Low-frequency estimation for moving-average Lévy processes

Vladimir Panov

Higher School of Economics (Moscow, Russia)

Joint work with Denis Belomestny

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Setup

Moving-average Lévy process is a stochastic integral

$$\boxed{Z_t = \int_{\mathbb{R}} \mathcal{K}(t-s)dL_s,} \qquad \text{where}$$

- **L_t is a Lévy process on $\mathbb{R}$**: (1) $X_0 = 0$ a.s.; (2) independent and stationary increments; (3) stochastic continuity: $X_{t+s} \xrightarrow{\mathbb{P}} X_t$ as $s \to 0$.

  Typical construction: $L_t = L^1_t \mathbb{I}\{t \geq 0\} - L^2_{(-t\text{-})}\mathbb{I}\{t < 0\}$, where L^1 and L^2 are two independent copies of a Lévy process on $\mathbb{R}_+$.
- **$\mathcal{K}(\cdot) : \mathbb{R} \to \mathbb{R}$ is a deterministic function**, $\mathcal{K} \in \mathcal{L}^1(\mathbb{R}) \cap \mathcal{L}^2(\mathbb{R})$.
  Rajput & Rosiński (1989): if $L_s \sim (\mu, 0, \nu)$, then Z_t exists iff

$$\int_{\mathbb{R}} \Bigg[\mathcal{K}(t-s)\mu + \int_{\mathbb{R}} \mathcal{K}(t-s)x \left(\mathbb{I}\{|\mathcal{K}(t-s)x| \leq 1\} - \mathbb{I}\{|x| \leq 1\} \right) \nu(dx) + \int_{\mathbb{R}} \left(\mathcal{K}(t-s)x\right)^2 \nu(dx) \Bigg] ds < \infty.$$

Typical example: $\mathcal{K}(x) = |x|^r e^{-\rho|x|}$ with $r > -1/2$ and $\rho > 0$.

Moving-average process: motivation

Lévy-driven Ornstein-Uhlenbeck process:

$$Z_t = e^{-\rho t} Z_0 + \int_0^t e^{-\rho(t-s)} dL_s \qquad \text{with} \quad \rho > 0.$$

Well-balanced Ornstein-Uhlenbeck process (*Schnurr & Woerner, 2011*):

$$\mathcal{K}(x) = e^{-\rho|x|} \quad \Longrightarrow \quad Z_t = \int_{\mathbb{R}} e^{-\rho|t-s|} dL_s.$$

More general set-up: $Z_t = \int_{\mathbb{R}} K(t,s) dL_s$, where, for instance,

$$K(t,s) := \frac{1}{\Gamma(1+\gamma)} \left[(t-s)_+^{\gamma} - (-s)_+^{\gamma} \right]$$

with $\gamma \in (0, 1/2)$ (fractional Lévy process).

Barndorff-Nielsen & Schmiegel (2009), Basse & Pedersen (2009), Podolskij (2014), Barndorff-Nielsen, Benth & Veraart (2015), Basse-O'Connor & Rosiński (2016)

Statistical problem

Stationarity of the process $Z_t = \int_{\mathbb{R}} \mathcal{K}(t-s) dL_s$:

Characteristic function $\Phi(u) := \mathbb{E} e^{\mathrm{i} u Z_t}$ is equal to

$$\boxed{\Phi(u) = \exp\left\{ \int_{\mathbb{R}} \psi\left(u \mathcal{K}(x)\right) dx \right\},}$$

where $\psi(u)$ is the characteristic exponent of L_t. For instance, if L_t has Lévy triplet (μ, σ, ν), and the jump part has finite variation, then

$$\psi(u) = t^{-1} \log\left(\mathbb{E}\left[e^{\mathrm{i} u L_t} \right] \right) = \mathrm{i} \mu u - \frac{1}{2} \sigma^2 u^2 + \int_{\mathbb{R}} \left(e^{\mathrm{i} u x} - 1 \right) \nu(x) dx.$$

Important: Z_t is an integral over $\mathbb{R}$.

Statistical problem: estimate $\nu(x)$ from $Z_0, Z_\Delta, ..., Z_{n\Delta}$ with fixed $\Delta > 0$.

Previous research:

- *Brockwell & Lindner (2012)* - inference for Lévy-driven OU and CARMA;
- *Cohen & Lindner (2013), Zhang, Lin & Zhang (2015)* - estimation of the parameters of the kernel function (empirical moments, least squares);
- *Glaser (2015), Basse-O'Connor, Lachieze-Rey & Podolskij (2015)* - limit theorems for the power variation (useful for high-frequency data).

Direct appoach

Special choice of $\mathcal{K}$. Let $\mathcal{K}$ be a symmetric kernel of the form:

$$\mathcal{K}_\alpha(x) := (1 - \alpha|x|)^{\frac{1}{\alpha}}, \quad |x| \le \alpha^{-1}$$

for some $\alpha \in (0,1)$. In this case, $\mathcal{K}'_\alpha(x) = -\mathcal{K}_\alpha^{1-\alpha}(x)$.

Direct relation between ψ and Φ.

$$\boxed{\psi(u) = \frac{1}{2} u^{1-\alpha} \left(u^\alpha \log(\Phi(u))\right)' = \frac{1}{2}\left(\alpha \log(\Phi(u)) + u\frac{\Phi'(u)}{\Phi(u)}\right).}$$

Assumptions on the model

$$L_t = \mu t + \sigma W_t + CPP_t^{(1)} \cdot \mathbb{I}\{t \ge 0\} + CPP_{-t}^{(2)} \cdot \mathbb{I}\{t < 0\}, \quad CPP_t^{(j)} := \sum_{k=1}^{N_t^{(j)}} \xi_k^{(j)},$$

where $N_t^{(1)}, N_t^{(2)}$, are 2 Poisson processes with intensity λ, $\xi_1^{(1)}, \xi_2^{(1)}, \ldots$ and $\xi_1^{(2)}, \xi_2^{(2)}, \ldots$ are i.i.d. r.v's with absolutely continuous distribution.

Direct approach: estimation procedure

1. **Estimation of Φ, ψ.**

$$\Phi_n(u) := \frac{1}{n} \sum_{j=1}^{n} e^{iuZ_{j\Delta}}, \qquad \psi_n(u) = \frac{1}{2} \left(\alpha \log(\Phi_n(u)) + u \frac{\Phi_n'(u)}{\Phi_n(u)} \right).$$

2. **Estimation of σ, λ, μ.** Since

$$\boxed{\psi(u) = \mathrm{i}\mu u - \frac{1}{2}\sigma^2 u^2 - \lambda + \mathcal{F}[\nu](u),}$$

we define

$$\begin{aligned}
(\sigma_n^2, \lambda_n) \quad &:= \quad \operatorname*{arg\,min}_{(\sigma^2,\lambda)} \int_0^\infty w^{U_n}(u)(\operatorname{Re}\psi_n(u) + \sigma^2 u^2/2 + \lambda)^2 \, du \\
\mu_n \quad &:= \quad \operatorname*{arg\,min}_{\mu} \int_0^\infty w^{U_n}(u)(\operatorname{Im}\psi_n(u) - \mu u)^2 \, du.
\end{aligned}$$

3. **Estimation of ν:**

$$\nu_n(x) := \mathcal{F}^{-1}\left[\left(\psi_n(\cdot) - i\mu_n[\cdot] + \tfrac{\sigma_n^2}{2}[\cdot]^2 + \lambda_n\right) w_\nu(\cdot/U_n)\right](x), \quad x \in \mathbb{R}.$$

Direct approach: theoretical results

Consider the class

$$\mathcal{T}_s = \mathcal{T}_s(\sigma^\circ, R) = \left\{ \sigma \in (0, \sigma^\circ), \ \int x^2 \nu(dx) \leq R, \ \left\| \nu^{(s)} \right\|_\infty \leq R \right\}$$

with some $\sigma^\circ, R > 0$.

Upper bound. If $\|\mathcal{F}(w_\sigma^1(u)/u^s)\|_{L^1} < \infty$, then it holds

$$\boxed{\lim_{A \to +\infty} \overline{\lim_{n \to +\infty}} \sup_{(\gamma,\sigma,\nu) \in \mathcal{T}_s} \mathbb{P}\left\{ \left| \sigma_n^2 - \sigma^2 \right| \geq A \cdot U_n^{-(s+3)} \right\} = 0.}$$

provided $U_n = \sqrt{\kappa \log(n)}$ with some constant $\kappa > 0$ depending on σ° and R.

Lower bound. For any $\sigma^\circ, R > 0$, there exists some $A > 0$ such that

$$\boxed{\underline{\lim}_{n \to +\infty} \inf_{\breve{\sigma}_n} \sup_{(\gamma,\sigma,\nu) \in \mathcal{T}_s} \mathbb{P}\left\{ \left| \breve{\sigma}_n^2 - \sigma^2 \right| \geq A \cdot (\log(n))^{-(s+3)/2} \right\} > 0,}$$

where the infimum is taken over all possible estimates $\breve{\sigma}_n, \breve{\gamma}_n, \breve{\lambda}_n$ of the parameters σ, γ, λ, and supremum - over all triplets from the class $\mathcal{T}_s$.

Mellin transform $\mathcal{M}[f](z) := \int_{\mathbb{R}_+} x^{z-1} f(x) dx, \; z \in \mathbb{C}$.

Analyticity: If $f(x) = \begin{cases} O(x^{-a-\varepsilon}) & \text{as } x \to 0+ \\ O(x^{-b+\varepsilon}) & \text{as } x \to +\infty \end{cases}$, where $\varepsilon > 0$, $a < b$, then $\mathcal{M}[f]$ is analytic in the region $\{z : \; a < \mathrm{Re}(z) < b\}$.

Parsenval formula: $\int_{\mathbb{R}_+} f(x) g(x) dx = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \mathcal{M}[f](z) \cdot \mathcal{M}[g](1-z) dz.$

Inverse Mellin transform: $f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} x^{-z} \cdot \mathcal{M}[f](z) dz.$

Multiplicative convolution:

$$f \star g(u) := \int_{\mathbb{R}_+} f(ux) g(x) dx \quad \Longrightarrow \quad \mathcal{M}[f \star g](z) = \mathcal{M}[f](z) \, \mathcal{M}[g](1-z).$$

Superposition of Mellin and Fourier transforms

$$\mathcal{M}[\mathcal{F}[f]](z) = \mathcal{M}\left[e^{ix}\right](z) \cdot \mathcal{M}\left[f\right](1-z).$$

Inference in general case: main idea

Denote $\Lambda(u) := \log(\Phi(u))$, and consider the Mellin transform of $\Lambda''(u)$:

$$\mathcal{M}\left[\Lambda''\right](z) = -\mathcal{M}\left[\mathcal{F}[\bar{\nu}]\right](z) \cdot \int_{\mathbb{R}} (\mathcal{K}(x))^{2-z}\, dx, \qquad \text{where } \bar{\nu}(x) = x^2 \cdot \nu(x).$$

$$\boxed{\mathcal{M}[\bar{\nu}](1-z) = \frac{\mathcal{M}[\Lambda''](1-z)}{Q(1-z)},} \qquad \text{with} \quad Q(z) := -\Gamma(z) e^{\mathrm{i}\pi z/2} \int_{\mathbb{R}} (\mathcal{K}(x))^{2-z}\, dx.$$

Estimation scheme

1. $Z_0, Z_\Delta, \ldots., Z_{n\Delta} \quad \longrightarrow \quad \Phi \quad \longrightarrow \quad \mathcal{M}[\Lambda'']$:

$$\mathcal{M}_n[\Lambda''](1-z) := \int_0^{U_n} \left[\frac{\Phi_n''(u)}{\Phi_n(u)} - \left(\frac{\Phi_n'(u)}{\Phi_n(u)} \right)^2 \right] u^{-z}\, du,$$

where $\Phi_n(u) := n^{-1} \sum_{k=1}^n \exp\{\mathrm{i} Z_{k\Delta} u\}$ and a sequence $U_n \to \infty$.

2. $\mathcal{M}[\Lambda''] \quad \longrightarrow \quad \mathcal{M}[\bar{\nu}] \quad \longrightarrow \quad \nu$:

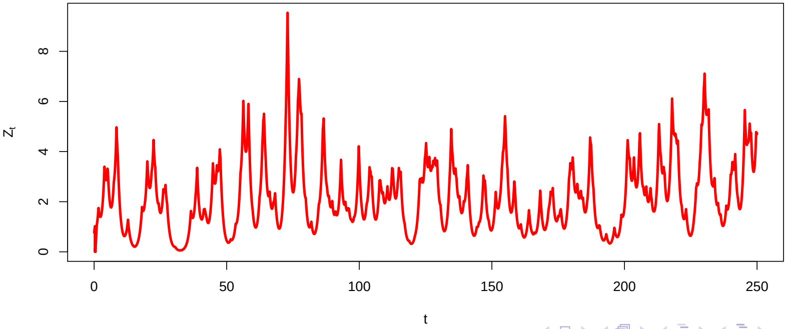
$$\bar{\nu}_n(x) := \frac{1}{2\pi \mathrm{i}} \int_{c-\mathrm{i}V_n}^{c+\mathrm{i}V_n} \frac{\mathcal{M}_n[\Lambda''](1-z)}{Q(1-z)} x^{-z}\, dz,$$

where $V_n \to \infty$ and $c \in (0,1)$.

Example

Consider the integral $Z_t := \int_{\mathbb{R}} \mathcal{K}(t-s)\, dL_s$, where

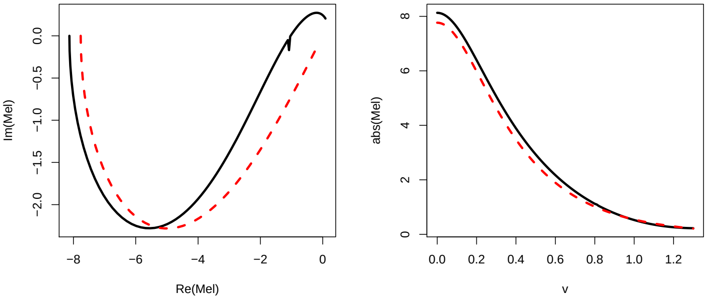
1. $\mathcal{K}(x) = e^{-|x|}, x \in \mathbb{R}$;
2. L_t is constructed from the compound Poisson process $L_t^{(1)} \overset{d}{=} L_t^{(2)} \overset{d}{=} \sum_{k=1}^{N_t} \xi_k$, where N_t is a Poisson process with intensity λ, and $\xi_1, \xi_2, \ldots$ are independent r.v.'s with standard exponential distribution.



Estimation of $\mathcal{M}[\Lambda']$: practice

We estimate the Mellin transform $\mathcal{M}[\Lambda'](1-z) = \int_0^\infty \frac{\Phi'(u)}{\Phi(u)} u^{-z}\, du$ at the points $z = c + \mathrm{i}v$, $v = -V_n..V_n$, with step $\delta = 2V_n/K$:

$$\mathcal{M}_n[\Lambda'](1-z) := \mathrm{i} \int_0^{U_n} \left[\frac{\mathrm{mean}(Z_{k\Delta} e^{\mathrm{i}uZ_{k\Delta}})}{\mathrm{mean}(e^{\mathrm{i}uZ_{k\Delta}})} - \mathrm{mean}(Z) e^{\mathrm{i}u} \right] u^{-z} du$$
$$+ 2\mathrm{i}\lambda\Gamma(1-z)\exp\{\mathrm{i}\pi(1-z)/2\}.$$

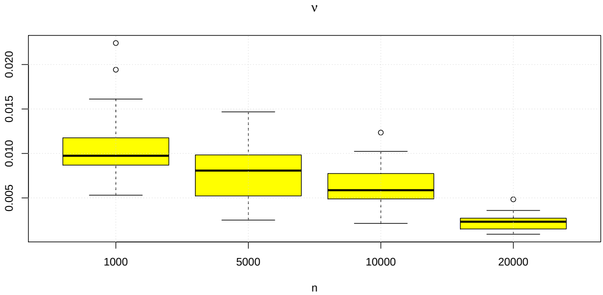


Estimation of $\nu(x)$: practice

Next, we estimate the Lévy density $\nu(x)$ by

$$\nu_n(x) := \frac{\delta}{2\pi x}\sum_{k=1}^{K} \mathrm{Re}\left\{ \frac{\mathcal{M}_n[\Lambda'](1-c-\mathrm{i}v_k)}{\widetilde{Q}(1-c-\mathrm{i}v)} x^{-(c+\mathrm{i}v_k)} \right\},$$

where $\widetilde{Q}(z) = \mathrm{i}\Gamma(z)\exp\{\mathrm{i}\pi z/2\}\int_{\mathbb{R}} (K(x))^{1-z}\,dx$, and measure the quality in terms of $\mathcal{R}(\nu_n) = \int_1^3 (\nu_n^\star(x) - \nu(x))^2\,dx$.



Theoretical study: main difficulty

Exponential inequality for i.i.d. (*Hoeffding's inequality, 1963*)
Let $X_1, X_2, ..$ are centered, i.i.d, and $\exists\, M : \ |X_1| \leq M$ a.s. Then for any $\zeta > 0$,

$$\mathbb{P}\Big\{\sum_{i=1}^{n} X_i \leq \zeta\Big\} \quad \geq \quad 1 - 2\exp\Big\{-\frac{\zeta^2}{2nM^2}\Big\}.$$

Dependence concept: α-mixing. For sigma-algebras $\mathcal{B}$ and $\mathcal{C}$,

$$\alpha(\mathcal{B},\mathcal{C}) \quad := \quad \sup_{B\subset\mathcal{B},C\subset\mathcal{C}} \left|\mathbb{P}\{B \cap C\} - \mathbb{P}\{B\}\mathbb{P}\{C\}\right|$$

A sequence $X_1, X_2, ...$ is said to be exponentially α-mixing, if

$$\alpha_k := \sup_{t\in\mathbb{Z}} \alpha\left(\sigma\left(X_s, s \leq t\right), \sigma\left(X_s, s \geq t+k\right)\right) \leq e^{-k\alpha^*} \longrightarrow 0, \qquad \text{as } k \to \infty.$$

Exponential inequalities (*Merlevède, Peligrad, Rio, 2009*)
Let $X_1, X_2, ..$ are centered, exponentially α-mixing, and $|X_k| \leq M, \forall k$ a.s. Then

$$\mathbb{P}\Big\{\sum_{i=1}^{n} X_i \leq \zeta\Big\} \quad \geq \quad 1 - \exp\Big\{-\frac{C\zeta^2}{nv^2 + M^2 + M\zeta\log^2(n)}\Big\}.$$

for all $\zeta > 0$ and $n \geq 4$, where $v^2 = \sup_i \left(\mathbb{E}[X_i]^2 + 2\sum_{j\geq i} \mathrm{Cov}(X_i, X_j)\right)$.

Mixing for multidimensional diffusions with jumps: *Masuda (2007)*

Assumptions on the Levy density ν

ν is supported on $\mathbb{R}_+$, fulfills $\int_0^1 x\nu(x)dx < \infty$, and for some $A > 0$, $\alpha \in (0,1), \gamma > 0, c \in (0,1)$, it holds

$$\max\left\{ \int_{\mathbb{R}} (1+|y|)^{\alpha} \left|\mathcal{F}[\overline{\nu}](y)\right| dy, \quad \int_{\mathbb{R}} e^{\gamma|u|} \left|\mathcal{M}[\overline{\nu}](c+\mathrm{i}u)\right| du \right\} \leq A,$$

Moreover, assume that for some $R > 0$,

$$\int_{x>1} e^{Rx}\nu(x)dx \leq A.$$

Typical example: $\nu(x) = \sum_{j=1}^{J} a_j x^{-\eta_j-1} e^{-\lambda_j x} \cdot \mathbb{I}\left\{x \geq 0\right\},$
where $a_j > 0,\ \eta_j < 1, \lambda_j > 0, \ \ \forall j \in 1..J.$

For instance, tempered stable process: $J = 1$ and $\eta_1 \in (0,1)$.

Theorem 1. Upper bound for $\overline{\nu}_n(x)$

Consider the event

$$\mathcal{A}_K := \left\{ \max_{j=0,1,2} \left\| \frac{\Phi_n^{(j)}(u) - \Phi^{(j)}(u)}{\Phi(u)} \right\|_{U_n} \leq K\varepsilon_n \right\},$$

for some $K > 0$, *where for any function* f *on* $\mathbb{R}$, $\|f\|_{U_n} = \sup_{u \in [-U_n, U_n]} |f(u)|$, *and* $\varepsilon_n \to 0$ *is such that*

$$K\varepsilon_n \left(1 + \|\Psi'_\sigma\|_{U_n}\right) \leq 1/2.$$

On the set $\mathcal{A}_K$, *the estimate* $\overline{\nu}_n(x)$ *satisfies*

$$\sup_{x \in \mathbb{R}_+} \{x^c |\overline{\nu}_n(x) - \overline{\nu}(x)|\} \leq \frac{1}{2\pi} \int_{\{|v| \leq V_n\}} \frac{\Omega_n}{|Q(1 - c - \mathrm{i}v)|} \, dv + \frac{A}{2\pi} e^{-\gamma V_n},$$

where

$$\Omega_n = 2K\varepsilon_n U_n^{1-c} + \left(A + \frac{2^\alpha A}{1-c}\right) \int_{\mathbb{R}} [\mathcal{K}(x)]^{c+1} [1 + U_n \mathcal{K}(x)]^{-\alpha} \, dx.$$

Assumptions on the kernel function $\mathcal{K}$

The kernel $\mathcal{K} \in \mathcal{L}^1(\mathbb{R}) \cap \mathcal{L}^2(\mathbb{R})$ satisfies

$$\sum_{j=-\infty}^{\infty} \left| \mathcal{F}[\mathcal{K}] \left(2\pi \frac{j}{\Delta} \right) \right| \leq K^*,$$

$$(\mathcal{K} \star \mathcal{K})(\Delta j) \leq \kappa_0 \, |j|^{\kappa_1} e^{-\kappa_2 |j|}, \qquad \forall j \in \mathbb{Z}$$

for some positive constants K^*, κ_0, κ_1 and κ_2, and moreover all eigenvalues of the matrix

$$\mathcal{M} = \Big((\mathcal{K} \star \mathcal{K})(\Delta(j-k)) \Big)_{k,j \in \mathbb{Z}}$$

are bounded from below and above by two finite positive constants.

Typical example: $\mathcal{K}(x) = \left(\sum_{r=0}^{R} \beta_r |x|^r \right) e^{-\rho |x|}$,
where $\beta_r \geq 0$ for all $r = 0, \ldots, R$, and $\rho > 0$.

Theorem 2. Probability of the event $\mathcal{A}_K$

Under the choice

$$\varepsilon_n = \sqrt{\frac{\log(n)}{n}} \cdot \exp\left(\frac{A}{2}\sigma^2 U_n^2 \int (\mathcal{K}(x))^2\, dx\right),$$

it holds for any $K > 0$

$$\boxed{\mathbb{P}\left\{\mathcal{A}_K\right\} \geq 1 - \frac{C_1}{\sqrt{K}} \frac{\sqrt{U_n} n^{(1/4) - C_2 K^2}}{\log^{1/4}(n)},}$$

where the positive constants C_1, C_2 *may depend on* K^*, A_R *and* κ_i, $i = 0, 1, 2$.

Upper bound for $\overline{\nu}_n(x)$: example

Consider the following $\mathcal{K}$ and ν :

$$\mathcal{K}(x) = \left(\sum_{r=0}^{R} \beta_r |x|^r \right) e^{-\rho|x|}, \qquad \nu(x) = \sum_{j=1}^{J} a_j x^{-\eta_j - 1} e^{-\lambda_j x} \cdot \mathbb{I}\{x \geq 0\}.$$

We have on $\mathcal{A}_K$

$$\sup_{x\in\mathbb{R}_+} \{x^c |\overline{\nu}_n(x) - \overline{\nu}(x)|\} \lesssim V_n^{\zeta} \left(\varepsilon_n U_n^{(1-c)} + U_n^{-\alpha} \right) + e^{-\gamma V_n},$$

with $\zeta = c + 1 + \mathbb{I}(R = 0)/2$.Therefore, under the choice

$$U_n = \theta \log^{1/2}(n), \qquad V_n = \varkappa \log(U_n),$$

with $\varkappa > \alpha/\gamma, \theta < \left(A \int (\mathcal{K}(x))^2 \, dx\right)^{-1/2}$, we get that the rates are logarithmic on $\mathcal{A}_K$:

$$\boxed{\sup_{x\in\mathbb{R}_+} \{x^c |\overline{\nu}_n(x) - \overline{\nu}(x)|\} \lesssim \log^{-\alpha/2}(n), \quad n \to \infty.}$$

Summary

- We consider stochastic integrals of the type

$$Z_t = \int_{\mathbb{R}} \mathcal{K}(t-s) dL_s,$$

  $\mathcal{K}$ - known deterministic function, L - unknown Lévy process on $\mathbb{R}$.
- Application of rather standard ideas is possible for
  $\mathcal{K}_\alpha(x) := (1-\alpha|x|)^{\frac{1}{\alpha}}, \quad |x| \leq \alpha^{-1}$ with some $\alpha \in (0,1)$.
- General case: Mellin transform for the derivative of $\Lambda(u) := \log(\Phi(u))$

$$\mathcal{M}\left[\Lambda''\right](z) = -\mathcal{M}\left[\mathcal{F}[\overline{\nu}]\right](z) \cdot \int_{\mathbb{R}} (\mathcal{K}(x))^{2-z} \, dx.$$

- Main challenge from theoretical point of view: mixing properties.
- D. Belomestny, V.Panov, and J.Woerner
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Thank you for your attention.