Applied Probability Workshop, Ilulissat 2016

Titles and abstracts, May 24, 2016

INVITED TALKS

Hansjörg Albrecher

Department of Actuarial Science, University of Lausanne, Switzerland

On capital injections and dividends in risk theory

Some recent advances in modelling capital injections and dividend payments for the surplus process of collective risk theory in insurance are presented. Several explicit formulas and identities are derived, some of which also extend tax identities developed earlier in the literature. Particular emphasis will be given on results for an underlying Lévy risk process.

This talk is based on joint research with J. Ivanovs.
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Søren Asmussen

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Studies in Improper Markov Renewal Equations

The asymptotics of the solution $Z(x)$ of the ordinary renewal equation $Z(x) = z(x) + \int_0^x Z(x-y)F(dy)$ depends crucially on $\|F\|$, the total mass of F. It is classical that $Z(x) \to C$ in the proper case $\|F\| = 1$ and that $Z(x) \sim Ce^{\gamma x}$ if either $\|F\| > 1$ (then $\gamma > 0$) or $\|F\| < 1$ and F is light-tailed (then $\gamma < 0$). For $\|F\| < 1$ and heavy-tailed F the results are surprisingly recent (Asmussen, Foss & Korshunov, 2003) and three different forms of the decay of $Z(x)$ occur according to the balance between $z(x)$ and the tail of F.

For the Markov renewal equation

$$Z_i(x) \;=\; z_i(x) + \sum_{j \in E} \int_0^x Z_j(x-y) F_{ij}(dy)$$

with a finite set of indices $i \in E$, the role of $\|F\|$ is taken by the spectral radius of the matrix $\boldsymbol{P}$ with ijth element $\|F_{ij}\|$. For the case $\rho \neq 1$, we present 1) an asymptotic expression for the analogue of γ when $\boldsymbol{P}$ is almost a transition matrix and 2) a Markov renewal version of the AFK 2003 result (as there, the concept of local subexponentiality plays a main role). The motivation arose from some computer performance models.

Joint work with Lester Lipsky, Steven Thompson and Julie Thøgersen.

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Mogens Bladt

University of Copenhagen and UNAM

Heavy Tails via Phase–Type Distributions: Construction, Applications and Estimation

In this talk we expose some of the recent and ongoing research regarding the use of infinite–dimensional phase–type distributions as model for heavy tailed phenomena. Phase–type distributions are by construction quite user–friendly for deriving exact and/or explicit formulae en stochastic modeling where probabilistic methods often can bring us further with less effort when compared to their analytic counterpart. However, they also contain a number of difficulties when for example it comes to statistical inference and they are too restrictive in certain applications due to being light tailed. We shall try to address the last issue by constructing a new class of distributions of infinite dimensional phase–type distributions, NPH, which are genuinely heavy–tailed but at the same time preserve the phase–type structure and the mathematical tractability. We ouline their construction, basic properties and applications in renewal and risk theory and show how they may be used by calibrating an infinite–dimensional hyper–exponential to a Pareto distribution. Finally we discuss issues regarding the estimation of distributions from the class NPH, which in principle can be performed with similar methods as for finite–dimensional distributions.

Jose Blanchet

Departments of IEOR, and Statistics, Columbia University, USA.

On Robust Risk Analysis

We consider the problem of maximizing an expectation over all probability measures which are within a given Optimal Transport distance of a target measure. This problem is solved explicitly for a large class of expectations of interest in great generality, and we show that the solution has a natural practical interpretation in terms of stress testing. Moreover, we provide an inference methodology for estimating the solution to these optimization problems, and compute the optimal rate of convergence of these estimators. The limit laws are non-conventional and interesting qualitative differences arise depending on the value of the dimension of the data d. By a careful choice of the distance we establish connections to popular machine learning algorithms, such as Lasso. Our results provide the foundation for data-driven robust stress testing.

This talk is based on joint work with K. Murthy and Y. Kang.

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Sergey Foss

Heriot-Watt University, Edinburgh and Novosibirsk State University

Walk in a Poisson Rain.

I will give an overview of a number of old and recent results related to a walk of a single or many walkers, either in one direction or according to the greedy policy.

The talk is partly based on a joint papers with L Rolla and V Sidoravicius, and with J Martin.

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Peter Glynn

Stanford University

TBA

Jevgenijs Ivanovs

Lausanne

Time Inhomogeneity in Longest Gap Problem with Applications to Computer Reliability

Consider an inhomogeneous Poisson process and let D be the first of its epochs which is followed by a gap of size $\ell > 0$. We establish a criterion for $D < \infty$ a.s., as well as for D being long-tailed and short-tailed, and obtain logarithmic tail asymptotics in various cases. These results are translated into the discrete time framework of independent non-stationary Bernoulli trials where the analogue of D is the waiting time for the first run of ones of length ℓ. The main motivation comes from computer reliability, where $D+\ell$ represents the actual execution time of a program or transfer of a file of size ℓ in presence of failures (epochs of the process) which necessitate restart.

Joint work with Søren Asmussen and Anders Rønn Nielsen.

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Offer Kella

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Two cute one dimensional studies

The first topic is based on the paper entitled “A Note On Integral Representations of the Skorokhod Map” written jointly with Patrick Buckingham and Brian Fralix, in which we present a very short derivation of the integral representation of the two-sided Skorokhod reflection Z of a continuous function X of bounded variation, which is a generalization of the integral

representation of the one-sided map featured in Anantharam and Konstantopoulos (2011) (and elsewhere). We also show that Z satisfies a simpler integral representation when additional conditions are imposed on X.
The second topic is based on a recent study with Liron Ravner entitled "Lowest priority waiting time distribution in an accumulating priority Lévy queue" in which we present a new method for deriving the waiting time distribution of the lowest class in an accumulating priority queue with positive Lévy input. Specifically, the priority of a customer is a function of their class and waiting time. The method relies on the construction of a workload overtaking process and showing an equivalence between the waiting time and the first hitting time of zero of a certain Lévy process.

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Thomas Mikosch

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The eigenstructure of the sample covariance matrix of heavy-tailed multivariate time series – some recent developments

In recent years the sample covariance matrix of high-dimensional vectors with iid entries has attracted a lot of attention. A deep theory exists if the entries of the vectors are iid light-tailed; the Tracy-Widom distribution typically appears as weak limit of the largest eigenvalue of the sample covariance matrix.

In the heavy-tailed case (assuming infinite 4th moments) the situation changes dramatically. Work by Soshnikov, Auffinger, Ben Arous and Péché shows that the largest eigenvalues are approximated by the points of a suitable non-homogeneous Poisson process. We follow this line of research.

We start by considering a p-dimensional time series with iid heavy-tailed entries where p is fixed. The eigenvalues of the corresponding sample covariance matrix are dominated by the diagonal elements, i.e., the sums of the squares of each row. The eigenvalues converge to independent stable limits and the eigenvectors are approximated by the canonical basis vectors. When p increases with n at polynomial rate similar behavior can be observed for the eigenvalues and eigenvectors of the sample covariance matrix. In this case, large deviation results of Nagaev-type help to show that the point process of the scaled eigenvalues converge to an inhomogeneous Poisson process. Weak convergence for the largest eigenvalues is a consequence.

We also consider the case when the multivariate time series constitute a heavy-tailed stochastic volatility random field. Also in this case, the behavior of the eigenvalues and eigenvectors of the sample covariance matrix is essentially dominated by the diagonal elements and limit theory is very similar to the iid case.

This is joint work with Richard A. Davis (Columbia Statistics), Johannes Heiny and Xiaolei Xie (Copenhagen)

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Leonardo Rojas-Nandayapa

TBA

Tomasz Rolski

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Extremes for Multivariate Brownian Motions

Consider a vector-valued Brownian motion (possible correlated) $(X_1(t), ..., X_n(t))$, $(t \geq 0)$ and let

$$\mathcal{P}(u) = \mathbf{P}\left(\exists_{t\geq 0}\forall_{i=1,\ldots,n}(X_i(t) - \mu_i t) > \alpha_i u\right),$$

where $\alpha_1, \ldots, \alpha_n > 0$ and $-\infty < \mu_1, \ldots, \mu_n < \infty$. We derive asymptotics of $\mathcal{P}(u)$ for $u \to \infty$ in form

$$\mathcal{P}(u) = Cu^{\alpha}e^{-\lambda u}.$$

The asymptotics is exact in the sense, that there appear some constants, called multidimensional Pickands' constants.

The talk is based on a joint work with Krzysztof Dębicki (University of Wroclaw), Enkelejd Hashorva (University of Lausanne), Lanpeng Ji (University of Lausanne).

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Peter Taylor
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Scale Matrices for Matrix-Analytic Models

Dating from the work of Neuts in the 1980s, the field of matrix-analytic methods has been developed to analyse discrete or continuous-time Markov chains with a two-dimensional state space in which the increment of a *level* variable is governed by an auxiliary *phase* variable. More recently, there has been considerable interest in general Markov additive models in which the level variable is continuous, and its dynamics have either bounded or unbounded variation. These models can be thought of as Markov-modulated Lévy processes.

From the Markov additive perspective, traditional matrix-analytic models can be viewed as special cases where increments in the level are constrained to be *lattice* random variables. For example, the class of $M/G/1$-*type Markov chains* in which transitions of the level variable are skip-free downward can have increments which are an integral multiple $k \geq -1$ of the distance between levels.

One object of interest in the study of one-sided scalar Lévy processes is the *scale function* which has a role in expressions for exit probabilities. The analogue of the scale function in a matrix-analytic model is interesting. If we are prepared to make strong nonsingularity assumptions about the matrices that define the model, then we can define a *scale matrix* in almost exactly the same way as in the scalar case. However, without these assumptions, it seems that there is no one object that has all the properties that we usually associate with the scale function.

In this talk, I shall discuss how we might define a scale matrix for a $M/G/1$-type Markov chain.

Joint work with Jevgenijs Ivanovs (University of Lausanne(and Guy Latouche (Université Libre de Bruxelles)

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CONTRIBUTED TALKS

Mohan Chaudhry
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Analytically elegant and computationally efficient results in terms of roots for the GIX/M/c queueing system

An elegant and simple solution to determine the distributions of queue length at different epochs and the waiting time for the model GIX/M/c is presented. In the past, the model GIX//M/c has been extensively analyzed using various techniques by many authors. The purpose of this paper is to present a simple and effective derivation of the analytic solution for pre-arrival epoch probabilities as a linear combination of specific geometric terms (except for the boundary probabilities when the number of servers is greater than the maximum batch size) involving the roots of the underlying characteristic equation. The solution is then leveraged to compute the waiting-time distributions of both first and arbitrary customers of an incoming batch. Numerical examples with various arrival patterns and batch size distributions are also presented. The method that is being proposed here not only gives an alternate solution to the existing methods, but it is also analytically simple, easy to implement, and computationally efficient.

Philip Ernst
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TBA

Suppose you have one unit of stock, currently worth 1, which you must sell before time T. The Optional Sampling Theorem tells us that whatever stopping time we choose to sell, the expected discounted value we get when we sell will be 1. Suppose however that we are able to see a units of time into the future, and base our stopping rule on that; we should be able to do better than expected value 1. But how much better can we do? And how would we exploit the additional information? The optimal solution to this problem will never be found, but in this paper we establish remarkably close bounds on the value of the problem, and we derive a fairly simple exercise rule that manages to extract most of the value of foresight.

Joint work with L.C.G. Rogers & Quan Zhou

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Kumiko Hattori

A Family of Self-Avoiding Processes on a Fractal

Non Markov processes are generally difficult to study, but on some fractal spaces self-similarity enables us to obtain rigorous results. We show that the 'erasing-larger-loops-first' method, which was first introduced for erasing loops from the simple random walk on the Sierpinski gasket, does work also for non-Markov random walks, in particular, self-repelling walks to construct a new family of self-avoiding walks on the Sierpinski gasket. The one-parameter family constructed in this method continuously connects the loop-erased random walk, first introduced by Lawler on integer lattices, and a self-avoiding walk which has the same asymptotic behavior as the 'standard' self-avoiding walk. We prove the existence of the scaling limit and study some path properties: The exponent governing the short-time behavior of the scaling limit varies continuously in the parameter. The limit process is almost surely self-avoiding, while it has path Hausdorff dimension strictly greater than 1. The theory of multi-type branching processes plays an important role in our study.

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Offer Kella

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Rodrigo Labouriau

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Statistics talk with graphical models as main ingredient.

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Patrick Laub

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Zbynek Pawlas

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Limit Theorems for Line Segment Processes

Line segment processes are one of the models used in stochastic geometry. They have been studied for a number of applications, where systems of segments randomly scattered in space occur. The segments may represent geological faults, textile structure or stress fibres in cells. They can have random length and random orientation (that are not necessarily independent). The simplest model is the Poisson line segment process where the segments are scattered in a completely random way. In this case the lengths and orientations are independent of segment locations. Such processes are called independently marked. We investigate asymptotic behaviour for different nonparametric estimators of summary characteristics of stationary independently marked line segment processes. The estimators are based on a single realization of the process observed within a bounded sampling window.

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Gustavo Schwenkler

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Efficient Parameter Estimation for Multivariate Jump-Diffusions

This paper develops an unbiased Monte Carlo estimator of the transition density of a multivariate jump-diffusion process. The drift, volatility, jump intensity, and jump magnitude are allowed to be state-dependent and non-affine. It is not necessary that the volatility matrix can be diagonalized using a change of variable or change of time. Our density estimator facilitates the parametric estimation of multivariate jump-diffusion models based on discretely observed data. Under conditions that can be verified with our density estimator, the parameter estimators we propose have the same asymptotic behavior as maximum likelihood estimators as the number of data points grows, but the observation frequency of the data is kept fixed. In a numerical case study of practical relevance, our density and parameter estimators are found to be highly accurate and computationally efficient.

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Seva Shneer

Global and Local Stability of Multi-Dimensional Markov Chains

We will look at multi-dimensional Markov chains where the components may be dependent but one of them has a predictable behaviour (is, in some sense, stable or unstable). We will be focused on finding conditions sufficient for the other components (or the entire chain) to be positive recurrent. Processes with the properties described in this talk appear naturally in a number of applications and we will discuss examples of these applications.

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Peter Straka

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A Numeric Scheme for Fractal Semi-Markov Processes

Anomalous Diffusion is a well-studied phenomenon in statistical physics. It is typically modeled by the scaling limit of a Continuous Time Random Walk (CTRW). These constitute a less-well known example of Semi-Markov processes which have infinitely many renewals in finite time intervals (the renewal set is a fractal). We study these Semi-Markov processes and show how they can be embedded into a higher-dimensional Markov process. We then derive master equations for this Markov process which allow for a discrete numeric scheme, and thus for the computation of finite-dimensional distributions of CTRW limit processes.

Joint work with Ricky Gill

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Robyn M. Stuart

University of Copenhagen

Cheeger Inequalities for Substochastic Markov Chains

The connection between the spectra of Markov chain transition matrices and the conductance of the Markov chain is well-established for stochastic Markov chains. In this work, we extend this relationship to the substochastic setting by deriving new bounds on the second eigenvalue of a substochastic Markov chain in terms of the conductance. Our results extend existing analogous results for stochastic Markov chains, and are consistent with these well-known results when the transition matrix is stochastic. If P is reversible with respect to the quasi-stationary distribution of the Markov chain, the bounds on the the second eigenvalue rely only on the largest eigenvalue and the conductance. We also derive some results for the case where P is not reversible, and we show that the bounds can be rearranged to give bounds on the conductance and metastability of a substochastic Markov chain, both in the situation where this Markov chain satisfies detailed balance condition as well as general substochastic transition matrices.

With Gary Froyland.

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Dawid Tarlowski

... Poland

Nonautonomous Dynamical Systems in Stochastic Global Optimization

We focus on discrete time inhomogeneous Markov recursions of the form $X_{t+1} = T_t(X_t; Y_t)$, where Y_t is an independent sequence. Given the continuous objective function $f : A \to [0; 1)$, where A is a metric space, we target the problem how to prove the convergence of the process X_t towards the set A^* of global minima of f(we assume $A^* \neq \emptyset$). Our methodology is to view the X_t as a nonautonomous dynamical system on Borel probability measures on A and to use the Lyapunov function technique. We present general convergence theorem and provide some practical examples, including the Simulated Annealing Algorithm.

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