

N. N. LEONENKO ¹

Superpositions of Ornstein-Uhlenbeck type processes: intermittency and multifractality

This is joint work with D. Denisov (Manchester University, UK), D. Grahovac (Osijek University, Croatia), A. Sikorskii (Michigan State University, USA) and M. Taqqu (Boston, University, USA).

This paper is motivated by the works of Barndorff-Nielsen [4,5] in which superpositions of stationary processes of Ornstein-Uhlenbeck (supOU) type with long-range dependence and self decomposable marginal distributions are constructed.

These processes may, in particular, have the normal inverse Gaussian distribution as one-dimensional marginal law [4]. We construct continuous time strictly stationary processes whose autocorrelation functions have a simple explicit form and exhibit long-range dependence [6].

We consider an additive functionals of supOU processes showing that they satisfied the property which we called intermittency [9,10]. This property can be interpreted also as multifractality with a scaling function of special form. To extend the analytical form of possible scaling functions we investigate the properties of multifractal products of supOU processes [1-3]. We present the general conditions for the L_q convergence [7,8] of cumulative processes to the limiting processes and investigate their q -th order moments and Rényi functions [11], which are nonlinear, hence displaying the multifractality of the processes as constructed. We also establish the corresponding multifractal scenarios for the limiting processes, such as log-normal, log-gamma, log-normal inverse Gaussian, log-tempered stable or log-normal tempered stable scenarios.

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¹Cardiff School of Mathematics, Cardiff University, Senghennydd Road, Cardiff CF24 4AG, United Kingdom

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