A Markov Chain Estimator of Multivariate Volatility from High Frequency Data

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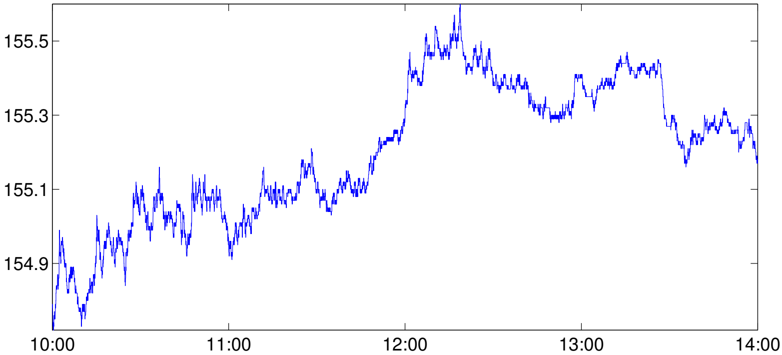
S&P 500 (March 18, 2013)



Realized Variance

- BSM

$$X_s = \int_0^s a_u \mathrm{d}u + \int_0^s \sigma_u \mathrm{d}B_u$$

- Realized Variance

$$RV = \sum_{i=1}^{n} x_{i,n}^2 \qquad x_{i,n} = X_{\frac{i}{n}} - X_{\frac{i-1}{n}},$$

estimates integrated variance

$$IV = \int_0^1 \sigma_u^2 \mathrm{d}u.$$

Realized Variance

- Barndorff-Nielsen & Shephard:

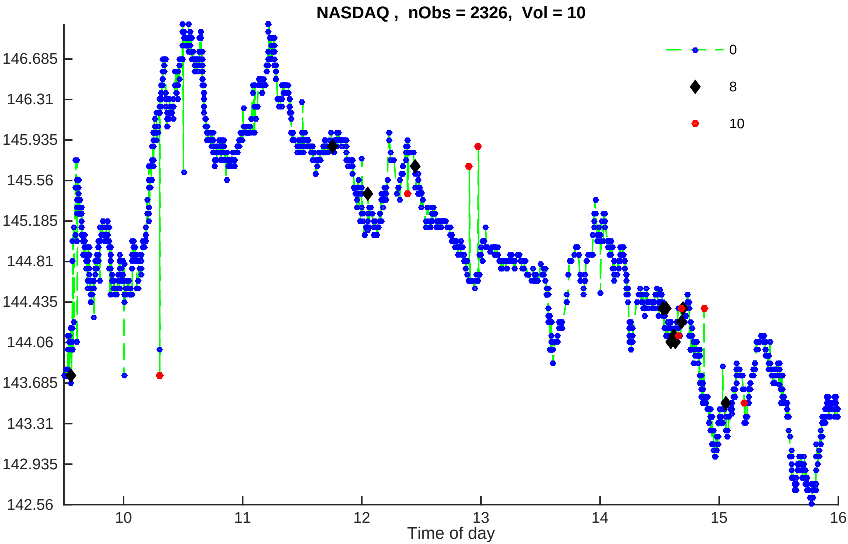
$$n^{1/2}(RV - \int_0^1 \sigma_u^2 \mathrm{d}u) \xrightarrow{Ls} MN(0, 2\int_0^1 \sigma_u^4 \mathrm{d}u).$$

- Feasible CLT

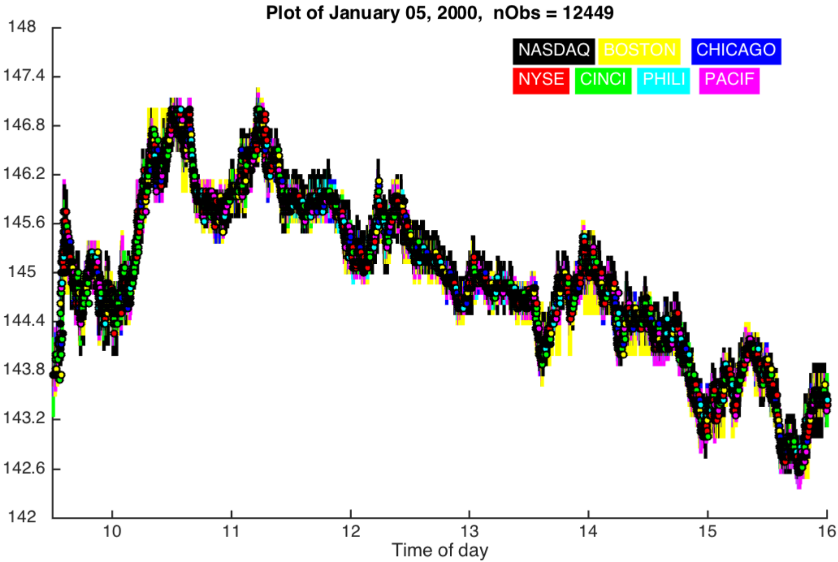
$$\frac{RV - \int_0^1 \sigma_u^2 \mathrm{d}u}{\sqrt{\frac{2}{3}\sum x_{i,n}^4}} \sim N(0,1).$$

Properties of High-Frequency Data

Dirty Facts about High-Frequency Data

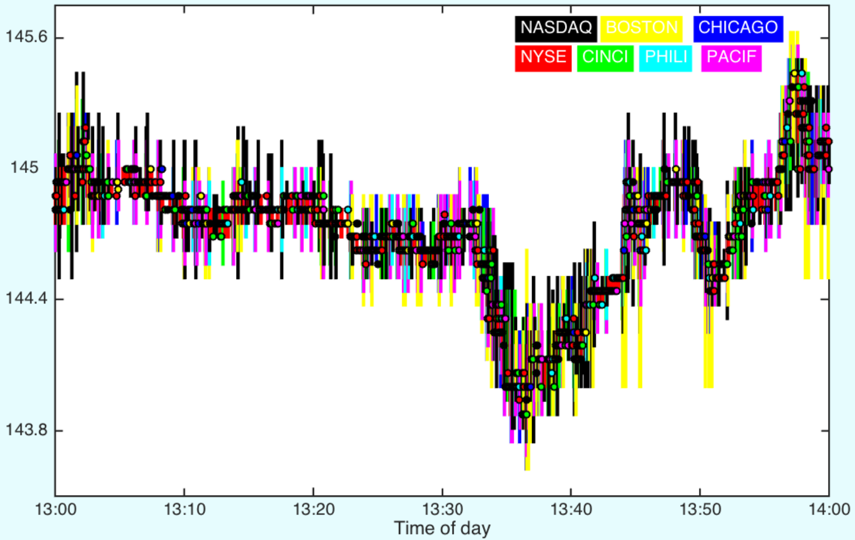


Prices



Plot of January 05, 2000, nObs = 12449
NASDAQ
BOSTON
CHICAGO
NYSE
CINCI
PHILI
PACIF
148
147.4
146.8
146.2
145.6
145
144.4
143.8
143.2
142.6
142
10
11
12
13
14
15
16
Time of day

Prices


NASDAQ
BOSTON
CHICAGO
NYSE
CINCI
PHILI
PACIF
145.6
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13:00
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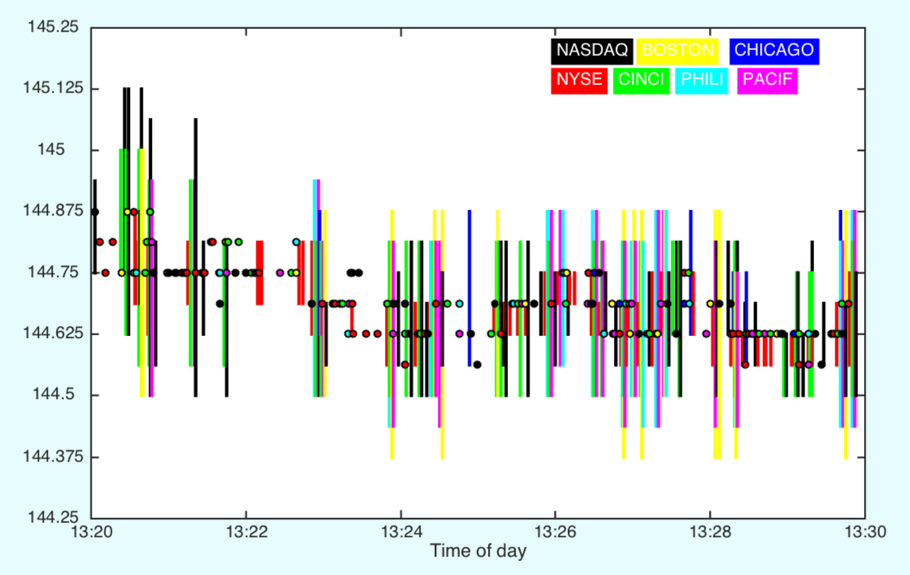
13:20
13:30
13:40
13:50
14:00
Time of day

Prices



145.25
145.125
145
144.875
144.75
144.625
144.5
144.375
144.25
13:20
13:22
13:24
13:26
13:28
13:30
Time of day
NASDAQ
BOSTON
CHICAGO
NYSE
CINCI
PHILI
PACIF

Empirical Features of Noise

- Hansen & Lunde (JBES, 2006)
  Noise is...
  - ... serially dependent
  - ... endogenous (especially mid-quotes)
  - ... small
  - ... has changed over time

Robust Estimators

Noise-Robust Estimators

- Realized Kernel: Barndorff-Nielsen et.al.
  - Univariate, iid noise. (EMA 2008)

$$n^{1/4}(RK - IV) \stackrel{Ls}{\to} MN\{0, 8\omega \left(\int \sigma_u^4 \mathrm{d}u\right)^{3/4}\}$$

  - Multivariate, general noise. $n^{1/5}$ (JoE 2011)
  - Subsampling RK. (JoE 2011)
  - Realized Kernels in Practice. (EJ 2009)
- Subsampling/Two-scale/Multiscale
- Pre-averaging.
- Many more....

Long Run Variance

- CLTs for time-series data

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n}(X_i-\mu)\xrightarrow{d}N(0,\Omega).$$

- With $\mu=0$. HF data $x_{i,n}=X_i/\sqrt{n}$.
  -

$$\begin{aligned}\tfrac{1}{n}\sum_{i=1}^{n}X_i^2 &= \sum_{i=1}^{n}x_{i,n}^2=RV\\ \sum_j k(\tfrac{j}{H_n})\tfrac{1}{n}\sum_{i=1}^{n}X_iX_{i-j} &= \sum_j k(\tfrac{j}{H_n})\sum_{i=1}^{n}x_{i,n}x_{i-j,n}=RK\end{aligned}$$

Martingale Representation of Time Series

- Consider
$$\sum_{s=1}^{t} X_s = \sum_{s=1}^{t} Y_s + \mu_t + U_t.$$
Martingale, deterministic term, stationary "noise".
-

$$\begin{aligned}\Omega = \mathrm{avar}\Big(\frac{1}{\sqrt{n}}\sum_{i=1}^{n} X_s\Big) &= \mathrm{avar}\Big(\frac{1}{\sqrt{n}}\sum_{s=1}^{n} Y_s + \frac{\mu_n + U_n}{\sqrt{n}}\Big). \\ &\simeq \frac{1}{n}\sum_{s=1}^{n} \mathrm{E}(Y_s^2)\end{aligned}$$

Towards the Main Result: Filtered Process

- General way to extract martingale from

$$X_t^\bullet = \sum_{s=1}^{t} X_s.$$

- Martingale component is:

$$Y_t^\bullet = \lim_{h\to\infty} \mathrm{E}(X_{t+h}^\bullet - \mu_{t+h} | \mathcal{F}_t),$$

where $\Delta\mu_t = \mathrm{E}(X_t)$.

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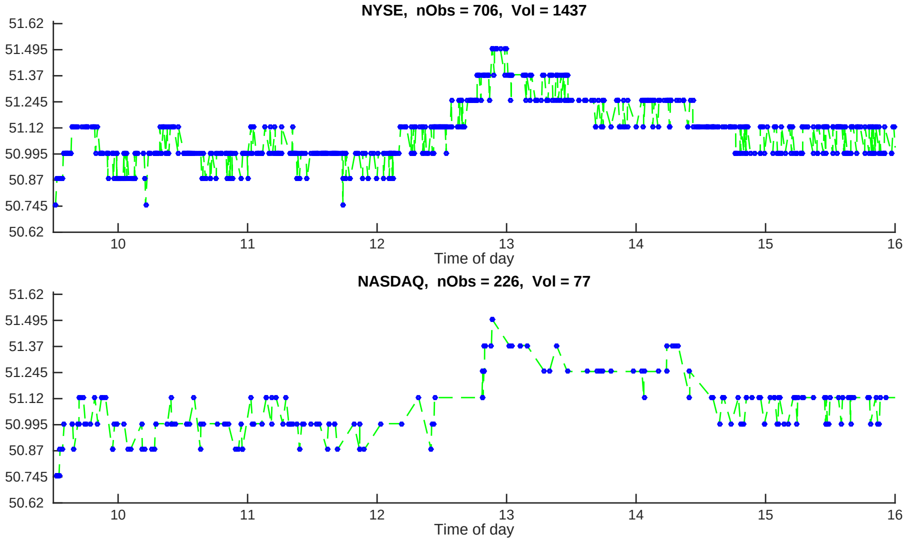
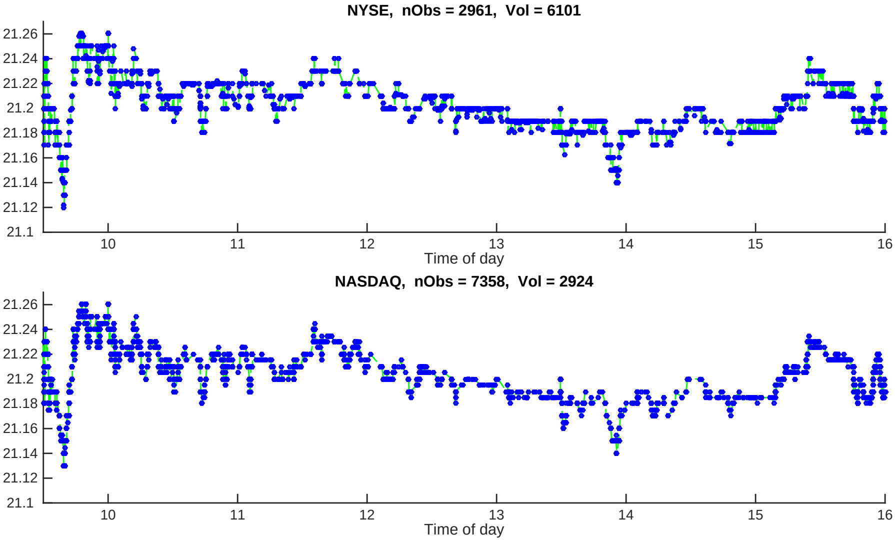
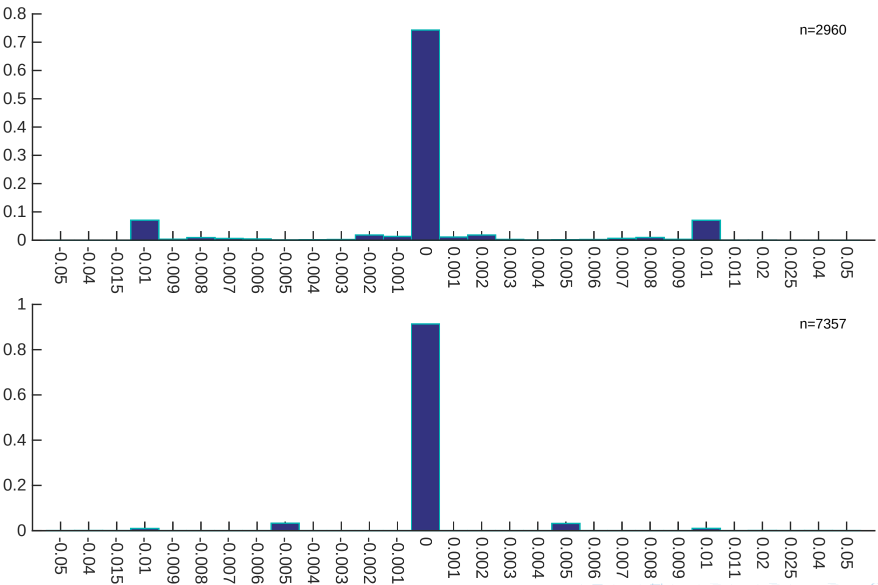
Prices 1995 (tick size = 1/8 $)



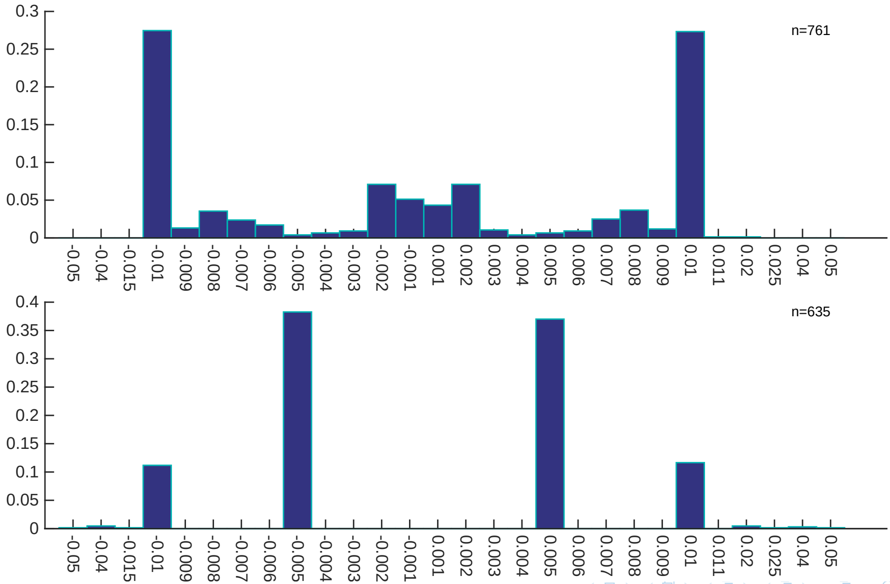
Prices 2013 (tick size = 1 cent)



Prices increments 2013 NYSE & NASDAQ



Prices increment 2013 NYSE & NASDAQ (without zeros)



Markov Framework

Markov Framework: Assumptions

- The d-dimensional process,

$$\{X_t\}_{t=1}^{n},$$

  - is **ergodic** and distributed as a
  - **homogeneous Markov chain**
  - of order $k < \infty$,
  - with $S < \infty$ states.

Markov Notation

- Transition matrix:

$$P_{r,s} = \Pr(X_{t+1} = x_s | X_t = x_r).$$

- Stationary distribution, $\pi \in \mathbb{R}^S$, solves

$$\pi' P = \pi'.$$

- Fundamental matrix

$$Z = (I - P + \Pi)^{-1}, \qquad \text{where} \quad \Pi = \iota\pi'.$$

- $\Lambda_\pi = \mathrm{diag}(\pi_1, \ldots, \pi_S).$

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Hansen (EL 2015): Martingale Decomposition

- For $X_t^{\bullet} = \sum_{s=1}^{t} X_s$:

$$X_t^{\bullet} = Y_t^{\bullet} + \mu_t + U_t,$$

where

- μ_t is a linear deterministic trend,
- $\{Y_t^{\bullet}, \mathcal{F}_t\}$ is a martingale – with $\mathcal{F}_t = \sigma(\{X_j^{\bullet}\}_{j \leq t})$.
- U_t is a bounded stationary process.

Hansen (EL 2015): Martingale Decomposition

- Martingale increments are:

$$Y_t = x' Z' e_{s_t} - x' Z' P' e_{s_{t-1}},$$

where e_s is the s-th unit vector, s_t is state at time t.

- So

$$\Omega = x' Z' (\Lambda_\pi - P' \Lambda_\pi P) Z x$$

- Deterministic term

$$\mu_t = t\mu.$$

"Noise" is

$$U_t = x'(I - Z)' e_{s_t}$$

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Covariance Structure

- We have with, $\mathbf{x} = (x_1, \ldots, x_S)'$,

$$\mathrm{var}(Y_t) = \underbrace{\mathbf{x}' Z' (\Lambda_\pi - P' \Lambda_\pi P) Z \mathbf{x}}_{=\Omega}$$

- Serial dependent noise

$$\mathrm{cov}(U_t, U_{t+j}) = \mathbf{x}'(I - Z)' \Lambda_\pi P^{|j|} (I - Z) \mathbf{x} = \mathbf{x}' Z' P' \Lambda_\pi P P^{|j|} Z \mathbf{x},$$

- Endogenous noise

$$\mathrm{cov}(Y_t, U_{t+j}) = \mathbf{x}' Z' (-\Lambda_\pi + P' \Lambda_\pi P) P^{j+1} Z \mathbf{x}, \qquad \text{for} \quad j \geq 0,$$

(zero for $j < 0$).

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- Estimate P and define

$$\hat{\Omega} = \mathbf{x}'\hat{Z}'(\Lambda_{\hat{\pi}} - \hat{P}'\Lambda_{\hat{\pi}}\hat{P})\hat{Z}\mathbf{x}.$$

- Hansen & Horel

$$\sqrt{n}\left\{\hat{\Omega} - \Omega\right\} \xrightarrow{d} \mathrm{N}(0, \Sigma_{\hat{\Omega}}),$$

as $n \to \infty$, where

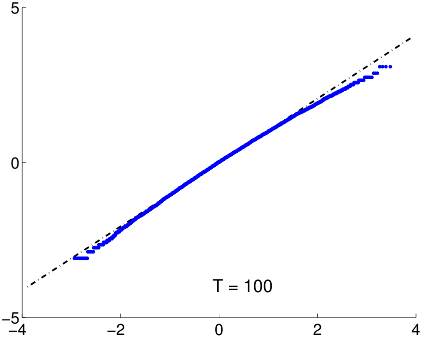
$$(\Sigma_{\hat{\Omega}})_{ij,kl} = \sum_{r,s,v} e_s' V_r e_v \frac{\partial \Omega_{ij}}{\partial P_{rs}} \frac{\partial \Omega_{kl}}{\partial P_{rv}},$$

with

$$\frac{\partial \Omega}{\partial P_{rs}} = \text{"nice expression in closed form"}.$$

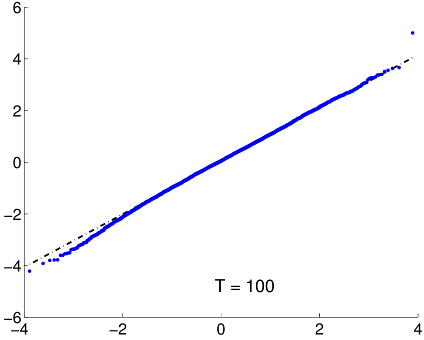
QQ Plots: CLTs for $\log \hat{\sigma}^2$ (T=100)

$$n^{1/2}\left\{\log \hat{\sigma}^2 - \log \sigma^2\right\} \xrightarrow{d} \mathrm{N}\left(0, \frac{\Sigma_{\hat{\sigma}^2}}{\sigma^4}\right), \qquad \tilde{t} = \frac{\log \hat{\sigma}^2 - \log \sigma^2}{\sqrt{\frac{1}{n}\hat{\Sigma}_{\hat{\sigma}^2}/\hat{\sigma}^4}} \overset{A}{\sim} \mathrm{N}(0,1)$$



QQ Plots: CLT $F(\hat{\rho})$ (T=100)

$$n^{1/2}[F(\hat{\rho}) - F(\rho)] \xrightarrow{d} \mathrm{N}(0, \tfrac{1}{4}\frac{\rho^2}{(1-\rho^2)^2}a'\Xi a).$$



- Asymptotic Scheme:

$$\mathbf{x} = \frac{1}{\sqrt{n}}\xi \qquad \text{with} \quad \xi \quad \text{fixed}.$$

- Realized variance

$$\sum_{i=1}^{n} x_{i,n}^2 = n\mathbf{x}'\Lambda_{\hat{\pi}}\mathbf{x} = \xi'\Lambda_{\hat{\pi}}\xi.$$

- Markov Chain estimator

$$\mathrm{MC}^{\#} = \xi'\hat{Z}'(\Lambda_{\hat{\pi}} - \hat{P}'\Lambda_{\hat{\pi}}\hat{P})\hat{Z}\xi.$$

Volatility of log-price?

- Typically we want to estimate volatility of $\log Y_t$.
- Prices are on a grid. Log-Prices are not.
- Turns out that $\mathrm{MC}^{\#}$ is estimating

$$\int_0^1 \sigma_u^2 Y_u^2 \mathrm{d}u.$$

Estimator from Filtered Log-Prices

- (univariate case)

$$\mathrm{MC}^* = \sum_{i=1}^{n} \left[\log \frac{X_i + e'_{s_i}(Z - I)\mathbf{x}}{X_{i-1} + e'_{s_{i-1}}(Z - I)\mathbf{x}} \right]^2,$$

  which is path dependent.

- Approximate estimator

$$\mathrm{MC} = \frac{\mathrm{MC}^{\#}}{\frac{1}{n}\sum_{i=1}^{n} X_i^2},$$

  empirically indistinguishable from MC^*.

Inhomogeneity?

- What if MC is inhomogeneous?
- E.g. first P^1 then P^2, etc.
- Estimate homogeneous MC of order k, with $k > c \log n$

Three Markov Estimators

- Full $d \times d$ matrix

$$\mathrm{MC} = D^{-1}\mathrm{MC}^{\#}D^{-1},$$

with $D = \mathrm{diag}\,(\delta_1, \ldots, \delta_d)$ and $\delta_j^2 = n^{-1}\sum_{t=1}^{n} X_{j,t}^2$ $j = 1, \ldots, d$.

- 1-composite (polarization-based). Correlation by:

$$\mathrm{MC}_{i,j}^{\#_1} = \tfrac{1}{4}(\mathrm{MC}_{X_i+X_j}^{\#} - \mathrm{MC}_{X_i-X_j}^{\#}),$$

- 2-composite: Piece together 2×2 estimators.

Enforcing PSD (composite estimators)

- Projections. E.g.

$$\min_{\Sigma} \|\Sigma - A\|_{\mathrm{Fro}} \quad \text{subject to} \quad \Sigma \geq 0,$$

Ledoit & Wolf also impose $\mathrm{diag}(\Sigma) = \mathrm{diag}(A)$.

- Let ω_{ij} be the standard errors of A_{ij}. Solve:

$$\min_{\Sigma} \sum_{i,j=1}^{d} \left(\frac{\Sigma_{ij} - A_{ij}}{\omega_{ij}} \right)^2 \quad \text{subject to} \quad \Sigma \geq 0.$$

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Simulation Study: Efficient Price

- Stochastic Volatility

$$\log Y_{i,t} = \log Y_{i,t} + \sigma_{i,t} V_{i,t}, \qquad i = 1, 2,$$

where

$$V_{i,t} = \gamma Z_{i,t} + \sqrt{1-\gamma^2} W_{i,t},$$

- with

$$\begin{pmatrix} Z_{1t} \\ Z_{2t} \\ W_{1t} \\ W_{2t} \end{pmatrix} \sim iid\mathrm{N}(0, \begin{pmatrix} 1 & & & \\ 0 & 1 & & \\ 0 & 0 & 1 & \\ 0 & 0 & \rho & 1 \end{pmatrix}).$$

Simulation Study (cont)

- Stochastic Volatility:

$$\sigma_{i,t} = \sqrt{\Delta}\left\{\exp\left(\beta_0 + \beta_1 \tau_{i,t}\right)\right\},$$

where

$$\tau_{i,t} = \exp(\alpha\Delta)\tau_{i,t-1} + \sqrt{\frac{\exp(2\alpha\Delta)-1}{2\alpha}} Z_{i,t}$$

and $\Delta = \frac{1}{N}$ with $N = 23,400$.

- $\sigma_{i,t}$, correlates with $Z_{i,t}$ (leverage effect if $\gamma \neq 0$).

Simulations: Observed Price

- Rounding errors:

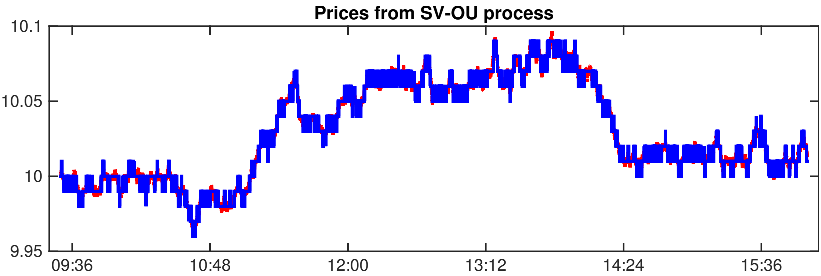
$$X_t = \delta[Y_t/\delta].$$

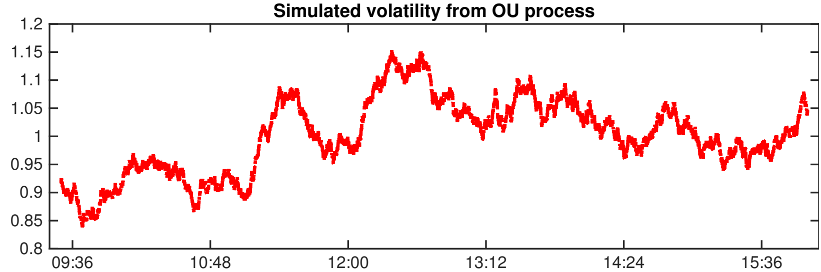
- Noise + Rounding errors

$$X_t = \delta[(Y_t + U_t)/\delta],$$

  with U_t are iid and uniformly distributed.
- Results: Estimator performs on par with other good robust estimator.

A Typical Simulated Sample Path





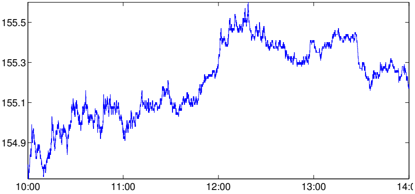
Empirical Analysis

Empirical Analysis

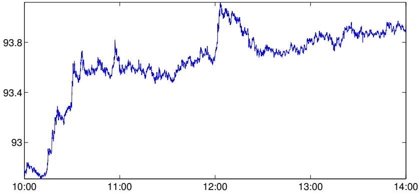
- SPY (ETF tracking S&P 500)
- 13 commodities (futures):
  - Crude Light (CL), Natural Gas (NG),
  - Gold (GC), Silver (SV), Copper (HG),
  - Live Cattle (LC), Lean Hogs (LH),
    Coffee (KC), Sugar (SB), Cotton (CT),
    Corn (CN), Soybeans (SY) and Wheat (WC).

March 18, 2013

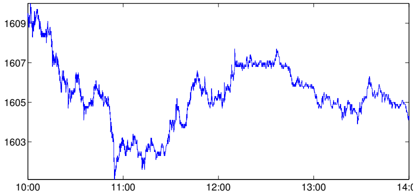
S&P 500



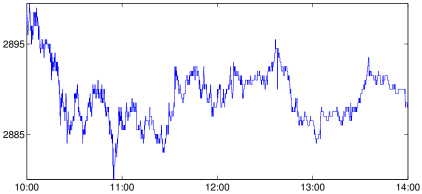
Crude Oil

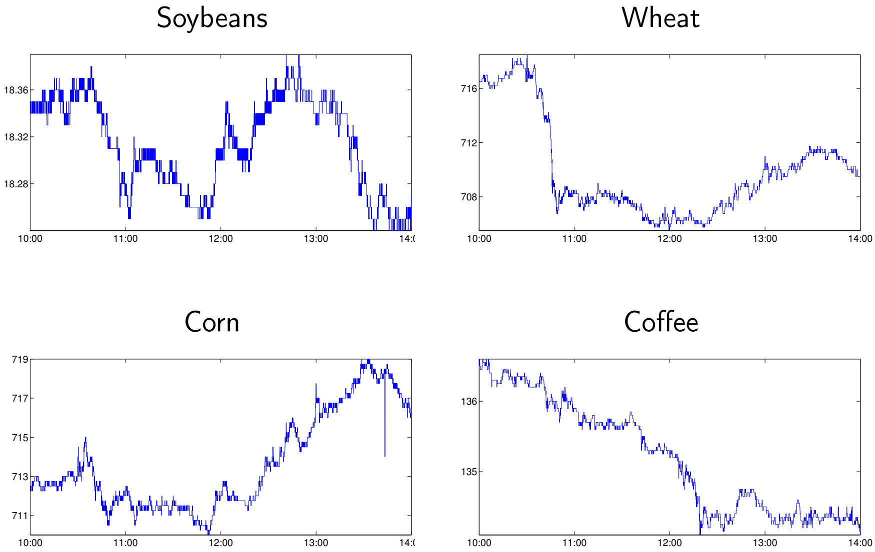


Gold



Silver




Soybeans
18.36
18.32
18.28
10:00
11:00
12:00
13:00
Wheat
716
712
708
10:00
11:00
12:00
13:00
14:00
Corn
719
717
715
713
711
10:00
11:00
12:00
13:00
Coffee
136
135
10:00
11:00
12:00
13:00
14:00

RV5min. March 18, 2013

SPY	CL	NG	GC	SV	HG	LC	LH	KC	SB	CT	CN	SY	WC
81.2													
0.48	**344**												
0.07	*0.15*	**805**											
-0.08	*-0.16*	*-0.01*	**146**										
-0.13	*-0.00*	*0.12*	*0.77*	**316**									
0.30	*0.24*	*0.34*	*0.03*	*0.20*	**270**								
-0.19	*-0.03*	*-0.25*	*-0.05*	*-0.09*	*-0.32*	**180**							
-0.00	*-0.15*	*-0.03*	*-0.01*	*0.02*	*0.03*	*0.05*	**618**						
-0.08	*-0.01*	*0.09*	*-0.20*	*-0.10*	*0.27*	*-0.09*	*-0.11*	**408**					
0.16	*-0.07*	*-0.11*	*-0.20*	*-0.17*	*0.05*	*-0.07*	*-0.02*	*0.10*	**275**				
0.24	*0.03*	*-0.09*	*0.04*	*0.04*	*0.07*	*-0.09*	*0.02*	*0.03*	*0.18*	**479**			
0.05	*0.01*	*-0.12*	*-0.14*	*-0.14*	*-0.13*	*0.21*	*-0.07*	*0.24*	*-0.07*	*0.16*	**641**		
0.09	*0.09*	*0.04*	*-0.09*	*-0.13*	*0.20*	*-0.02*	*-0.17*	*0.15*	*0.08*	*0.23*	*0.56*	**416**	
-0.10	*0.05*	*0.06*	*-0.11*	*-0.05*	*0.02*	*0.25*	*-0.00*	*0.38*	*0.12*	*0.10*	*0.69*	*0.48*	**553**

RV10min. March 18, 2013

SPY	CL	NG	GC	SV	HG	LC	LH	KC	SB	CT	CN	SY	WC
69													
0.59	**342**												
-0.04	*0.22*	**579**											
0.03	*-0.43*	*0.21*	**78**										
-0.20	*-0.26*	*0.36*	*0.48*	**140**									
0.18	*0.28*	*0.38*	*0.06*	*0.26*	**317**								
0.03	*0.04*	*-0.25*	*-0.16*	*-0.16*	*-0.45*	**165**							
0.14	*0.06*	*-0.21*	*-0.05*	*-0.06*	*0.31*	*0.12*	**418**						
-0.23	*-0.03*	*0.21*	*-0.19*	*-0.07*	*0.33*	*-0.09*	*-0.22*	**447**					
-0.04	*-0.10*	*-0.11*	*-0.02*	*-0.07*	*0.13*	*0.01*	*0.13*	*0.19*	**242**				
0.19	*0.21*	*0.09*	*-0.14*	*-0.13*	*-0.09*	*0.31*	*0.07*	*-0.13*	*0.42*	**503**			
-0.02	*-0.02*	*-0.06*	*-0.17*	*-0.13*	*-0.17*	*0.45*	*-0.21*	*0.19*	*-0.06*	*0.09*	**700**		
-0.14	*0.05*	*0.13*	*-0.10*	*-0.03*	*0.13*	*0.24*	*-0.15*	*0.22*	*-0.03*	*0.15*	*0.71*	**464**	
-0.14	*0.03*	*0.07*	*-0.13*	*-0.07*	*0.10*	*0.39*	*0.01*	*0.45*	*0.24*	*0.22*	*0.70*	*0.72*	**861**

MC1. March 18, 2013

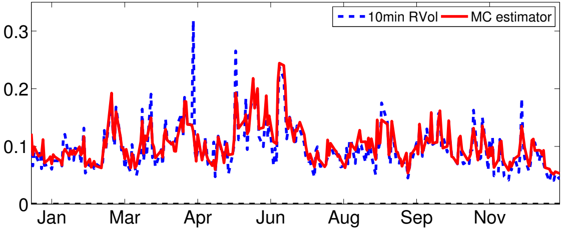
SPY	CL	NG	GC	SV	HG	LC	LH	KC	SB	CT	CN	SY	WC
117													
0.35	**392**												
-0.11	*-0.04*	**922**											
-0.09	*0.07*	*-0.01*	**117**										
0.02	*0.16*	*-0.02*	*0.72*	**380**									
0.35	*0.27*	*0.05*	*0.04*	*0.08*	**365**								
0.02	*0.05*	*-0.07*	*-0.04*	*0.13*	*-0.04*	**142**							
-0.02	*-0.06*	*0.01*	*0.03*	*0.06*	*-0.05*	*0.14*	**607**						
0.09	*0.10*	*0.05*	*-0.08*	*-0.00*	*-0.01*	*0.02*	*0.01*	**422**					
-0.03	*0.01*	*-0.02*	*0.05*	*-0.01*	*-0.10*	*0.04*	*-0.01*	*0.03*	**345**				
0.01	*0.02*	*-0.09*	*-0.14*	*0.19*	*0.03*	*-0.01*	*-0.04*	*0.19*	*-0.09*	**545**			
-0.02	*0.03*	*0.08*	*0.12*	*-0.00*	*-0.01*	*0.05*	*-0.00*	*-0.00*	*0.01*	*0.26*	**476**		
0.01	*0.01*	*-0.01*	*0.03*	*0.01*	*-0.06*	*-0.08*	*0.01*	*0.10*	*-0.02*	*0.15*	*0.49*	**365**	
0.01	*0.01*	*-0.00*	*0.09*	*-0.05*	*-0.02*	*0.11*	*-0.04*	*-0.10*	*0.09*	*0.16*	*0.63*	*0.34*	**408**

MC2. March 18, 2013

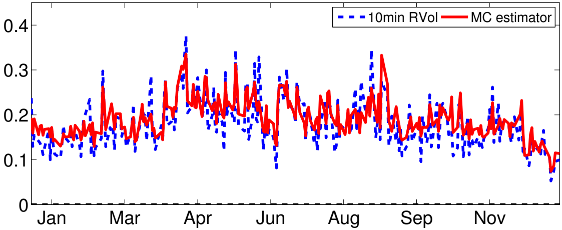
SPY	CL	NG	GC	SV	HG	LC	LH	KC	SB	CT	CN	SY	WC
117													
0.38	**392**												
-0.04	0.01	**922**											
-0.11	-0.01	0.00	**117**										
-0.04	0.10	-0.07	0.71	**380**									
0.37	0.33	0.15	0.07	0.06	**365**								
0.07	0.03	-0.09	-0.02	0.02	-0.13	**142**							
-0.02	-0.02	-0.01	0.10	0.03	0.04	0.06	**607**						
0.06	0.07	0.12	-0.12	-0.15	0.06	-0.08	0.14	**422**					
0.01	-0.02	-0.10	-0.02	0.07	0.00	0.02	0.02	0.06	**345**				
0.17	0.19	-0.08	-0.15	-0.13	0.18	-0.05	0.74	0.01	0.22	**545**			
-0.04	-0.06	0.05	0.00	-0.09	-0.02	0.04	-0.05	-0.03	0.03	0.16	**476**		
0.05	0.06	-0.02	0.02	-0.00	-0.09	-0.09	0.04	0.14	-0.08	0.04	0.57	**365**	
0.00	-0.01	-0.04	-0.01	-0.10	-0.04	0.13	0.07	-0.10	0.08	0.14	0.73	0.42	**408**

2013: RV10min & Markov

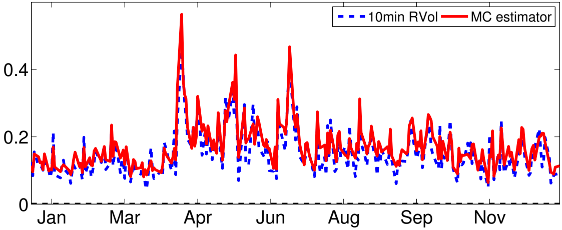
S&P 500



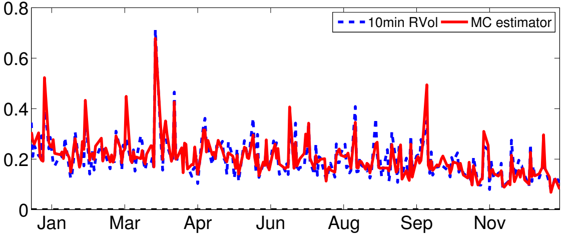
Crude Oil



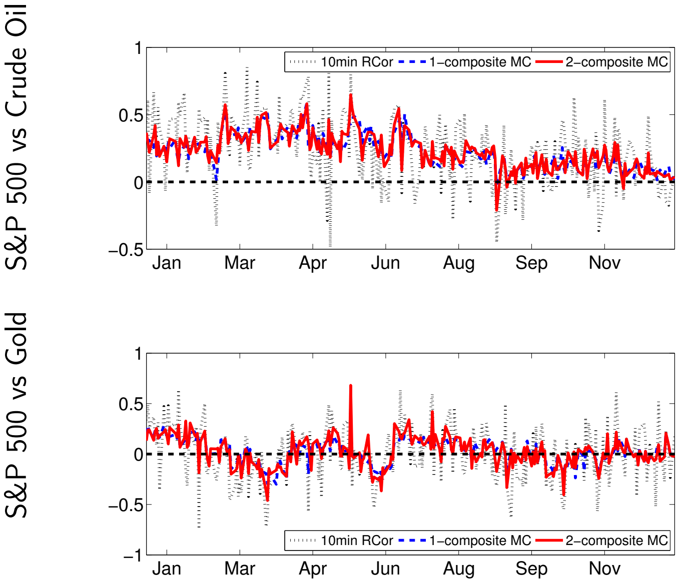
Gold


10min RVol
MC estimator
0.4
0.2
0
Jan
Mar
Apr
Jun
Aug
Sep
Nov

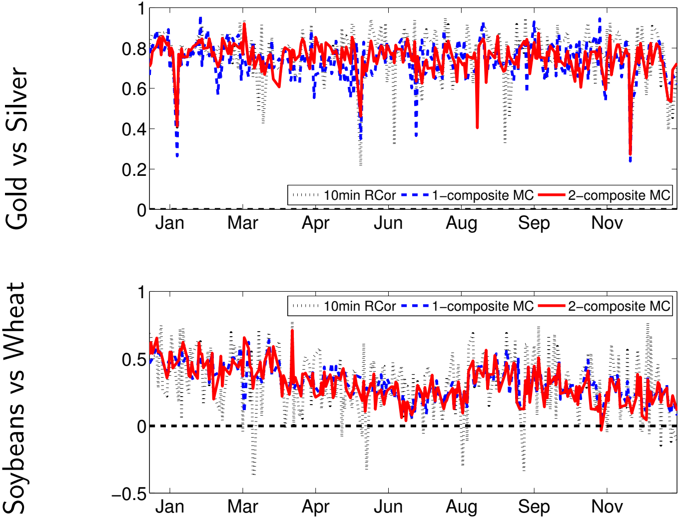
Wheat


10min RVol
MC estimator
0.8
0.6
0.4
0.2
0
Jan
Mar
Apr
Jun
Aug
Sep
Nov

2013: Correlations: RC10min & Markov



2013: Correlations: RC10min & Markov



Summary

- Markov Chain methods for Estimating Multivariate Volatility from High-Frequency Data
  - Three estimators: Full and two composite
  - Projection of covariance matrix (using standard errors)
  - Simulations: On par with Realized Kernel
  - Empirical
- Piece of larger project
  - Martingale Decomposition ✓
  - Inference about Long-Run Variance (✓)
  - Volatility estimation from high frequency data (with Horel) (✓)
  - Multivariate volatility (with Horel, Lunde, Archakov) ✓