Lévy-based Tempo-Spatial Modelling; with Applications to Turbulence

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Abstract

This paper discusses certain types of tempo-spatial models constructed from Lévy bases. The dynamics are described by a field of stochastic processes $X = \{X_t(\sigma)\}$, on a set $\mathcal{S}$ of sites σ, defined as integrals

$$X_t(\sigma) = \int_{-\infty}^{t} \int_{\mathcal{S}} f_t(\rho, s; \sigma) Z(\mathrm{d}\rho \times \mathrm{d}s)$$

where Z denotes a Lévy basis. The integrands f are deterministic functions of the form $f_t(\rho, s; \sigma) = h_t(\rho, s; \sigma)\mathbf{1}_{A_t(\sigma)}(\rho, \sigma)$ where $h_t(\rho, s; \sigma)$ is of a special kind and $A_t(\sigma)$ is a subset of $\mathcal{S} \times \mathbb{R}_{\leq t}$.

We first consider OU (Ornstein-Uhlenbeck) fields $X_t(\sigma)$ representing several extensions of the concept of OU processes (processes of Ornstein-Uhlenbeck type), with the main focus on the potential of $X_t(\sigma)$ for dynamic modelling. Applications to dynamical spatial processes of Cox type are briefly indicated.

The second part of the paper discusses the modelling of tempo-spatial correlations of SI (stochastic intermittency) fields of the form

$$Y_t(\sigma) = \exp\{X_t(\sigma)\}.$$

This form allows for explicit calculation of expectations $\mathrm{E}\{Y_{t_1}(\sigma_1) \cdots Y_{t_n}(\sigma_n)\}$, which we use to characterise correlations. SI fields can be thought of as a dynamical, continuous and homogeneous generalisation of turbulent cascades. In this respect we construct an SI field with tempo-spatial scaling behaviour that accords with the energy dissipation observed in turbulent flows. Some parallels of this construction are also briefly sketched.