A class of spatio-temporal and causal stochastic processes, with application to multiscaling and multifractality

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Abstract

We present a general class of spatio-temporal stochastic processes describing the causal evolution of a positive-valued field in space and time. The field construction is based on independently scattered random measures of Lévy type whose weighted amplitudes are integrated within a causality cone. General n-point correlations are derived in closed form. As a special case of the general framework, we consider a causal multiscaling process in space and time in more detail. The latter is derived from, and completely specified by, power-law two-point correlations, and gives rise to scaling behaviour of both purely temporal and spatial higher-order correlations. We further establish the connection to classical multifractality and prove the multifractal nature of the coarse-grained field amplitude.