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Infinite divisibility of sums of Gaussian squares

Let (X_1, X_2, X_3, X_4) be a mean zero four-dimensional Gaussian vector with positive definite covariance matrix Σ. Then the vector (X_1^2, X_2^2) is always infinitely divisible but (X_1^2, X_2^2, X_3^2) may fail to be. Interestingly, we regain infinite divisibility when we consider $(X_1^2, X_2^2 + X_3^2)$. A next natural step is then to consider the vector $(X_1^2 + X_2^2, X_3^2 + X_4^2)$. Additionally, studying infinite divisibility of $(X_1^2 + X_2^2, X_3^2 + X_4^2)$ may be a step in understanding infinite divisibility in the second Wiener chaos.

We will take two different approaches to this problem: first we present a readily calculated inequality on Σ that ensures infinite divisibility and second, we give a more theoretical necessary and sufficient condition for infinite divisibility. Finally, we consider some numerical considerations to gain intuition about the problem.