Abstracts
Progress in High Dimensional Probability
An International Conference in Honour of Jørgen Hoffmann-Jørgensen

Department of Mathematics, Aarhus University

May 27–30 2013

Programme: Starts Monday May 27 at 9.45 (registration from 9.15) and ends Thursday May 30 at 12.

Invited speakers:

- Victor H. de la Peña (Columbia University). See page 15.
- Goran Peskir (The University of Manchester). See page 16.
- Iosif Pinelis (Michigan Technological University). See page 17.
- Markus Riedle (King's College London). See page 18.
- Jan Rosiński (University of Tennessee). See page 19.
- Flemming Topsøe (University of Copenhagen). See page 20.
- Aad van der Vaart (Leiden University). See page 21.
- Michel Weber (Université Louis-Pasteur and C.N.R.S.). See page 22.
- Jon A. Wellner (University of Washington). See page 23.
- Wojbor Woyczynski (Case Western Reserve University). See page 24.

Large deviations for additive functionals of Markov chains

Alejandro de Acosta, Department of Mathematics, Case Western Reserve University

For a Markov chain $\{X_j\}$ with general state space S and $f : S \to \mathbb{R}^d$, the large deviation principle for $\{n^{-1} \sum_{j=1}^{n} f(X_j)\}$ is proved under a condition on the chain which is weaker than uniform recurrence but stronger than geometric recurrence and an integrability condition on f, for a broad class of initial distributions. This result is extended to the case when f takes values in a separable Banach space. Assuming only geometric ergodicity and under a non-degeneracy condition, a local large deviation result is proved for bounded f. A central analytical tool is the transform kernel, whose required properties, including new results, are established. The rate of function in the large deviation results is expressed in terms of the convergence parameter of the transform kernel.

Lévy processes with two-sided reflection

Søren Asmussen, Aarhus University

Doubly reflected (at two barries 0, b) stochastic processes come up in a number of applications, for example storage problems and data transmission. We look at a number of aspects for the Lévy-driven case, in particular construction (Skorokhod problems versus more pragmatic approaches), stationary distributions (given in terms of the scale function), representation of barrier overflow rates, and large buffer asymptotics ($b \to \infty$).

The talk is based on joint work with Lars Nørvang andersen, Peter Glynn and Mats Pihlsgård

The Itô–Nisio theorem for non-separable function spaces and applications

Andreas Basse-O'Connor, Aarhus University

Jørgen Hoffmann-Jørgensen has made great contributions to the study of sums of independent random variables with values in linear spaces which is also the theme of this talk. The classical Itô–Nisio theorem states that a sum of independent random variables with values in a separable Banach space converges a.s. in norm if and only if it converges in the weak topology in law. For non-separable Banach spaces the situation is much more complicated. For example, the Itô–Nisio theorem breaks down for ℓ^∞ and to the best of my knowledge there are only two known examples where it is valid, namely the space of functions of bounded variation BV_1 and the space of càdlàg functions $D[0,1]$. In this talk we will list several new examples including a class of p-variation spaces. These results relies on a new Itô–Nisio theorem for non-separable linear spaces which will be presented in Jørgen's talk together with some techniques from functional analysis. As an application we show convergence in p-variation norm of various Karhunen-Loève type representations of stochastic processes. The talk is based on joint work with Jørgen Hoffmann-Jørgensen and Jan Rosiński.

The best approximation of fractional Brownian motions by Gaussian Markov processes

Dedicated to my advisor Jorgen Hoffmann-Jorgensen

Vladimir T. Dobric, Lehigh University

Given a fractional Brownian motion process $(Z_H(t))_{t\in[0,\infty)}$with Hurst index H and its natural filtration $(\mathcal{F}_t)_{t\in[0,\infty)}$, we want to identify a Gaussian Markov process $(GM_H(t))_{t\in[0,\infty)}$, which among all Gaussian Markov process $\mathcal{G}_H$ with the same filtration as fractional Brownian motion, the best approximates $(Z_H(t))_{t\in[0,\infty)}$ in the sense that uniformly over time the relative L^2 difference between two processes is minimal, that is, we want to find the minimum point of

$$D_H^2 = \inf_{GM_H\in\mathcal{G}_H} \sup_{t\in[0,\infty)} \frac{E\left(Z_H(t) - GM_H(t)\right)^2}{EZ_H^2(t)}.$$

This problem is solved by using the duality theory developed by Dobric-Ojeda in which $(Z_H(t))_{t\in[0,\infty)}$ is imbedded into the fractional Gaussian field $Z = (Z_H(t))_{(t,H)\in[0,\infty)\times(0,1)}$ whose covariance is given by

$$E\left(Z_H\left(t\right)Z_{H'}\left(s\right)\right) = a_{H,H'}\left\{\frac{\left|t\right|^{H+H'} + \left|s\right|^{H+H'} - \left|t-s\right|^{H+H'}}{2}\right\}.$$

The process canonically defined by $X_{H,H'}\left(t\right) = \mathbb{E}\left(Z_H\left(t\right) \mid \mathcal{F}_t^{H'}\right)$, $t \geq 0$, is a martingale when (H, H') is a dual pair $(H + H' = 1)$, otherwise $X_{H,H'}$ contains a martingale component respect to $(\mathcal{F}_t^H)_{t\in[0,\infty)}$. For a dual pair the process $X_{H,H'}$ is, up to a multiplicative constant, the fundamental martingale of Molchan. Since every Gaussian Markov process in $\mathcal{G}_H$ can be written as $(\psi(t)M(t))_{t\in[0,\infty)}$, where ψ is a deterministic function and $(M(t))_{t\in[0,\infty)}$ is a martingale respect to $(\mathcal{F}_t)_{t\in[0,\infty)}$, the problem of finding the best Gaussian Markov process reduces to

$$D_H^2 = \inf_{M_H\in\mathcal{M}_H} \sup_{t\in[0,\infty)} \frac{E\left(Z_H(t) - \psi_H(t)M_H(t)\right)^2}{EZ_H^2(t)}$$

where $\mathcal{M}_H$ is the set of all Gaussian martingales respect to $(\mathcal{F}_t^H)_{t\in[0,\infty)}$ and $\psi_H(t)$ is the orthogonal projection of $Z_H(t)$ onto $M_H(t)$. The main part of

this work is to establish that the best approximation is unique and it is the martingale component of $X_{H,H'}$, where H' is determined as a unique zero of a rather complex function that contains differences of Hurwitz zeta functions (generalizations of Riemann zeta functions). Although determining H' requires numerical methods, it can be theoretically proved that if $H > 1/2$ then $H + H' > 1$, and if $H < 1/2$ then $H + H' < 1$. Therefore, the best approximation martingale is not the fundamental martingale of fractional Brownian motion but rather an interesting new fractional Brownian motion martingale. Surprisingly, D_H is for $H \in [0.4, 1]$ less then 0.1, which open doors to many applications, most notably in financial mathematics.

Update on Uniform Central Limit Theorems

R. M. Dudley, MIT

The first edition of the book appeared in 1999. A manuscript of the second edition was sent to the publisher in March 2013. Some of the main changes between the two editions are:

The second edition will no longer include double-starred sections having statements without proofs.

Added, with proofs, are:

The Bretagnolle–Massart theorem on specific constants for the Komlós–Major–Tusnády rate of convergence of classical empirical processes to the Brownian bridge;

The Massart theorem giving the precise constant in the Dvoretzky–Kiefer–Wolfowitz inequality; one step in the proof is replaced by a numerical verification;

Talagrand's generic chaining criterion for boundedness of Gaussian processes;

Haussler's sharp bound for covering numbers of Vapnik–Červonenkis classes of sets (as proved in van der Vaart and Wellner's book *Weak Convergence and Empirical Processes*);

A characterization of the uniform Glivenko–Cantelli property of a class of functions, given in a paper by Dudley, Giné, and Zinn, 1991;

Giné and Zinn's characterization of uniform Donsker classes of functions as finitely uniformly pregaussian;

Bousquet, Koltchinskii, and Panchenko's proof that the convex hull of a uniform Donsker class is uniform Donsker.

The Itô-Nisio theorem for non-separable linear spaces

Jørgen Hoffmann-Jørgensen, Aarhus University

The Itô-Nisio theorem states that an infinite sum of independent random vectors taking values in a separable Banach space converges a.s. in norm if and only if it converges weakly in law. This is not true for general non-separable spaces. However, at the Banff-conference in 2011 A. Basse-O'Connor and J. Rosinski, showed that the Itô-Nisio thorem holds for $D[0;1]$ with its sup- norm and for $BV_1[0;1]$ with its usual norm. At the conference I claimed that this could not be a coincidence and that the results must be special cases of a Itô-Nisio theorem for a certain class of non-separable linear spaces.

In a joint work with Andreas Basse-O'Connor and greatly inspired by the paper of Andreas and Jan Rosinski, we have found two large classes of non-separable linear spaces satisfying the Ito-Nisio theorem, where $D[0;1]$ belongs to the first class and $BV_1[0;1]$ belongs to the second class. Strangely enough, the two classes of linear spaces are quite different in nature. In the lecture, I will present the two classes and the precise form the Itô-Nisio theorem. The results is based on a maximal inequality of the Lévy-Ottaviani type which will be presented in the lecture. In a subsequent lecture, Andreas will show how this applies to Banach spaces and give a series of applications to norm convergence of Karuhnen-Loéve expansions.

Asymptotics for the Length in Some Longest Common and/or Increasing Subsequence Problems

Christian Houdré, Georgia Institute of Technology

I will first provide a panorama of various recent and not so recent results (due to various authors) on the asymptotics, in mean, variance and limiting law, for the length of some subsequence problems. Then, I will describe a recent result in the following framework: Let $X_1, X_2, \ldots, X_n, \ldots$ and $Y_1, Y_2, \ldots, Y_n \ldots$ be two independent sequences of iid random variables taking their values in a common ordered alphabet. Let LCI_n be the length of the longest common and increasing subsequence of $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$. As n grows without bound, and when properly centered and normalized, LCI_n is shown to converge, in distribution, towards a Brownian functional that we identify. Very classical high dimensional tools are at the root of this result.

Oracle inequalities for high dimensional vector autoregressions

Anders Bredahl Kock, Aarhus University

This paper establishes non-asymptotic oracle inequalities for the prediction error and estimation accuracy of the LASSO in stationary vector autoregressive models. These inequalities are used to establish consistency of the LASSO even when the number of parameters is of a much larger order of magnitude than the sample size. We also give conditions under which no relevant variables are excluded.

Next, non-asymptotic probabilities are given for the Adaptive LASSO to select the correct sparsity pattern. We then give conditions under which the Adaptive LASSO reveals the correct sparsity pattern asymptotically. We establish that the estimates of the non-zero coefficients are asymptotically equivalent to the oracle assisted least squares estimator. This is used to show that the rate of convergence of the estimates of the non-zero coefficients is identical to the one of least squares only including the relevant covariates. We illustrate our finite sample results by simulations as well as an application to a standard macroeconomic data set.

Low Rank Estimation on Graphs

Vladimir Koltchinskii, School of Mathematics, Georgia Institute of Technology

Estimation of large low rank matrices based on samples of linear measurements has been intensively studied in the recent years. The estimation methods are based on nuclear norm minimization in the noiseless case and on nuclear norm penalization in the noisy case. The analysis of these methods relies on a variety of tools from several areas of mathematics (convex optimization, high-dimensional probability, nonasymptotic theory of random matrices, nonparametric statistics). In this talk, we consider a version of noisy matrix completion in which a symmetric kernel is defined on the vertex set of a weighted graph and the goal is to estimate the kernel based on its i.i.d. noisy observations at randomly picked couples of vertices. The underlying assumption is that the kernel is low rank and, at the same time, smooth on the graph. The smoothness is characterized by discrete Sobolev type norms defined in terms of graph Laplacian. We obtained minimax lower bounds (in the classes of symmetric kernels of a given rank and given smoothness) on the error (measured by the Hilbert Schmidt norm) of an arbitrary estimator of the target kernel and developed estimation methods for which such bounds are attained up to log factors. The results show that, when the dimension of the problem is very large, the error rates are becoming nonparametric and they depend both on the rank of the target kernel and its smoothness.

Intersection local times, loop soups and permanental Wick powers

Michael B. Marcus, City University of New York

Several stochastic processes related to transient Levy processes with potential densities u(x,y)=u(y-x), that need not be symmetric nor bounded on the diagonal, are studied. Some of the results obtained give the known results when u(x,y) is symmetric.

Real valued processes on a space of measures V endowed with a metric d, are considered and sufficient conditions for the continuity of these processes on (V,d) are obtained. The processes studied include n-th order intersection local times of transient Levy processes and permanental chaoses, which are the loop soups of these intersection local times. The permanental chaos processes are used to define permanental Wick powers. This terminology is used because, when u(x,y) is symmetric, these are standard Wick powers, a class of n-th order Gaussian chaoses. It is shown that permanental Wick powers can also be obtained as loop soups.

On α-covariance for random vectors without second finite moment

Vygantas Paulauskas, Vilnius University

In the talk we introduce the notion of α-covariance as the measure of dependence for a wide class of random variables with infinite variance. This class include stable random vectors and random vectors in their domain of attraction, stable processes and stable random elements in separable Banach spaces. We compare α-covariance with covariation and codifference which are mainly used for the last three decades as measures of dependence for heavy tailed distributions.

From Boundary Crossing of Non-Random Functions to Boundary Crossing of Stochastic Processes

Victor H. de la Peña, Columbia University

In this talk I will introduce moment bounds for first crossing times that can be thought of as natural extensions of the concept of boundary crossing of non-random functions to the case of arbitrary stochastic processes. The results were inspired in part by the development of decoupling inequalities for randomly stopped processes with independent increments.

Joint work with Mark Brown and Tony Sit.

Jørgen Hoffmann-Jørgensen and Probability at Aarhus

Goran Peskir, The University of Manchester

I will present reminiscences of probability at Aarhus and the landmark role that Jørgen Hoffmann-Jørgensen has played in its development for over half the century.

On the nonuniform Berry–Esseen bound

Iosif Pinelis, Michigan Technological University

Due to the effort of a number of authors, the value c_u of the absolute constant factor in the uniform Berry–Esseen (BE) bound for sums of independent random variables has been gradually reduced to 0.4748 in the iid case and 0.5600 in the general case; both these values were recently obtained by Shevtsova. On the other hand, Esseen had shown that c_u cannot be less than 0.4097. Thus, the gap factor between the best known upper and lower bounds on (the least possible value of) c_u is now rather close to 1. The situation is quite different for the absolute constant factor c_{nu} in the corresponding nonuniform BE bound. Namely, the best correctly established upper bound on c_{nu} in the iid case is about 30 times the corresponding best known lower bound, and this gap factor is greater than 100 in the general case. In the present paper, improvements to the prevailing method (going back to S. Nagaev) of obtaining nonuniform BE bounds are suggested. Moreover, two new methods are presented, of a rather purely Fourier kind, each based on a family of smoothing inequalities, which work better in the tail zones. As an illustration, a quick proof of Nagaev's nonuniform BE bound is given.

Ornstein-Uhlenbeck processes driven by cylindrical Lévy processes

Markus Riedle, King's College London

In this talk we consider the stochastic Cauchy problem in a Banach space driven by a cylindrical Levy process. Here, a cylindrical Levy process is understood in the classical framework of cylindrical random variables and cylindrical measures, and thus, it can be considered as a natural generalisation of cylindrical Wiener processes or white noises. The first part of the talk is devoted to develop a stochastic integration theory for deterministic, operator-valued integrands with respect to cylindrical Levy processes and to provide necessary and sufficient conditions for a function to be integrable. In the second part, we apply the developed theory to derive the existence of a solution for the Cauchy problem and to consider spatial and temporal regularity and irregularity properties of the solution.

CLT for Wiener chaos via asymptotic independence

Jan Rosiński, University of Tennessee

The celebrated fourth moment theorem of Nualart and Peccati (2005) states that, for homogeneous Wiener chaoses of a fixed order, the convergence of the second moments to 1 and of the fourth moments to 3 implies the convergence of chaoses in distribution to the standard normal law. On the other hand, Rosinski and Samorodnitsky (1999) observed that homogeneous Wiener chaoses are independent if and only if their squares are uncorrelated.

In this talk we relate both results and show that the fourth moment theorem follows from an analogous criterion for the asymptotic independence of Wiener chaoses. Furthermore, we derive a multidimensional version of the fourth moment theorem, applicable in the study of stochastic processes, and give new bounds on the rate of convergence. We show applications to the limit theory of short and long range dependent stationary Gaussian time series involving Gaussian and non Gaussian limits. If time permits, we will also mention an extension to non-Gaussian discrete chaos. This talk is based on a joint work with Ivan Nourdin.

Jørgen Hoffmann-Jørgensen - a man to learn from

Flemming Topsøe, Department of Mathematical Sciences, University of Copenhagen

My time with Jørgen goes back to the sixties and continued through the sixties and early eighties. We focused mainly on the endeavour, jointly with Jens Peter Reus Christensen, to write a comprehensive treatise on "Topology, Measure and Borel Structure", starting with foundational issues from axiomatic set theory and comprising "all" which a working functional analyst or probabilist needed to know from measure theory in a broad sense.

Numerous were our meetings, at times with guests, in Aarhus or in Copenhagen. Typically, Jørgen lectured to us, presenting his view, mostly based on new comprehensive handwritten notes. Jørgen introduced us to axiomatic set theory and to elements of descriptive set theory. I understood how important these issues were, not only for professional mathematicians but also for the students of mathematics, something I carried with me to Copenhagen as background for much of my teaching there. I shall touch upon some of the lessons learned from the point of view of teaching through the collaboration with Jørgen.

Through a 15-year long period, I was occupied with matters related to the research infrastructure and we never finished our project as anticipated. However, Jørgen expanded on our approach, leading to the publication of numerous research papers and of his well known monograph "Probability with a View Towards Statistics".

At the "High Dimensional Probability"-conference at Sandbjerg Castle in 2002, contributions from me and from Peter Harremoës on the importance of information theory for probability were included in the programme. During the last part of my talk I will take this issue up and indicate, if only briefly, some of the issues which have an impact on probability. It is consistent with the view expressed in 1962 by Ingarden and Urbanik, and later followed up by Kolmogorov, that "information should preceede probability, not the other way round". Elements of game theory will be important in order to support this view as will philosophically inclined considerations involving notions of cognition such as "truth", "belief", "knowledge" and "perception". I may illustrate the relevance of all this by indicating some concrete examples possibly with a longer excursion into isotonic regression and/or other problems from probability, geometry and information theory proper.

Gaussian small ball probabilities and Bayesian contraction rates

Aad van der Vaart, Leiden University

Gaussian measures on Banach spaces are popular as priors in nonparametric Bayesian analysis: an unknown functional parameter is modelled as a realization from a Gaussian measure (the "prior"), observed data is considered to be realized according to a statistical model defined by this parameter, and the conditional distribution of the functional parameter given the data (the "posterior") is considered the Bayesian solution to the problem. To investigate the usefulness of this solution one can investigate whether it recovers an unknown true functional parameter if the amount of data increases indefinitely. In many statistical settings this turns out to depend on the exponent of the (noncentered) small probability of the Gaussian prior around the true functional parameter. Besides the basic result we discuss so-called adaptative Bayesian inference using a scale of Gaussian measures, linked by their reproducing kernel Hilbert spaces. [Joint work with Harry van Zanten.]

Convergence properties of systems of dilated functions: classical results and recent advances

Michel Weber, Université Louis-Pasteur and C.N.R.S.

Let f be a 1-periodic function, and let $f_n(x) = f(nx), n = 1, 2, \ldots$ be the associated dilated functions. The study of the convergence properties in mean or almost everywhere of series of averages $\sum_k c_k f_{n_k}$ or averages $\frac{1}{N}\sum_{n=1}^{N} f_n$ goes back to the 1930's. We present classical results, notably their connection with the theory of Dirichlet series, and recent advances.

Rates of convergence for the MLE's of log-concave and s-concave densities

Jon A. Wellner, Department of Statistics, University of Washington, Seattle WA

I will discuss Nonparametric Maximum Likelihood Estimates of log-concave and $s-$concave densities. In particular it will be shown that the MLE's of log-concave and s-concave densities on $\mathbb{R}$ converge at the rate $n^{2/5}$ with respect to Hellinger distance. The proofs rely on recent progress concerning entropy bounds for classes of convex functions due to Dryanov (2009) and Guntuboyina and Sen (2013) and some new "slicing metrics" together with a "dual induction" argument. In the case of $\mathbb{R}^d$ with $d \geq 2$ I will briefly discuss some recent progress and sketch a variety of open problems. (Based on joint work with Charles Doss.)

Interplay Between the Nonlinear and Nonlocal Components of Diffusions

Wojbor A. Woyczynski, Case Western Reserve University, Cleveland, Ohio

One of the motivations of our program was to develop understanding of the interplay between the nonlinear and nonlocal components in evolution equation driven by the infinitesimal generators of processes with jumps, such as Levy processes and flights. In the process we also studied the probabilistic approximations (propagation of chaos) for several extensions of the classical quasilinear and strongly linear PDEs, including the conservation laws, Hamilton-Jacobi, porous medium equation, and reaction-diffusion type equations for Darwinian evolutionary population models where the hydrodynamic limits may still preserve some "background" random noise. Some critical phenomena have been discovered.