

Work done by total stresses

$$-\left(p_{ni}\frac{\partial\overline{\upsilon}_m}{\partial x_i}+p_{mi}\frac{\partial\overline{\upsilon}_n}{\partial x_i}\right)$$

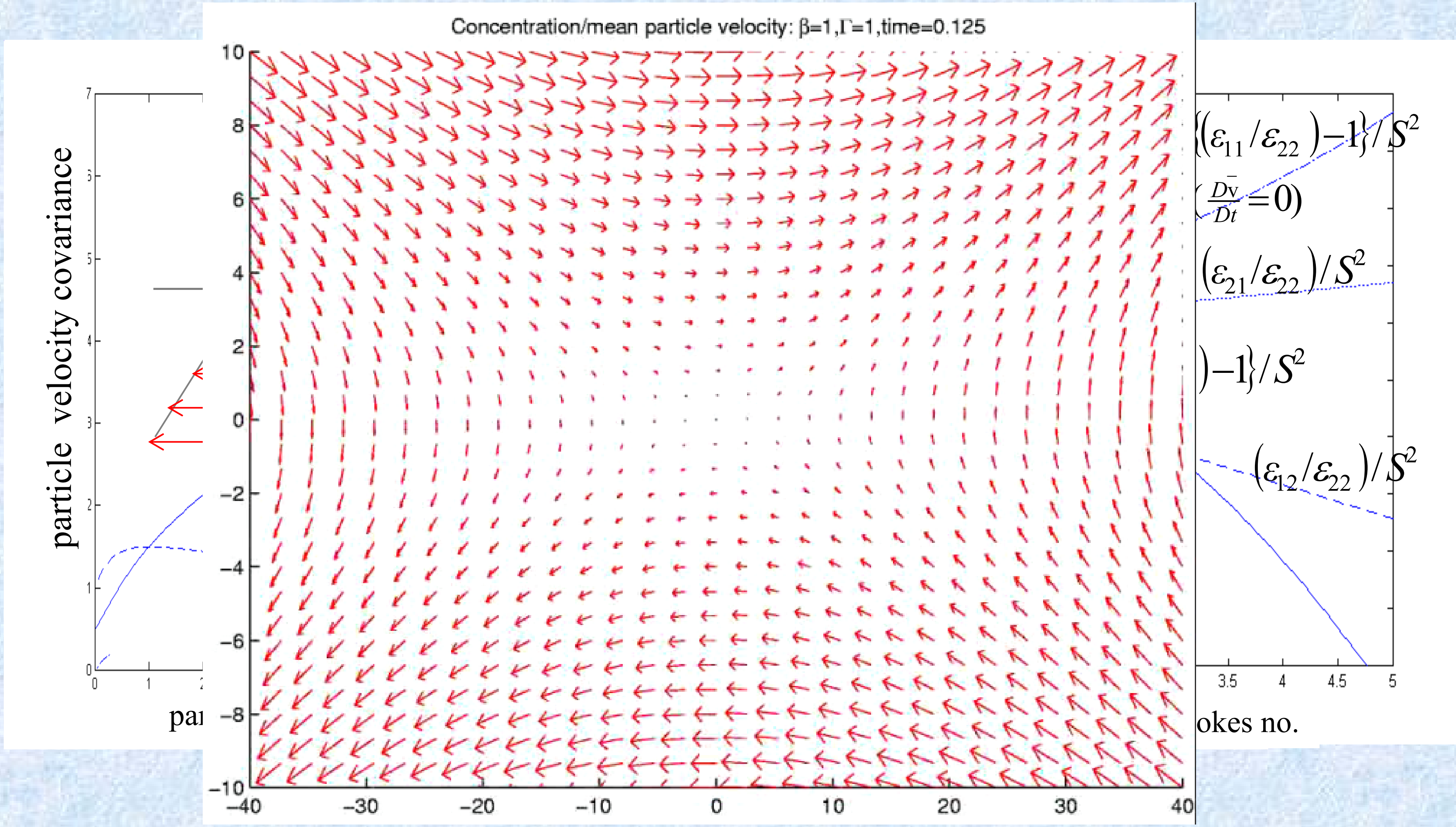
$$p_{ni} = \overbrace{\langle\rho\rangle\overline{\upsilon'_n\upsilon'_i}}^{\text{kinetic stresses}}+\underbrace{\langle\rho\rangle\overline{\lambda}_{ni}}_{\text{stresses from turbulent forces}}$$

$$-2\beta\langle\rho\rangle\left(\overline{\upsilon'_m\upsilon'_n}-\mu^S_{mn}\right)$$

$$\mu^S_{mn} = \tfrac{1}{2}\left[\overline{u'_m\mathrm{v}'_n}+\overline{\mathrm{v}'_m u'_n}\right]$$

Particle-fluid velocity covariances

Dispersion in a simple shear



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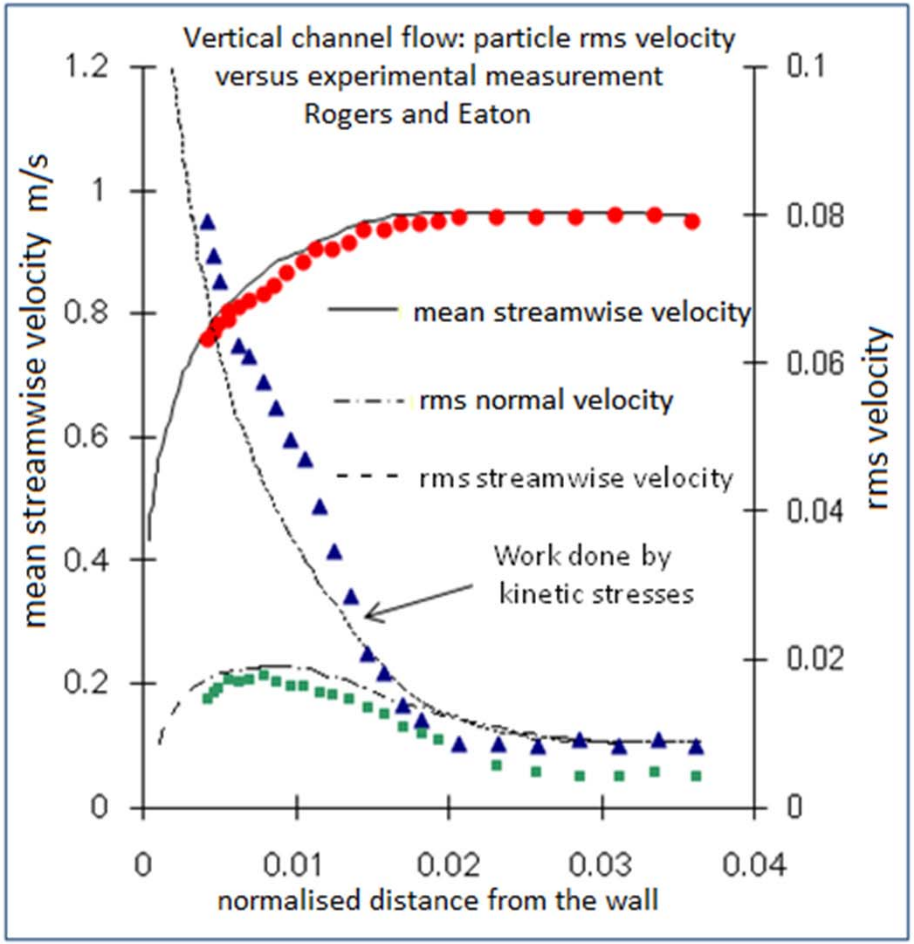
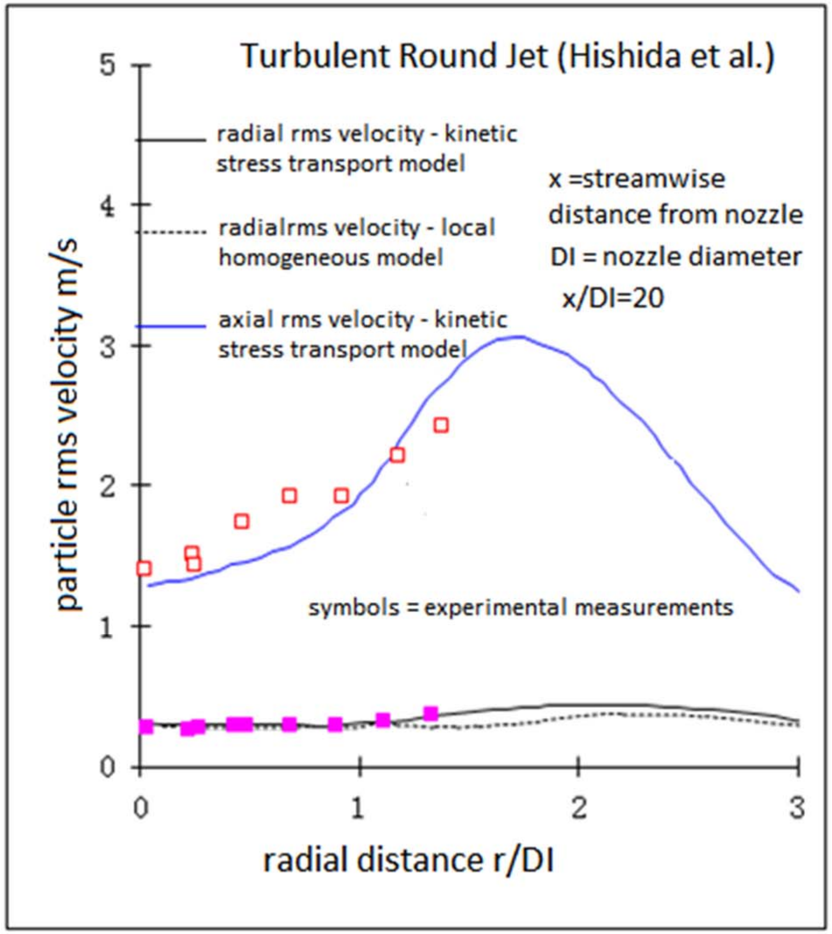
Predictions versus experimental measurements

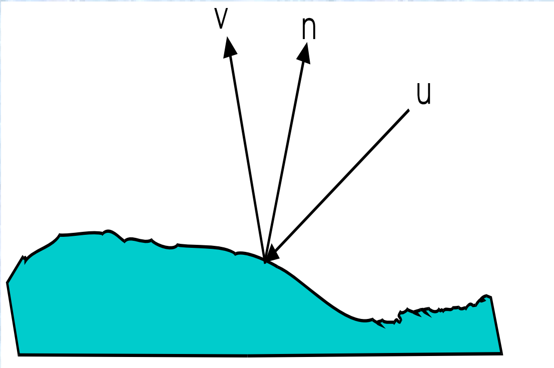
Simonin et al.





Near-wall behaviour

- influence of particle wall interactions
  - scattering & absorption/deposition/ bounce /resuspension



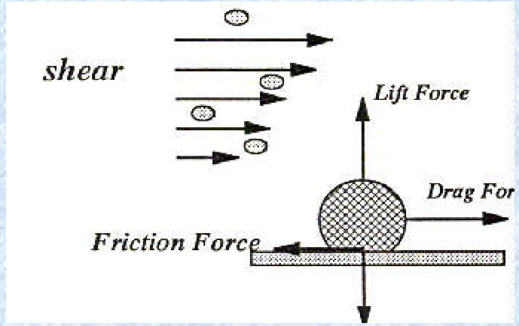
boundary conditions

$$\underline{v}\,p(\vartheta,\underline{x},t) = \int_{\underline{u}.\underline{n}\le 0} \underline{u}\,P(\underline{u},\underline{x},t)\,\Theta(\underline{v}|\underline{u})\,d\underline{u}$$

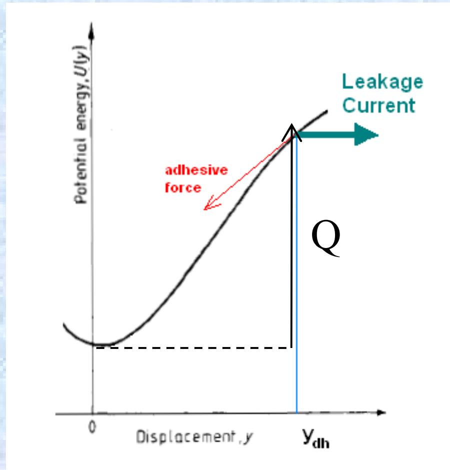
$$\Theta(\underline{v}|\underline{u}) = \textit{distribution of } \underline{v} \textit{ given } \underline{u}$$

$$p = \omega \exp - Q/2\langle PE \rangle$$

- *p, the resuspension rate constant*
- *ω , the typical frequency of vibration,*
- *Q height of adhesive potential well,*
- *<PE> average potential energy of particle in the well.*

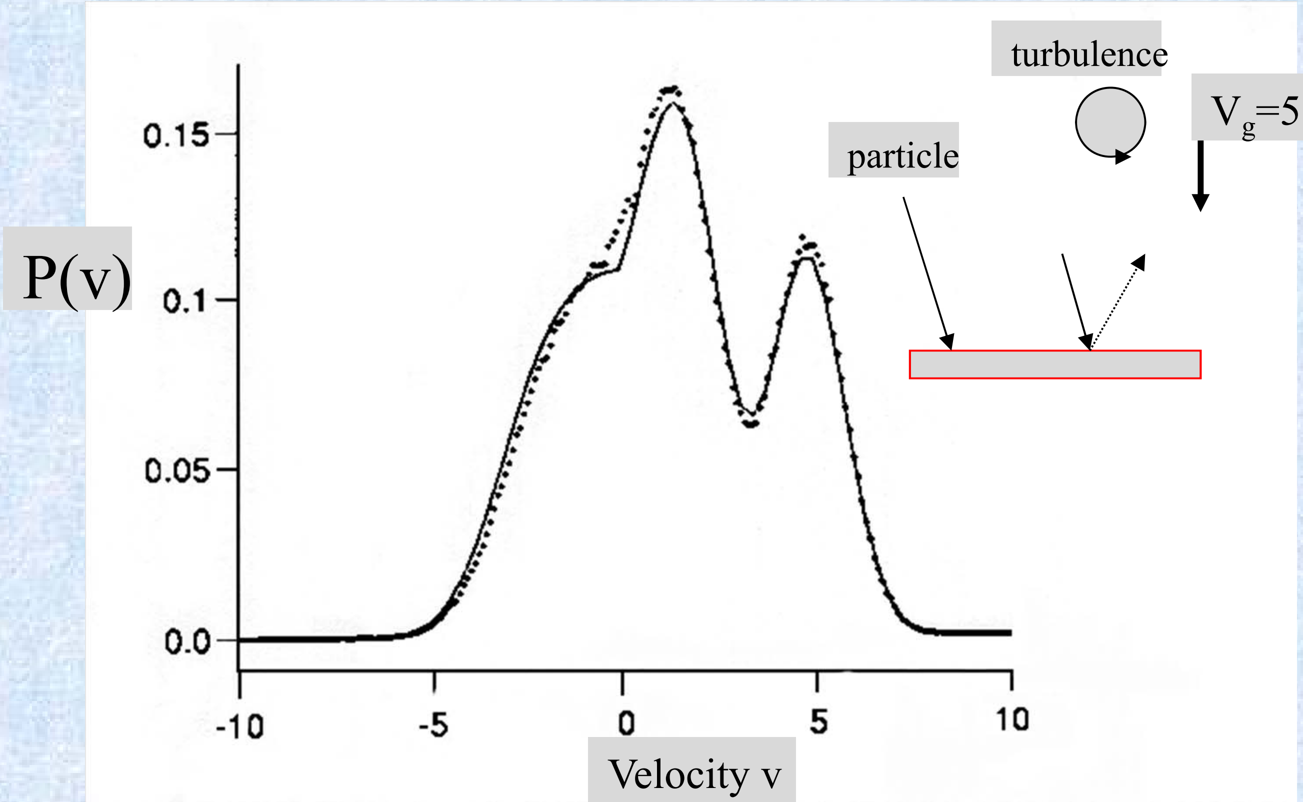


particle resuspension



Particle escape from potential well

Particle wall impacts with absorption



Critical impact velocity v_c=5, settling velocity v_g=5
(normalized on particle rms velocity for perfect reflection)

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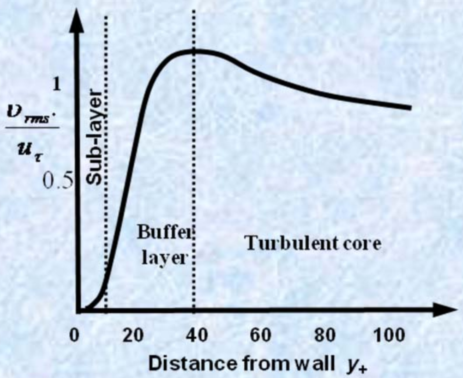
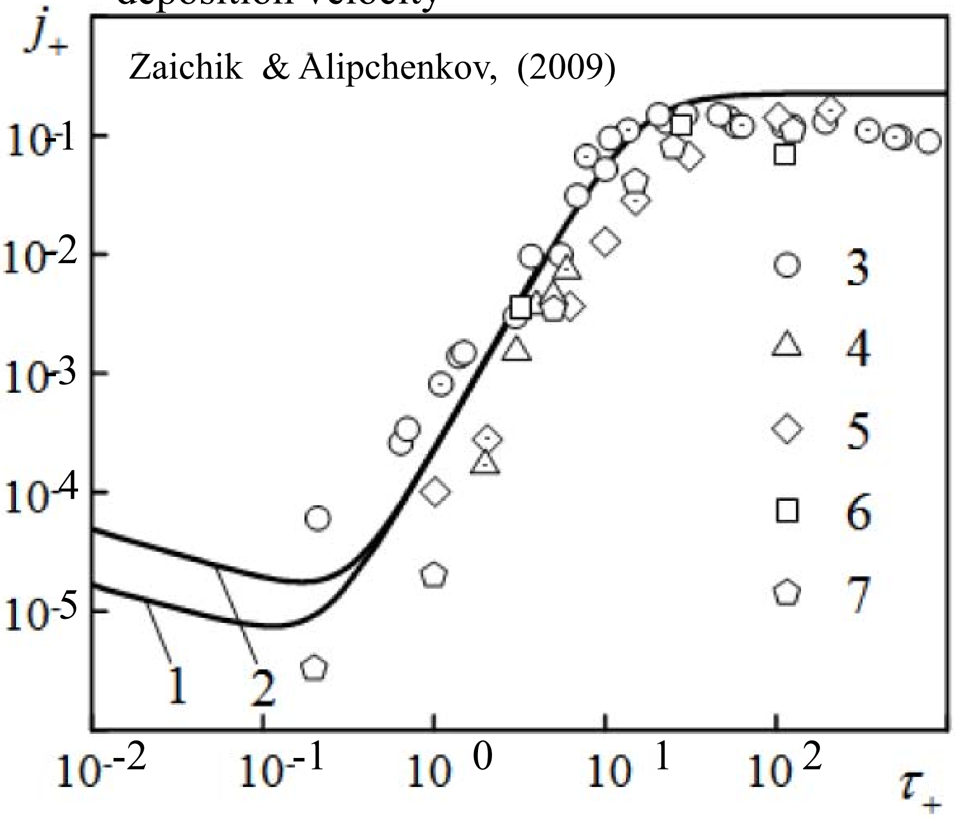
Deposition in turbulent boundary layer



- *rapid decay of turbulence near the wall*
- *particle not in local equilibrium with flow*
- *break down of gradient transport*
- *comparison of PDF results with experimental results*

deposition velocity



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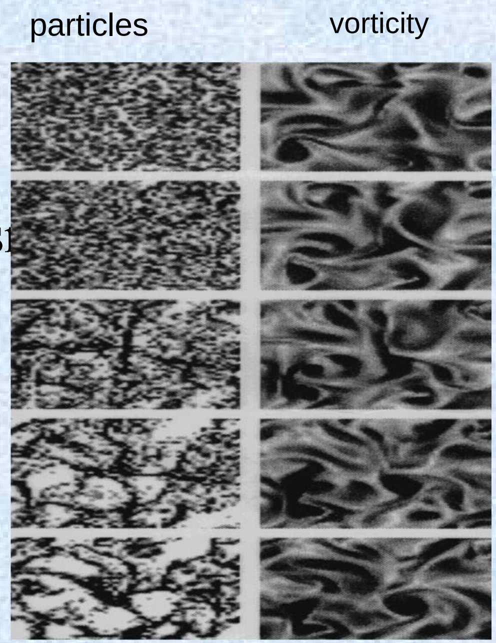
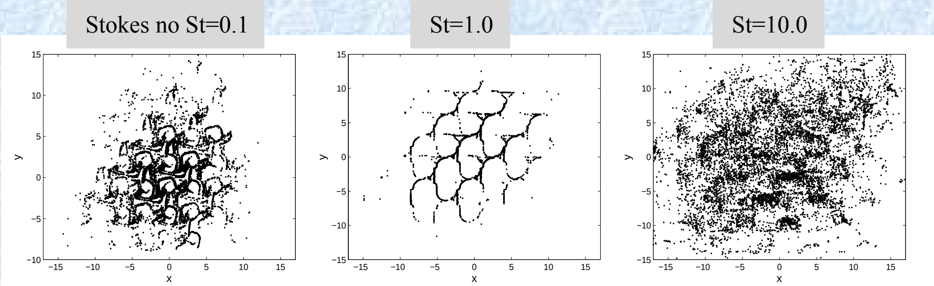
Two-particle dispersion

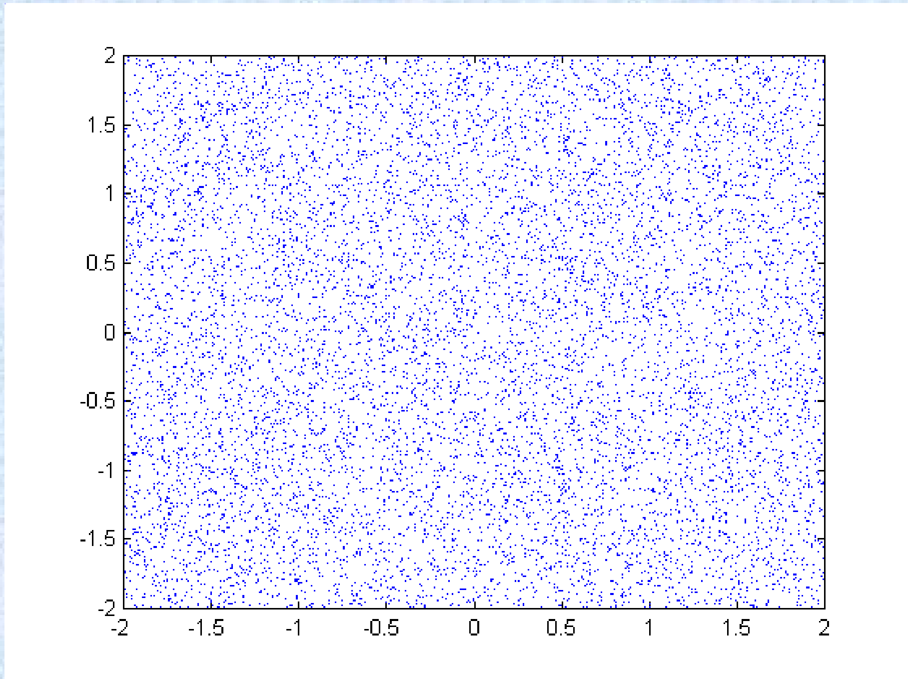
- Segregation/clustering
  - Full Lagrangian approach
    - Compressibility of a particle flow
    - Singularities in particle concentraiosn
- Random uncorrelated motion
- 2-particle PDF approach
  - Radial distribution
  - Collision kernel

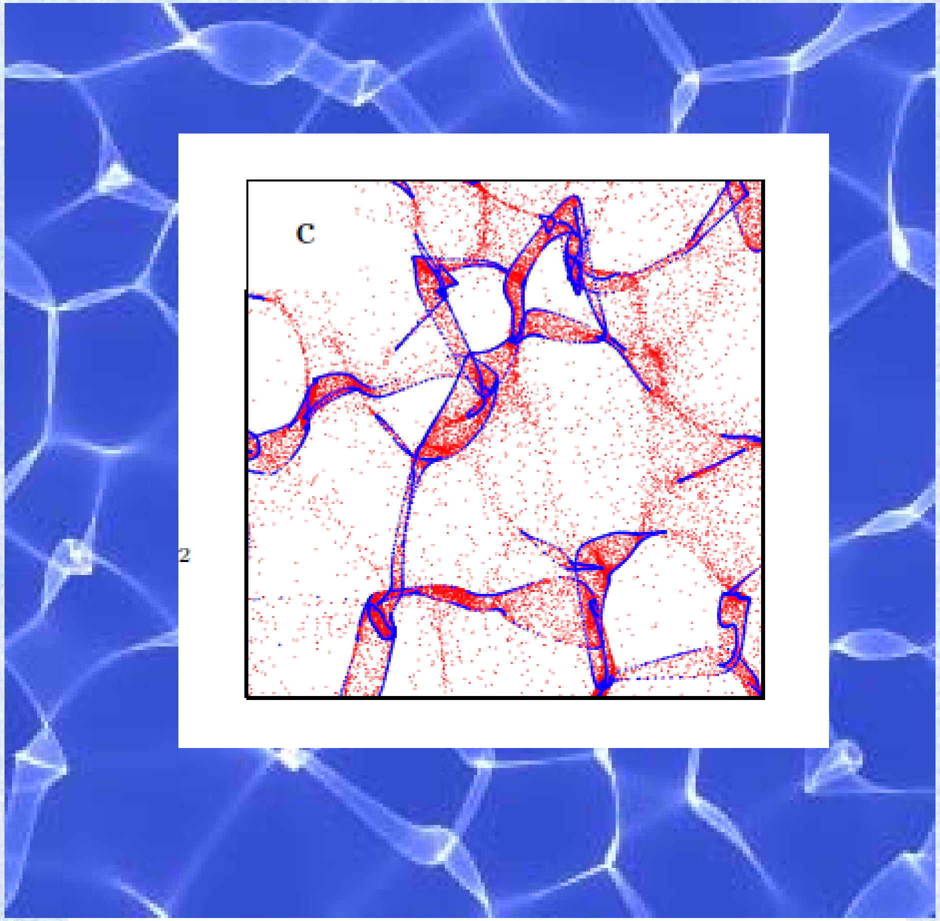


Wang & Maxey JFM 1993

Unmixing by turbulent flows

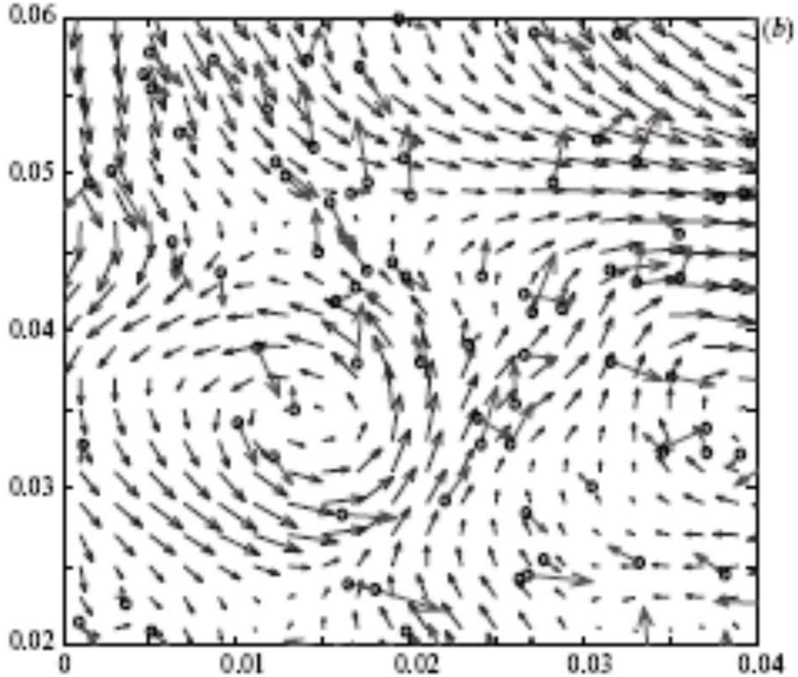


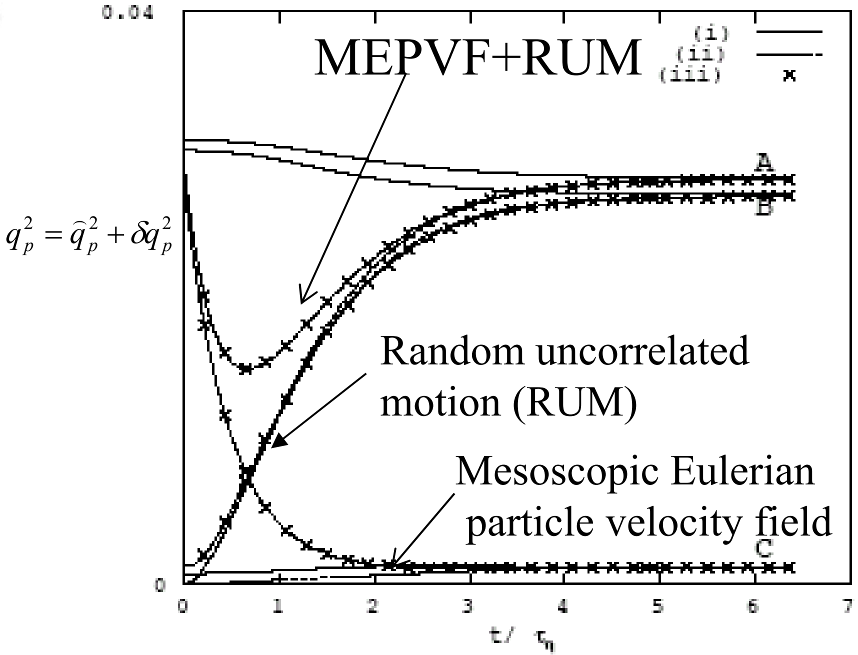




Caustics (Wilkinson & Mehlig 2007)

Random uncorrelated motion (RUM)





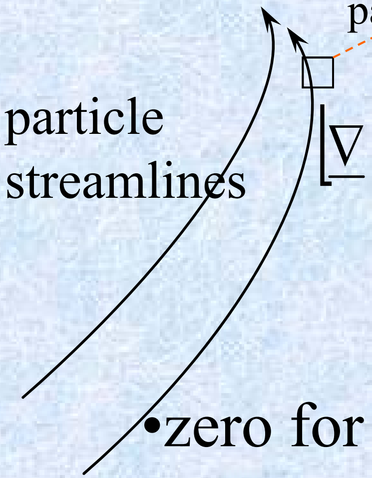
Février et. al JFM, 2005

$$\mathbf{v}_p^{(m)}(t) = \tilde{\mathbf{v}}_p(\mathbf{x}_p^{(m)}(t), t, H_f) + \delta\mathbf{v}_p^{(m)}(t).$$

Compressibility of a particle flow

Falkovich, Elperin,Wilkinson, Reeks

Compressibility (rate of compression of elemental particle volume along particle trajectory)



$$\left\lfloor \underline{\nabla} \cdot \underline{\upsilon}_p (\underline{y}, s) \right\rfloor_{y=Y(\underline{x},t|s)}$$

Divergence of the particle velocity field along a particle trajectory

- zero for particles which follow an incompressible flow
- non zero for particles with inertia
- measures the change in particle concentration

Measurement of the compressibility

Deformation of elemental volume $\rightarrow$ $J_{ij} = \dfrac{\partial x_{p,i}(\underline{x}_0, t)}{\partial x_{0,j}}; \; J = \left|\det J_{ij}\right|$

$\delta V(t)/\delta V(0)$ – volume fraction of elemental volume of particles along a particle trajectory

compessibility $\nabla \cdot \underline{\mathrm{v}}_p\left(\underline{x}_0, t\right) = J^{-1}\dfrac{dJ}{dt} = \dfrac{d}{dt}\ln J$ compression

can be obtained directly from solution of particle eqns. of motion

- $\underline{x}_p(t), \underline{\mathrm{v}}_p(t), J_{ij}(t), J(t))$ - Full Lagrangian Method (FLM)

- Avoids calculating the compressibility via the particle velocity field
- Can determine the statistics of ln J(t) easily.

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Particle trajectories in a periodic array of vortices

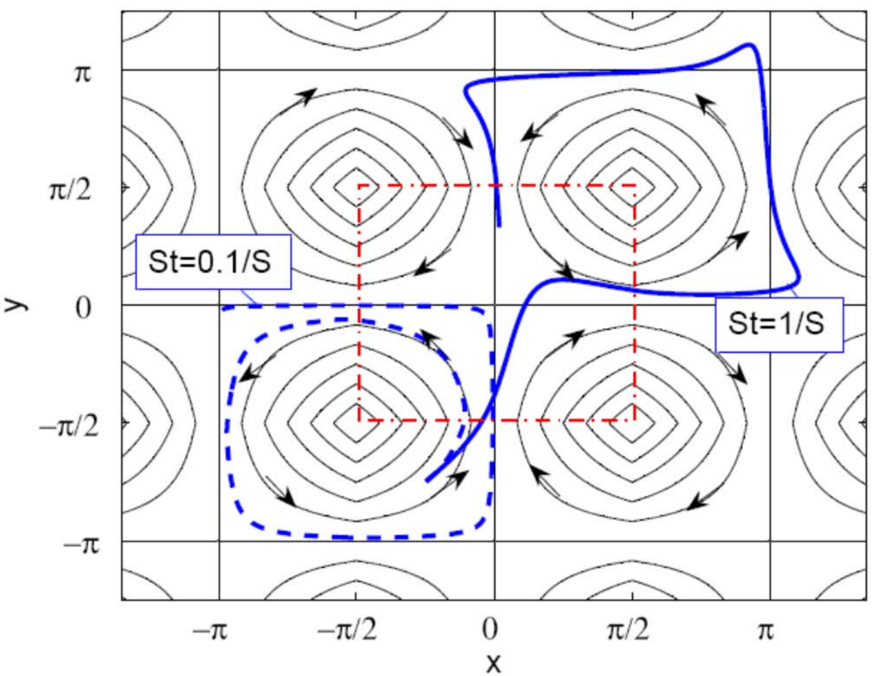

FIGURE 1. Particle trajectories in a frozen field of periodic vortices, for $St = 0.1/S$ (dashed blue line; heavily damped case) and $St = 1/S$ (solid blue line; lightly damped case), where S represents the strain rate in the flow. The two particles are released in $(x, y) = (-\pi/4, -3\pi/4)$ with a velocity equal to the local carrier flow velocity at time $t = 0$, and traced for a time $t = 20$. The highlighted area (red dash-dotted line) designates the basic element out of which the entire flow field is constructed.

Deformation Tensor J

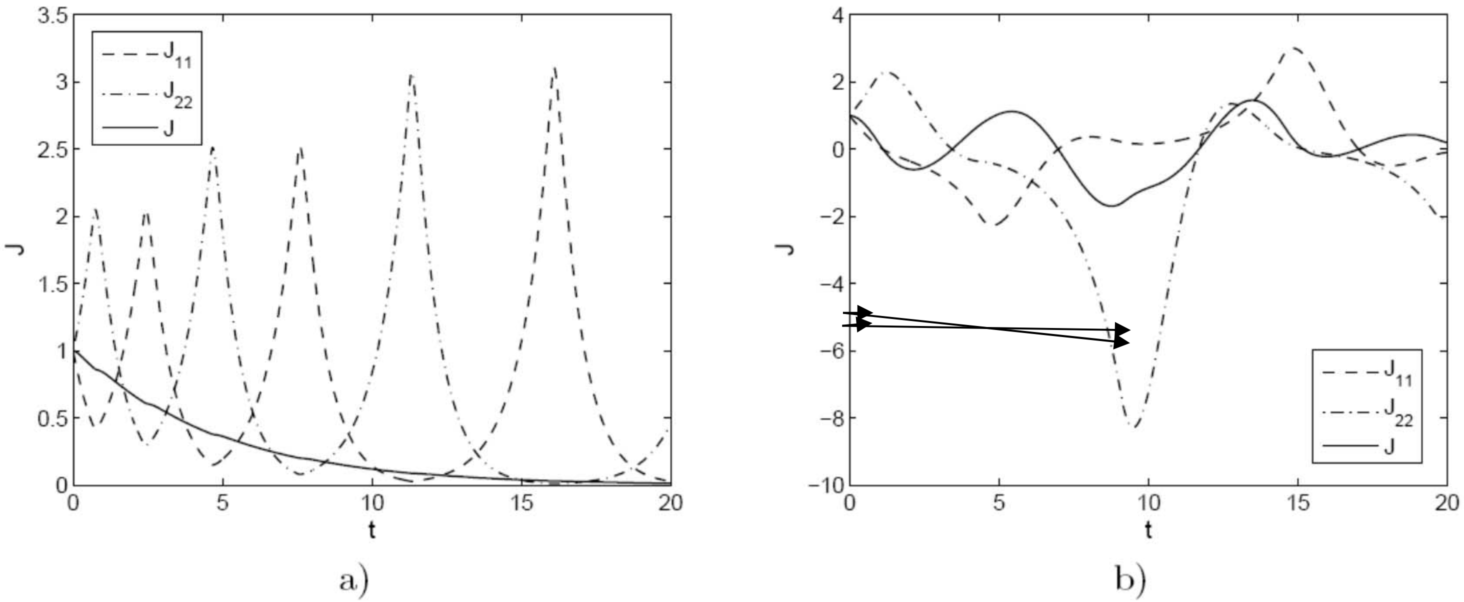

FIGURE 2. Values of the deformation J and the components of the deformation matrix J_{11} and J_{22} in the frozen field of periodic vortices depicted in Fig. 1, for (a) the heavily damped case $St = 0.1/S$, and (b) the lightly damped case $St = 1/S$. The values of J_{11}, J_{22} and J are calculated along the same two particle trajectories as plotted in Fig. 1. The strain rate is taken as $S = \sqrt{\overline{S^2}} = \sqrt{12}/\pi$.

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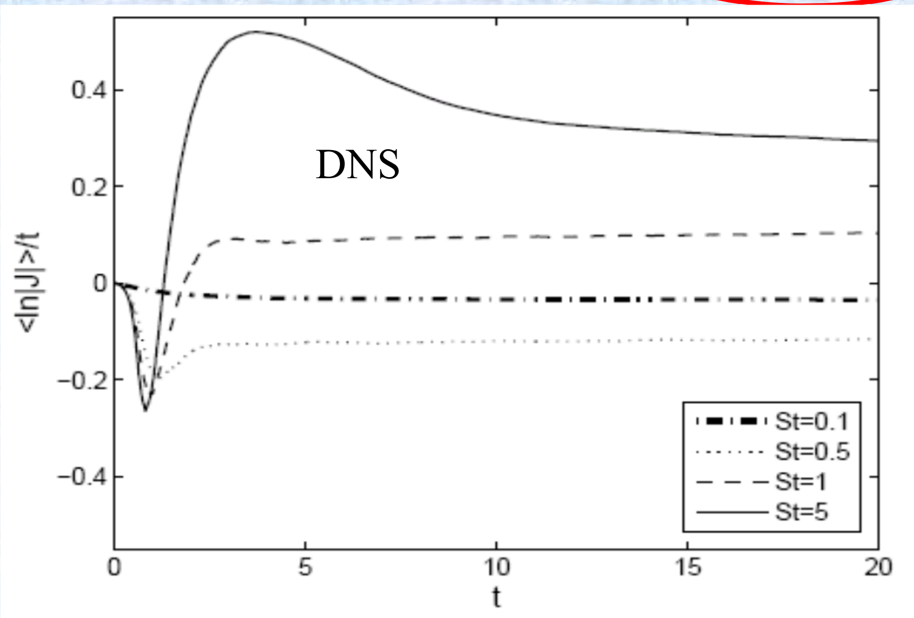
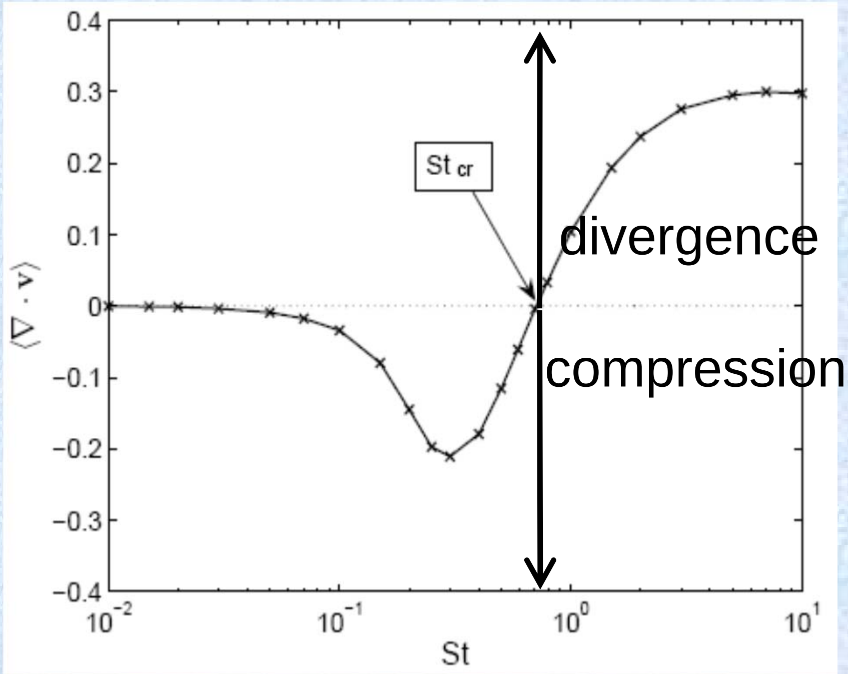
Particle average compressibility

Compressibility of the pvf: $$\frac{d}{dt} < \ln|J| > = < \nabla \cdot \mathbf{v} >$$





For a given flow field, there is a threshold St below which the segregation goes on indefinitely with time, and above which the dilation prevails over segregation.

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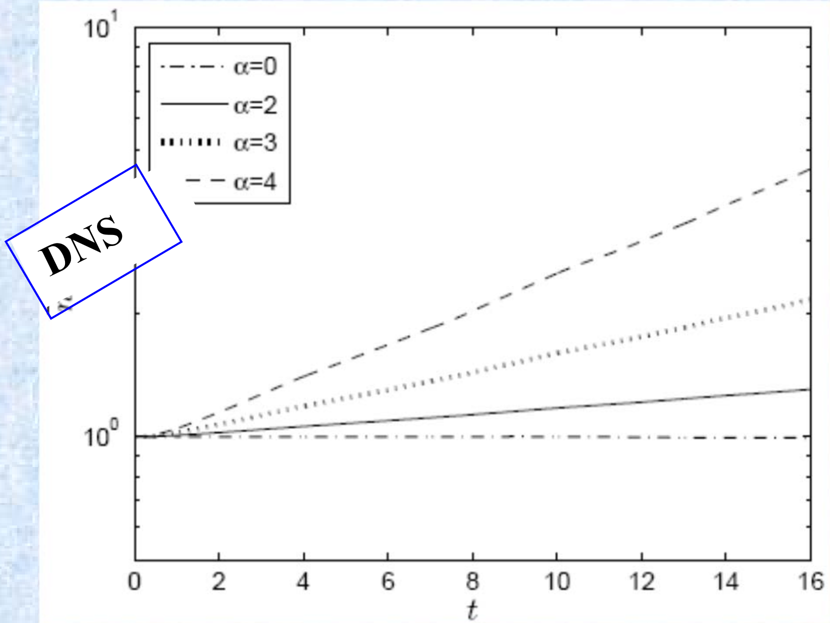
Moments of particle number density

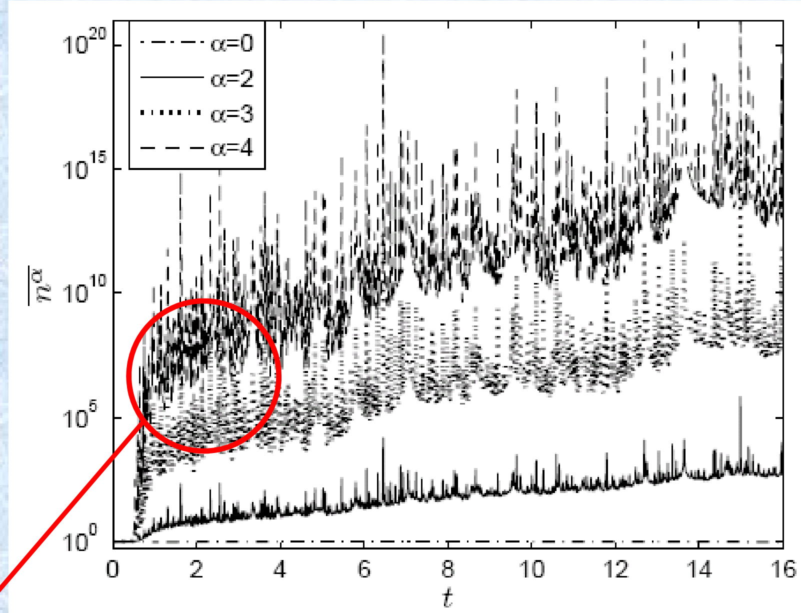
$$|J(t)| \approx n^{-1}(t) \longrightarrow n^{\alpha} \approx |J|^{-\alpha} \longrightarrow \langle n^{\alpha} \rangle \approx \langle |J|^{-\alpha} \rangle$$

$$\overline{n^{\alpha}} = \langle n^{\alpha-1} \rangle = \langle |J|^{1-\alpha} \rangle$$

St=0.1



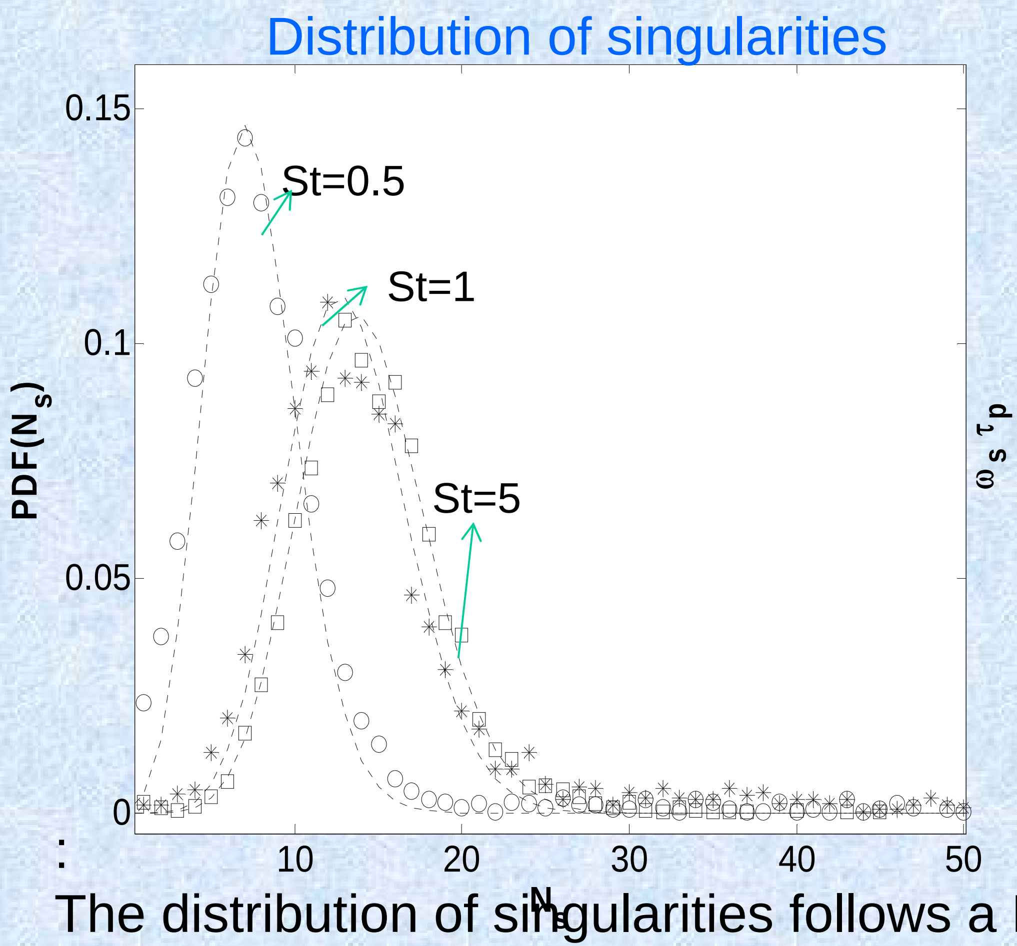
St=0.5

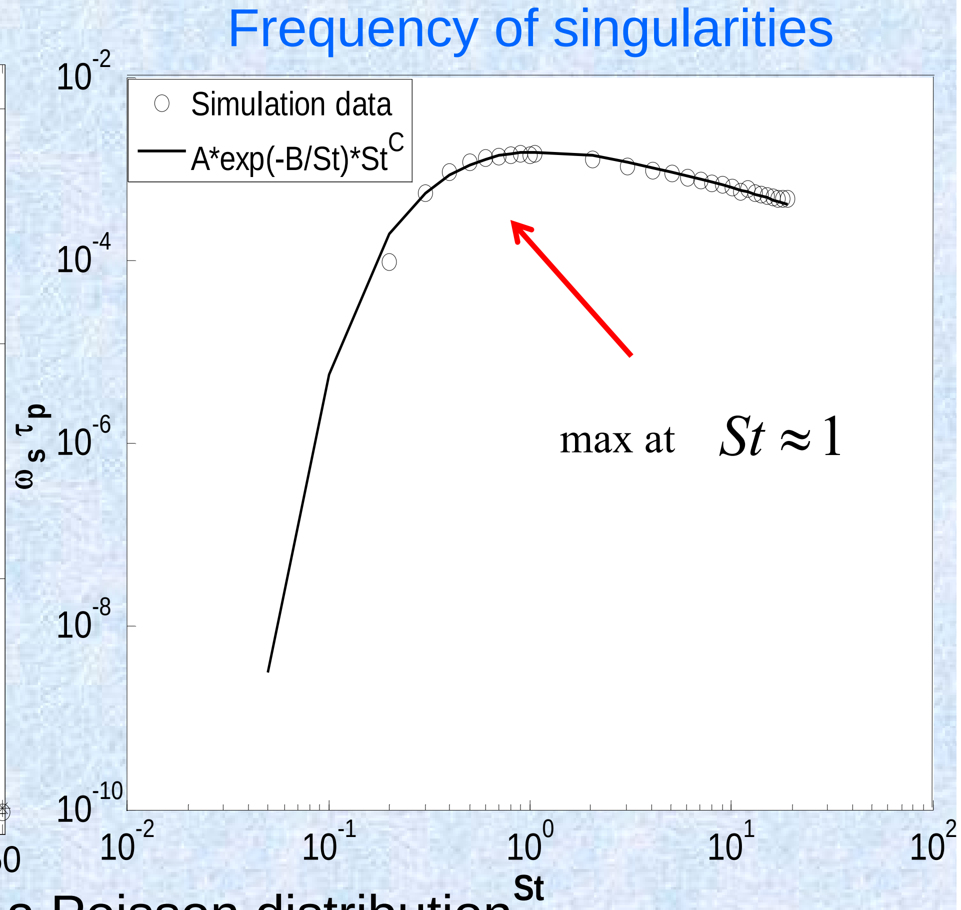


- Particle number density is spatially strongly intermittent
- The segregation goes on with time!
- The peaks reveal the presence of singularities!

Singularities in the ptcl concentration field

Singularities correspond to |J|=0 events





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The distribution of singularities follows a Poisson distribution

The maximum frequency of singularity events occurs for $St \approx 1$

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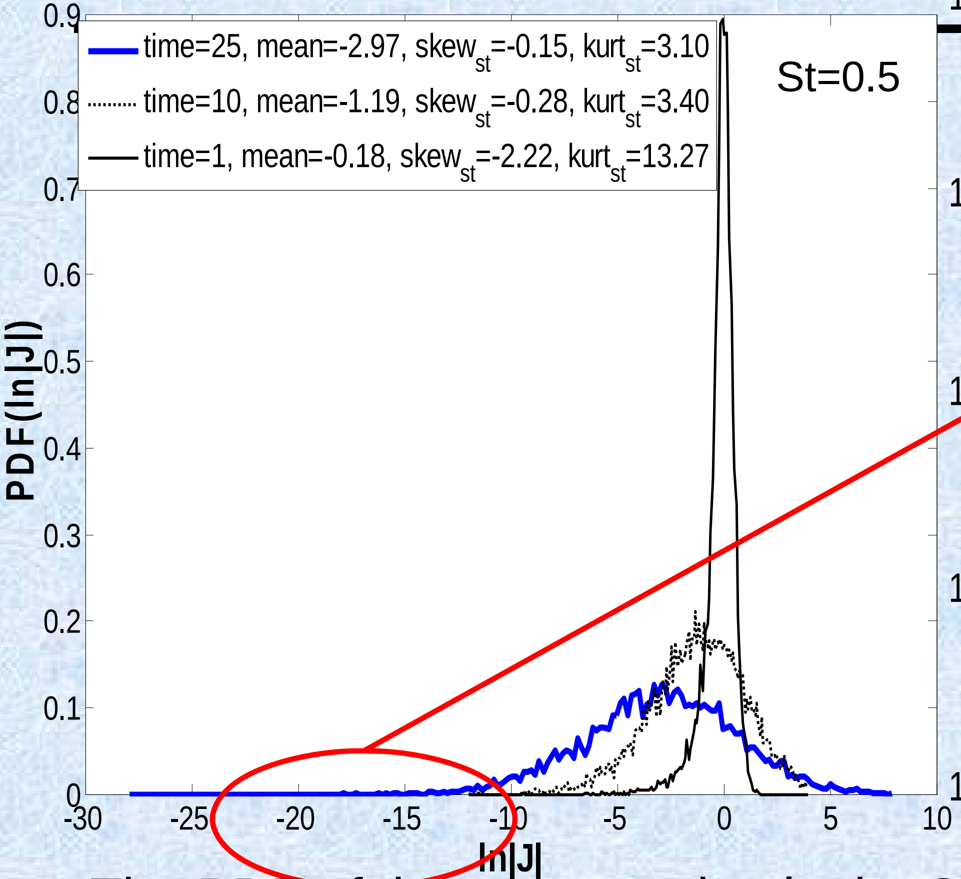
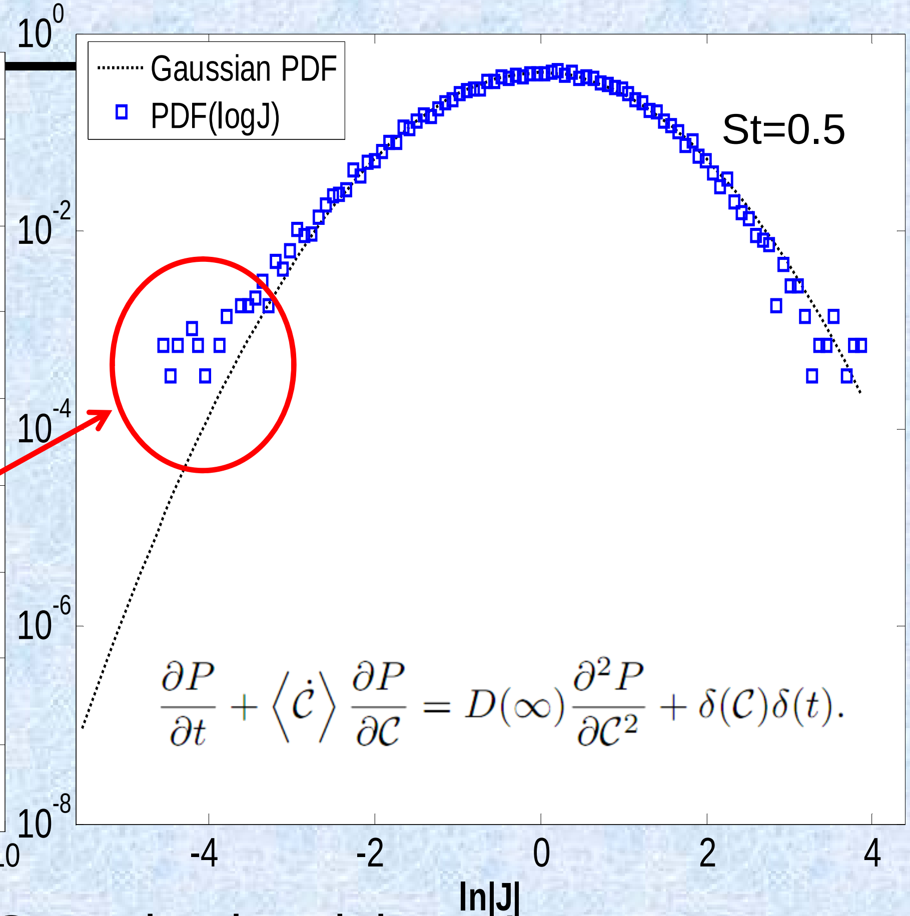
Statistics of the compression C=ln|J|





$$\frac{\partial P}{\partial t} + \left\langle \dot{\mathcal{C}} \right\rangle \frac{\partial P}{\partial \mathcal{C}} = D(\infty)\frac{\partial^2 P}{\partial \mathcal{C}^2} + \delta(\mathcal{C})\delta(t).$$

- The PDF of the compression looks Gaussian but..it is not!
- Singularities correspond to ln|J|-> -inf..what is the cause for the deviation from Gaussianity visible on the left tail of the curve?

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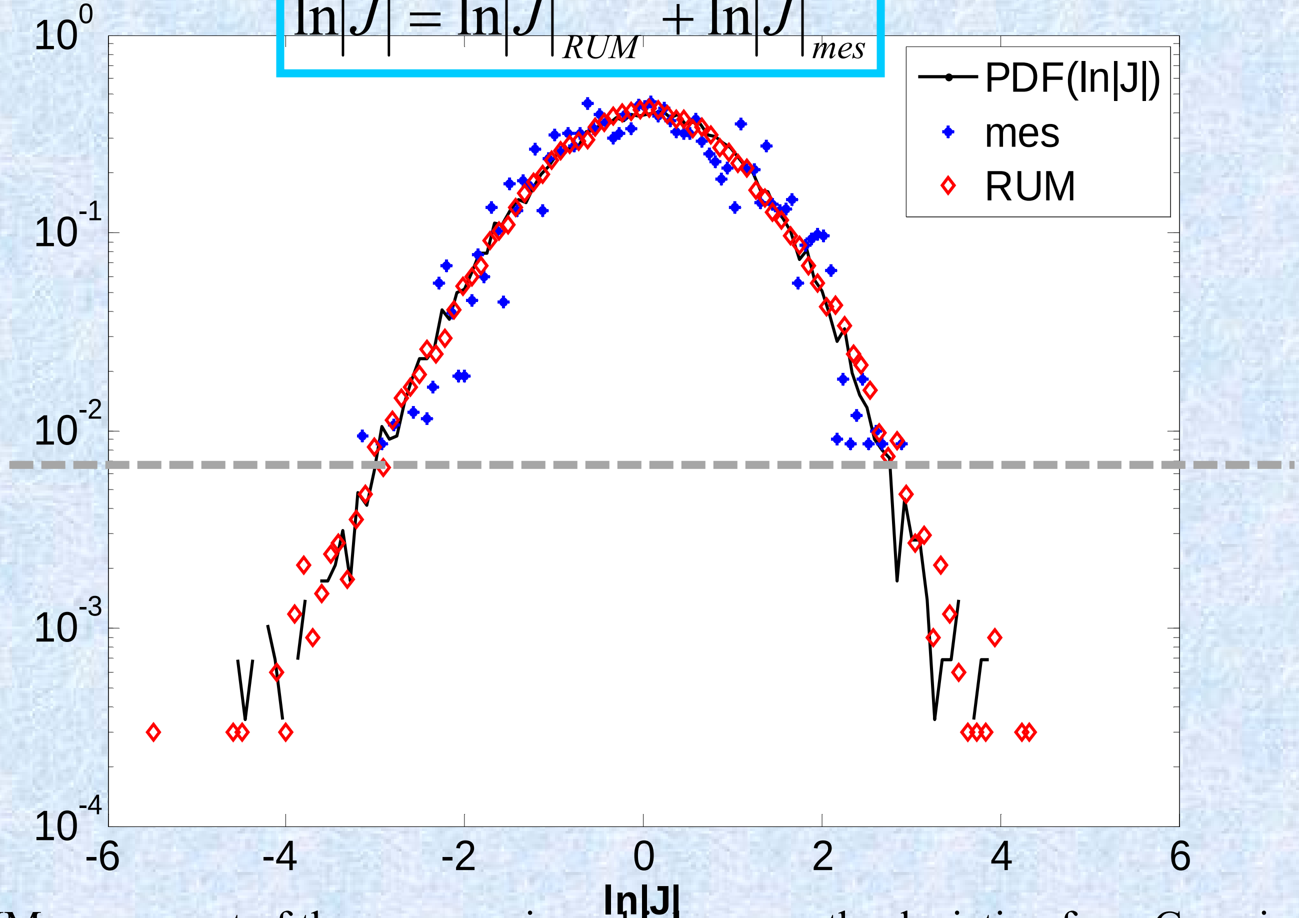
The effect of RUM on C=ln|J|

$$\ln|J| = \ln|J|_{RUM} + \ln|J|_{mes}$$



- It is the RUM component of the compression which causes the deviation from Gaussianity!

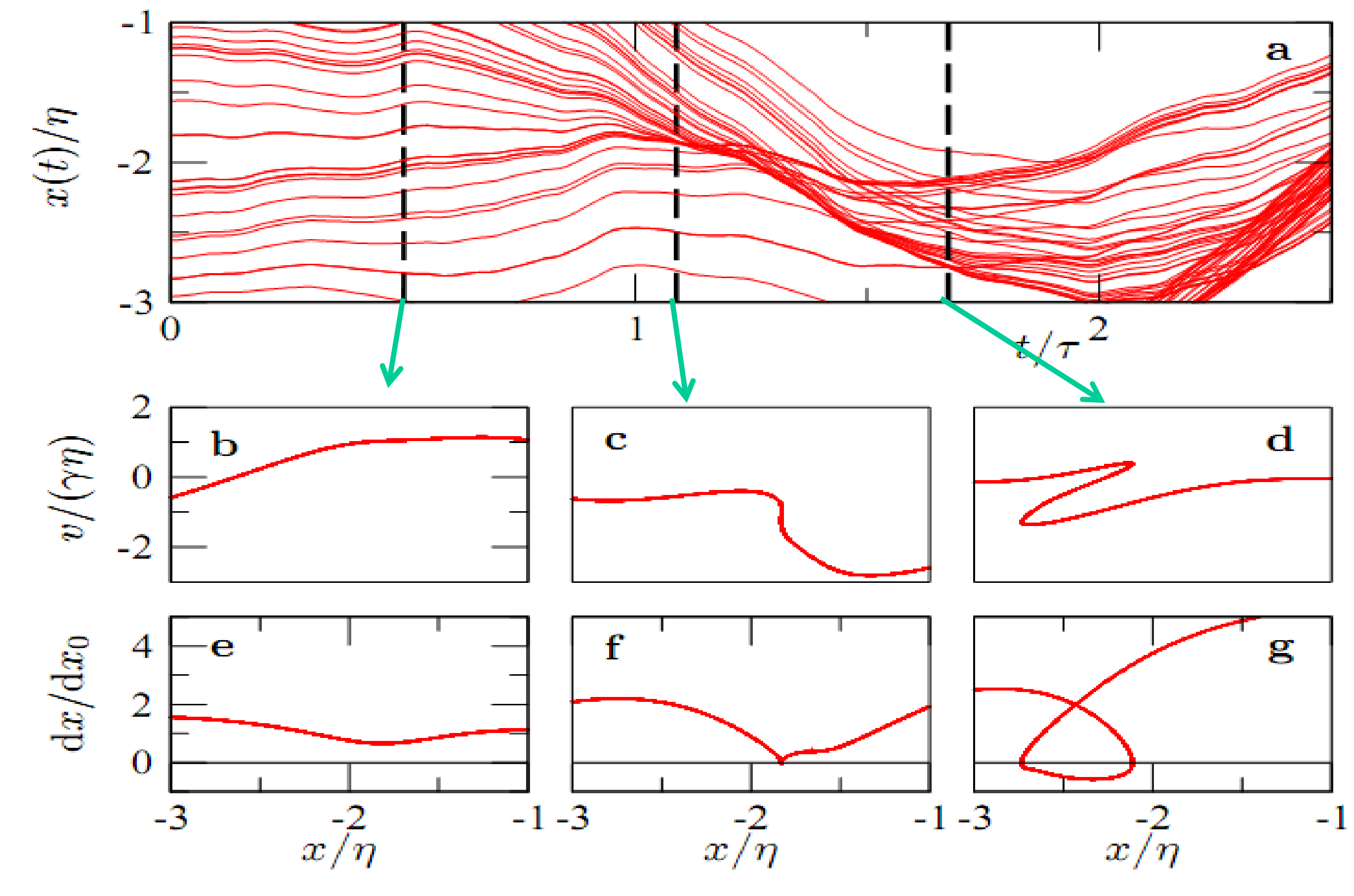
=>Singularities and RUM are intrinsically related

Singularities and RUM

Trajectories

Velocity (RUM)

|J| (singularities)



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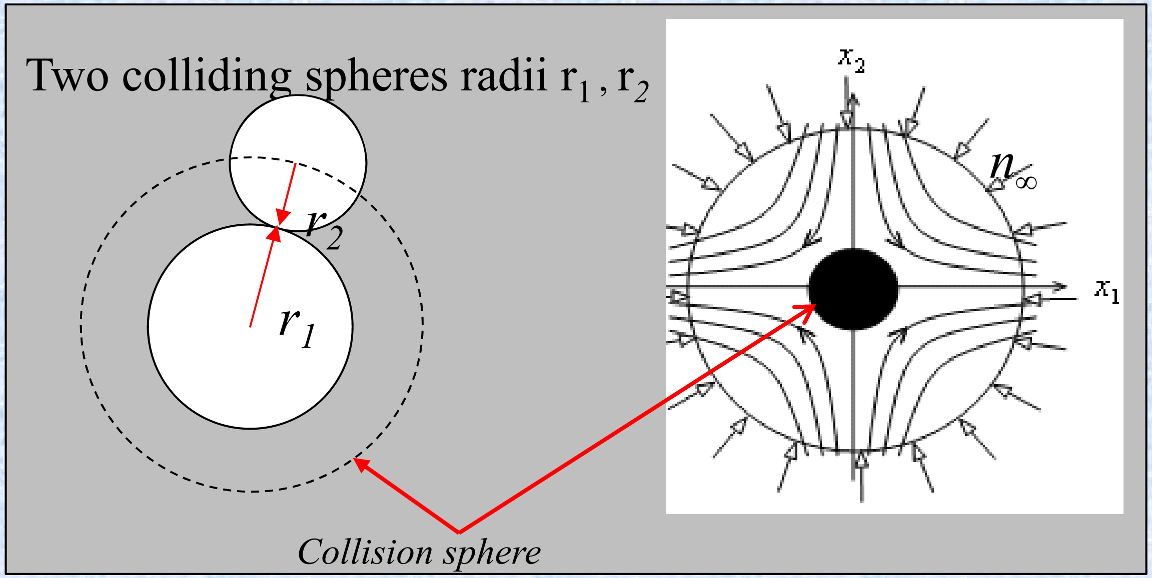
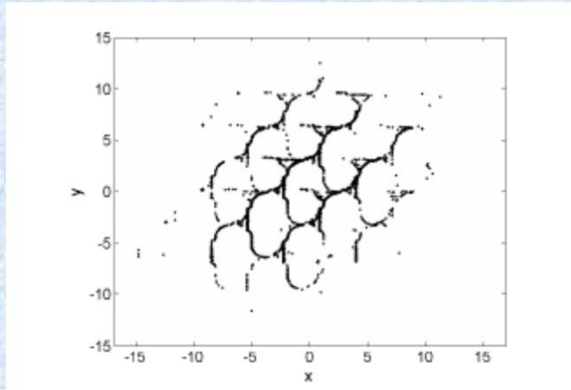
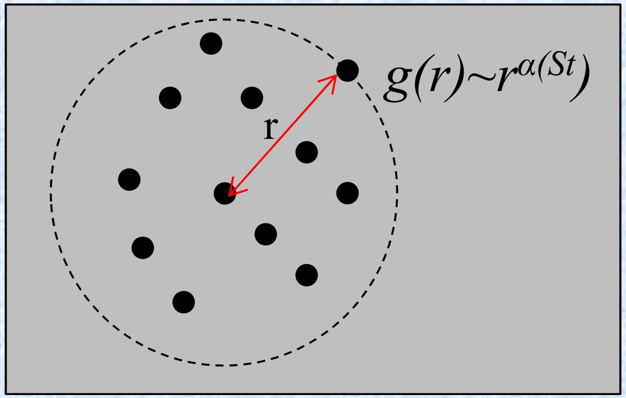
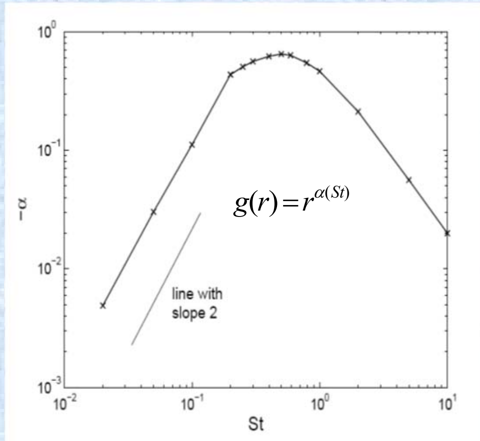
Pair dispersion and segregation









Agglomeration of inertial particles

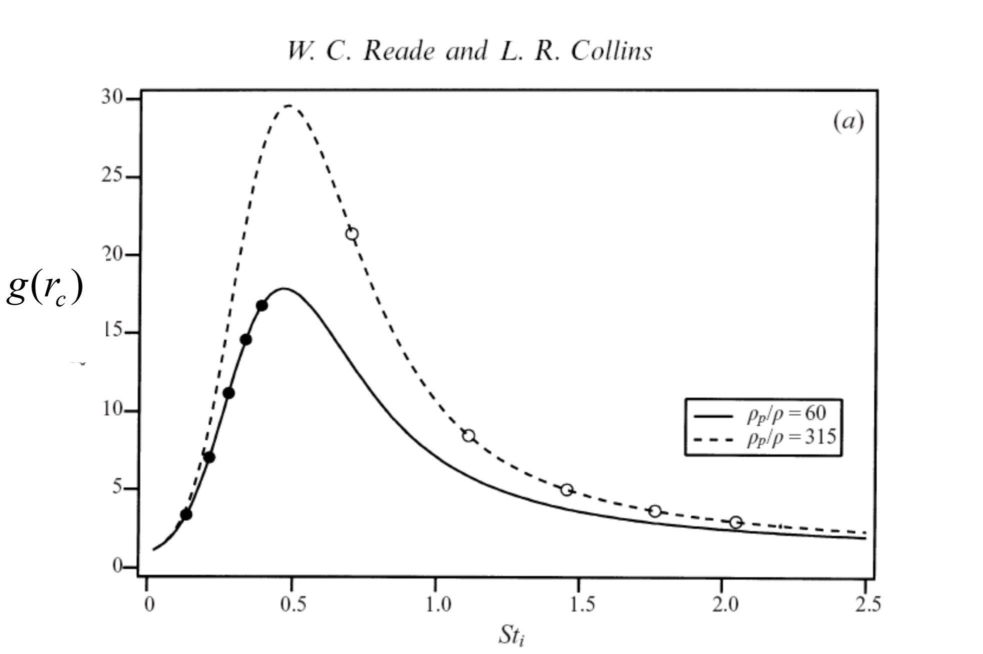
Sundarim & Collins(1997) , Reade & Collins (2000): measurement of rdfs and impact velocities as a function of Stokes number *St*

$$K(r_1, r_2) = 4\pi r_c^{\ 2}\ g(r_c, St)\ \overline{w_r}(St)$$

Net relative velocity between colliding spheres along their line of centres

RDF at r_c

$g(r_c)$



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PDF Equation for relative dispersion

(Zaichik and Alipchenkov 2003,2009)

$\underline{w}$ = relative velocity between identical particle pairs, distance $\underline{r}$ apart

$\Delta\underline{u}(\underline{r},\underline{t})$ = relative velocity between 2 fluid pts, distance $\underline{r}$ apart at time t

Structure functions $\left\langle \Delta u_r^{\,2}(r) \right\rangle, \left\langle \Delta u_\theta^{\,2}(r) \right\rangle \sim (r/\eta_K)^2 \quad (r/\eta_K) << 1$

Eqns of Motion $\dfrac{d\underline{r}}{dt} = w \quad ; \quad \dfrac{d\underline{w}}{dt} = \beta\left(\Delta\underline{u}(\underline{r},t) - \underline{w}\right)$

$$\frac{\partial P}{\partial t} + \underline{w}\cdot\frac{\partial P}{\partial \underline{r}} - \beta\frac{\partial}{\partial \underline{w}}\cdot \underline{w}P(\underline{w},\underline{r},t) = -\frac{\partial}{\partial \underline{w}}\cdot \beta\overline{\Delta\underline{u}P}$$

Net turbulent Force (diffusive)

$\beta = St^{-1}$

convection

$$\left(\frac{\partial}{\partial \underline{\mathrm{w}}}\cdot\underline{\mu} + \frac{\partial}{\partial \underline{r}}\cdot\underline{\lambda}\right)P(\underline{\mathrm{w}},r,t)$$

mass $$\frac{\partial\langle\rho\rangle}{\partial t} + \frac{\partial}{\partial x_i}\langle\rho\rangle\overline{w} = 0$$

momentum $$\langle\rho\rangle\frac{D}{Dt}\overline{\mathrm{w}}_i = -\frac{\partial}{\partial r_j}\langle\rho\rangle\overline{w_i' w_j'} - \beta\overline{\mathrm{w}}_i\langle\rho\rangle + \langle\rho\rangle\beta\overline{\Delta u_i'}$$

$$\longrightarrow g(r) \approx r^{-\alpha_1 St^2} \quad St << 1$$

PDF Equation predictions

Zaichik and Alipchenkov, NJP 2009

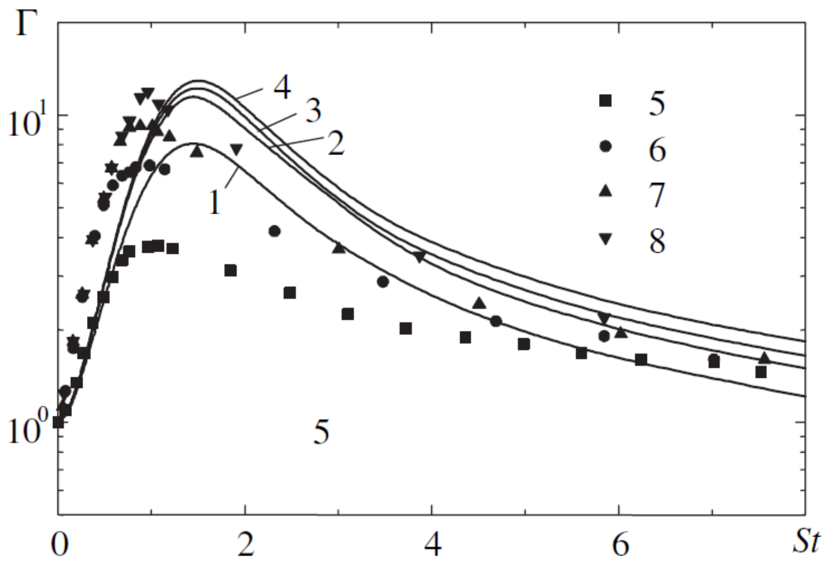


Figure 8. Influence of particle inertia on the RDF for $\bar{r} = 1$: 1–4, predictions; 5–8, DNS [8]; 1, 5, $Re_\lambda = 24$; 2, 6, $Re_\lambda = 45$; 3, 7, $Re_\lambda = 58$; 4, 8, $Re_\lambda = 75$.

mean of magnitude of relative velocity $\langle |w_r| \rangle$

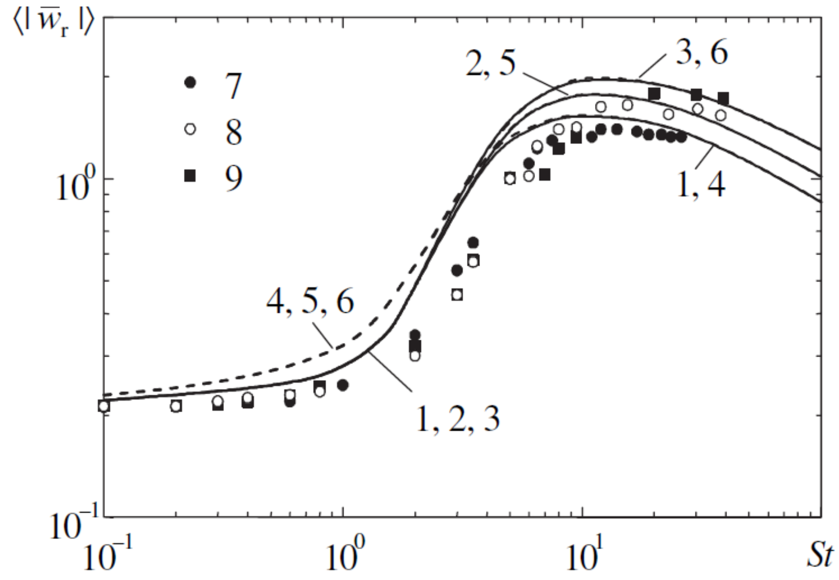
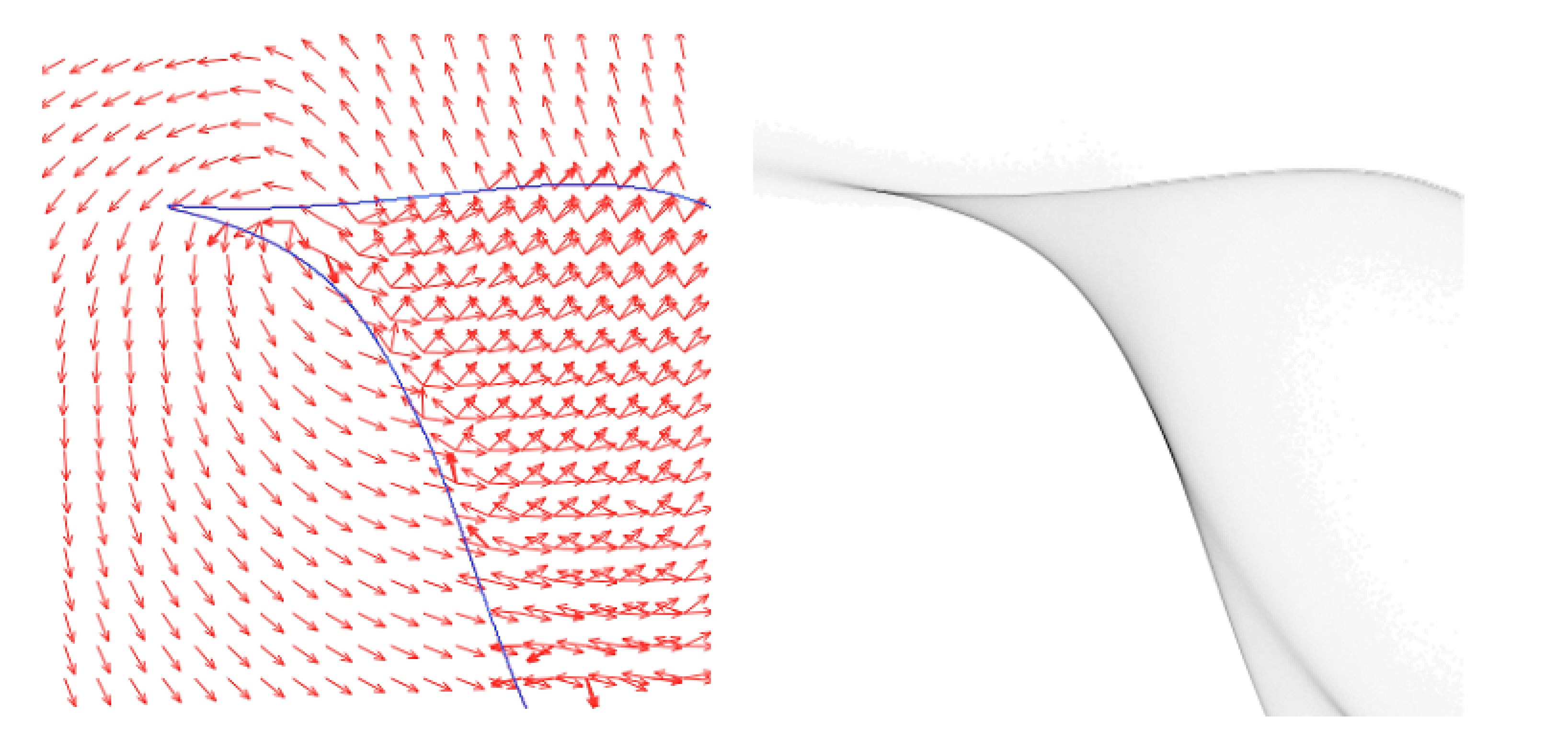


Figure 5. Influence of particle inertia on the mean relative velocity magnitude for $\bar{r} = 1$: 1–3, predictions with $\bar{d} = 0$; 4–6, predictions with $\bar{d} = 1$; 7–9, DNS [8]; 1, 4, 7, $Re_\lambda = 45$; 2, 5, 8, $Re_\lambda = 58$; 3, 6, 9, $Re_\lambda = 75$.

Caustics



Multivalued particle velocities in a 2-D random flow

Particle density

Summary & conclusions

- Transport of inertial particles turbulent flows
  - Methods and approaches
    - PDF approach / FLM
  - One particle dispersion
    - homogeneous and simple shear flows
    - inhomogeneous – turbulent boundary layer
  - Two particle dispersion
    - Unmixing of particle flows
      - compressibility of a particle flow
      - Singularities /interemittency
      - Formation of caustics

THANKS FOR YOUR ATTENTION

Any questions?