Modern Methods in Insurance Pricing and Industrial Statistics

Book of Abstracts

Buali Sina University

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Book of Abstracts

Conference on
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Sep 3-5, 2017
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Preface

Dear Participants, Colleagues, and Friends,

WELCOME to the conference on Modern Methods in Insurance Pricing and Industrial Statistics. We are delighted to celebrate the continued success of our society, and its work to draw together the international community of statisticians, both academics and industry professionals, who share our goal of making statistics the foundation for decision making in business and related applications.

We have put together an interesting programme with a number of keynote lectures, talks and posters, covering a broad range of topics in insurance pricing and industrial statistics.

There will be eight keynote lectures opening the three conference days and five parallel sessions with 84 talks. The talks will be held either in English or in Farsi. A large number of posters - 33 altogether - will be presented.

On Monday afternoon, there will be an excursion to the water cave and to Lalejin, the center of pottery and ceramic productions in the Middle East, which will be completed by a dinner in a traditional restaurant.

We would like to thank the local organizing committee who made a great effort in making every participant's attendance to the conference as pleasant as possible, as well as all of you who will contribute to the success of the conference by presenting talks, posters or leading workshops before and during the conference.
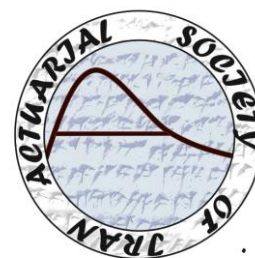
Hamedan is a great city, and the host institution has been gracious and generous. We hope you will have a wonderful time and enjoy a productive conference.

Rahim Mahmoudvand, Søren Asmussen and Kristina Lurz
On behalf of the MIPIS 2017 Programme Committee

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Keynote Speakers

- Hansjörg Albrecher, University of Lausanne, Switzerland
- Søren Asmussen, Aarhus University, Denmark
- Alireza Edalati, Actuary at VHV Insurance AG, Germany
- Amin Hassanzadeh, Shahid Beheshti University, Iran
- David Rios Insua, Rey Juan Carlos University, Spain
- Gholamreza Nakhaeizadeh, Karlsruhe Institute of Technology, Germany
- Jens Perch Nielsen, CASS Business School, United Kingdom
- Amir Safari, Insurance Research Center, Iran
- Ali Shojaie, University of Washington

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KEYNOTE LECTURES

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An Adversarial Risk Analysis Framework to Cyber security

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Abstract

Cyberthreats affect all kinds of organizations, from private companies to public administrations, including also critical infrastructures. Risk analysis is an essential tool to address them as it allows organizations to deal with the threats affecting them, prioritize the defence of their assets and decide what countermeasures should be implemented. Many risk analysis methods are present in cybersecurity control models, compliance frameworks and international standards. However, most of them are less than satisfactory, focusing on risk matrix based approaches. After outlining one of the main existing proposals in this field as an example, a comprehensive framework for risk analysis in cybersecurity will be proposed including the presence of adversarial threats and the use of insurance as part of the security portfolio. A case study outlining the proposed framework is presented.

Keywords

Risk analysis, cybersecurity.

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Bayesian Ideas for Premium Strategies

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Abstract

The traditional control parameters in determining the strategy of an insurance company are dividends and reinsurance arrangements. Premiums are obviously of equal practical importance, but theoretical studies are rarer. We use here the traditional Bayesian view of the risk parameters of the insured (say the Poisson rate of generating car accidents) to be randomly fluctuating in the portfolio. This allows quantifying the influence of the premium level on the portfolio size and thereby approaching control problems such as minimizing the ruin probability. Also deductibles are in part involved. Further a game theoretic perspective via differential games in a competitive market is outlined.

Keywords

Bayesian approach, Insurance Company, Dividend, Reinsurance.

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Cat Risks and Insurance in Iran

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Abstract

Iran is one of the most natural disaster-prone countries in the world. Flood, earthquake, drought, storms could result is devastating damages, casualties, physical and mental damages as well as financial loss to victims. This can change victims' life and make it difficult or in some cases impossible to get recovered.

Historical experiences show that following the occurrence of natural disasters, not only people suffer from the tragic consequences but also governments suffer from the imposed financial pressures. Basically, after a catastrophe, governments often intervene in the compensation of catastrophe victims which causes significant budgetary pressures. Relying on state financial supports within circumstances in which the country faces large destruction of property and loss of life, is not a reasonable solution. Instead, insurance is an appropriate financial mechanism which can efficiently compensate for significant amount of losses. Now the question is, when an insurance product can safely covers a catastrophic event? As I mentioned earlier, catastrophes are associated with huge financial losses. Therefore in order to attract insures' interest in offering products that cover catastrophes, a high standard and reliable risk assessment is needed. Insurers should have some clear idea regarding the risk type, extent of damages and the estimation over the probable financial loss. For example since 1950, natural disasters caused two insurance companies become insolvent in Canada. We can find other examples in different countries as well. In 1906, an earthquake struck San Francisco through which around 80 percent of San Francisco was destroyed and 12 insurance companies failed. Hurricane Andrew is another example of a catastrophe which happened in 1992, Florida and as a result nine insurance companies failed.

In fact hurricane Andrews in 1992 and the Northridge Earthquake in 1994 changed the insurance industry's view of natural catastrophes. Since then, catastrophe modelling became a fundamental element of almost every insurance company. Basically in case of catastrophes, since they occur rarely, unfortunately we do not access significant historical data and loss information. Therefore traditional actuarial techniques are not suitable. The basic components that almost all recent catastrophic models contain are "Hazards" which roughly speaking is characteristics of a particular risk such as earthquake. "Data exposure" which mainly is the location of each property at risk. "Vulnerability" which quantifies the extent of damages on the property at risk. And finally "Loss" which estimates the financial loss that the property may face based on a given hazard. However it is important to note that it is also critical to come up with a premium which is to some extent affordable for customers.

Keywords

Catastrophe, Insurance Industry, Iran.

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Improved customer in on-line customer communication and its statistical implications

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Abstract

One of the crucial challenges of modern financial advice is to find a reliable and fast way of assessing the risk profile of a pension customer. We will go through the implications of failed financial advice and we introduce a new hedging methodology that can minimise the risk of such failure. A future vision is to add market timing based on statistical estimation of the dynamics of the market. The talk will describe these ongoing statistical challenges and give guidance to how such market timing could be combined with our new communication and hedging strategy.

Keywords

Pension customer, risk assessment, hedging, Statistical challenges.

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Inference in High Dimensions with the LASSO

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Abstract

In recent years, a great deal of interest has focused on conducting inference on the parameters in a linear model in the high-dimensional setting. We review the challenges and two main ideas behind the recently proposed methods, one that has focused on inference based on a sub-model selected by the lasso, and the other that has focused on inference using a debiased version of the lasso estimator. We then consider a simple and very naive two-step procedure for this task, in which we (i) fit a lasso model in order to obtain a subset of the variables; and (ii) fit a least squares model on the lasso-selected set. Conventional statistical wisdom tells us that we cannot make use of the standard statistical inference tools for the resulting least squares model (such as confidence intervals and p-values), since we peeked at the data twice: once in running the lasso, and again in fitting the least squares model. However, we show that under a certain set of assumptions, with high probability, the set of variables selected by the lasso is deterministic. Consequently, the naive two-step approach can yield confidence intervals that have asymptotically correct coverage, as well as p-values with proper Type-I error control. Furthermore, this two-step approach unifies two existing camps of work on high-dimensional inference.

Keywords

Confidence interval, Lasso, p-value, Post-selection inference, Significance testing.

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From Statistics to Machine Learning and Big Data Mining: Application Cases from Automotive Industry

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Abstract

In the last three decades, the modern technologies like machine learning, data mining and big data mining have been applied in different commercial and industrial fields, specially, in the data reach automotive industry. These modern technologies benefit, however, from statistics which is a relative older discipline.

The amount of the structured and unstructured data that are generated and collected in the automotive industry is huge with ascending trend. The data are produced during different phases of the vehicle lifecycle like development, production, sales and aftersales. A part of the generated data in the social networks can be attractive for the automotive industry as well.

Supply chain management, quality and safety management, customer relationship management and some aspects of risk management are some areas for application of data mining and big data mining in the automotive industry. Data mining and big data mining will play even a more important role in future of the automotive industry, specially, in conjunction with the topics like autonomous driving, Industry 4.0 and emission reduction management.

The talk outlines the application potential of data mining and big data mining in the automotive industry by presenting a couple of application cases and gives a perspective about the future development.

Keywords

Machin Learning, Big data, Data mining.

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Modelling Joint Lifetimes of Couples by Using Bivariate Phase-type Distribution

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Abstract

Many insurance products and pension plans provide benefits which are related to couples, and thus under influence of the survival status of two lives. Some studies show the future lifetime of couples is correlated. Three reasons are available to confirm this fact: (1) catastrophe events that affect both lives, (2) the impact of spousal death and (3) the long-term association due to common life style. Regardless of these reasons, this dependence could have a financial impact on insurance companies and pension plans providers. In this article, we use a health index called physiological age to require a model based on Markov process that reflects reasons of impacts. Under this model, future joint life time of couples follow a bivariate phase-type distribution (BPH). The model has physical interpretation and closed-form expressions for some quantities and tractable computation for other ones. We use the model to pricing some products, relevant to couples annuities and life insurances, to show the effect of dependence on pricing.

Keywords

Bivariate Phase-type Distribution, Pension plans.

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On the Optimality of Reinsurance Forms

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Abstract

Reinsurance is an important ingredient in the risk management of insurance companies. Over the last decades there has been an enormous academic activity on identifying optimal reinsurance forms under given objectives and constraints. At the same time, in practice additional factors and constraints play a role that is sometimes difficult to formalize. In this talk some recent developments in trying to narrow the gap between academic research and practical viewpoints on the topic will be discussed, with a particular emphasis on the role of capital. This will also lead to the identification of the potential attractiveness of some non-standard reinsurance forms

Keywords

Reinsurance, academic research, practice.

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The German Insurance Industry – Review and Perspectives

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Abstract

The German insurance industry-having 539 active insurance companies and investment portfolio of 1,509 Milliard Euro in year 2015-is one of the most developed and competitive insurance market of the world. Beside insurance companies there are various parties contributing to this market such as independent auditors, software companies, societies and associations, regulatory institutions etc.

The aim of this talk is to give an overview of different aspects of this market, especially clarify the role of actuaries in this industry. Therefore it contains a statistical summary of the insurance market, presents different legal forms of insurance companies, the most used software and technologies and the well-known societies.

Keywords

Germany, Insurance Industry, Software, Market.

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A Non-Parametric Approach to Clustering Multiple Multivariate Time Series

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Abstract

Many Time series data are ubiquitous. As such, they have motivated many works in machine learning and data analysis fields for classification and clustering of temporal data. While several clustering methods for univariate time series exist, and a few for multivariate series, most are based on distance and/or dissimilarity measures that do not utilize the time dependency information that is inherent to time series data. To explore and highlight the main dynamic structure of a set of multivariate time series, we extend the use of standard variance-covariance matrices for non-time series data in principal component analysis to the use of cross-autocorrelation matrices at time lags k = 1,2,.... This is also achieved by combining the principles of both canonical correlation analysis and principal component analysis for time series to obtain a new type of covariance/correlation matrix for a principal component analysis to produce a so-called principal component time series. Simulations and real data are used to study the effectiveness of the new procedure to show the benefits of the extended autocorrelation and cross-autocorrelation functions in exploring the main structural features of multiple time series.

Keywords

Multiple Multivariate Time Series, Principal Component Analysis

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Deep Neural Networks Revolution

Vahid Partovi Nia

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Abstract

Statisticians look at prediction as a wighted average over extended attribute space. Examples range from highly interpretable models like ARIMA, GARCH, linear regression, to more sort of black box techniques such as principal component regression, kernel method, smoothing splines, regression tree, random forest, and project pursuit. However, a deep neural network provides a totally different insight: repeated projection-cuts trained over a large set of data. Neural networks has been around for a while, but cheap parallel computing hardware such the GPUs and cloud servers has given this method a considerable push ahead, specially for big data. I argue that deep learning is not only an artificial intelligence tool, but can be considered as a strong candidate in all modeling problems that aims at promoting model prediction accuracy over model interpretation power. It is still difficult to answer why deep learning works so well, but I will provide some intuitions that emphasizes on projection-cut as a remedy to the curse of dimensionality. The performance of deep learning on certain applications is largely ahead of other methods, and this is why most industries are interested to invest on adopting this method for their prediction challenges. I see a great potential for deep learning in insurance industry to automate customer experience, fraud investigation, claim processing, and so on.

Keywords

Deep learning, convolutional neural network, projection pursuit regression, recurrent networks, stochastic gradient descent.

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Insurance and Finance: New Problems, New Directions

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Abstract

Over the last decades, there has been a consistently increasing growth of insured losses due to nature-related events which led the insurance and finance industries to develop alternative risk transfer products. This talk aims to propose a Natural Multi-Hazard Risk Assessment framework for the quantification of financial and insurance risks arising from multiple natural hazards. To set up this framework one needs to study four major topics related to risk assessment: modeling natural hazard risks, insurance/re-insurance markets, financial tools and financial markets. The proposed framework manages the multi-hazard risk by transferring it to insurance and financial markets by designing new products that alleviate part of the risk by implementing securitization mechanisms to achieve access to adequate funds within financial markets.

Keywords

Insurance, Finanace, Risk Assesment.

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Model Selection in an Automated Forecasting System

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Abstract

A common field where forecasting is applied in industrial companies is demand forecasting. Not only are forecasts updated regularly, e.g. weekly or monthly, but there is often a high number of time series to be forecasted, e.g. the demand of all products of the company in every market the company serves. Looking at each time series individually is not feasible, which is why an automated forecasting system is needed. To be able to deal with all possibly occurring cases, the forecasting system needs to contain a sufficient selection of forecasting methods appropriate for the different kinds of time series. Both classical time series methods as well as machine learning (ML) methods are useful. For machine learning methods, k-fold cross validation (see [1]) can be applied for model selection and tuning of hyperparameters, as ML methods do not rely on a specific order and completeness of the observations as long as the predictors are correctly assigned. k-fold cross validation, however, is not applicable for most time series methods, and [2] encourages the use of information criteria for model selection, if available. Furthermore, the forecast accuracy measure used for model selection plays a crucial role in what method is considered the best and not all forecast accuracy measures fit all kinds of time series. We describe and discuss our experiences in model selection for forecasting and present some ideas of how to deal with the challenges arising from the different types of time series and forecasting methods. We encourage to sometimes refrain from the best forecasting method in terms of an accuracy measure in order to maintain major time series characteristics.

Keywords:

Automated forecasting system; Time series; Machine learning; Accuracy measures

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Robust Singular Spectrum Analysis: an Application to the Energy Sector

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Abstract

Singular spectrum analysis (SSA) is a non-parametric method for time series analysis and forecasting that incorporates elements of classical time series analysis, multivariate statistics, multivariate geometry, dynamical systems and signal processing, and can be seen as an alternative to the classical methods. Although SSA has proved its usefulness and advantages over classical methods, one of the steps of the SSA algorithm is the singular value decomposition (SVD) of the trajectory matrix which, being a least squares method, is highly sensitive to data contamination and the presence of even a single outlier, if extreme, may draw the leading principal component towards itself resulting in possible misinterpretations and in turn lead to bad practical decisions.

In this paper we propose a robust alternative to SSA, which uses a robust SVD algorithm, that that overcomes the potential fragility of its classical version when the data are contaminated. The SSA and the robust SSA are compared in terms of quality of the model fit and forecasting. This is done by using Monte Carlo simulations and real data from the energy sector.

Keywords

Singular spectrum analysis; robust singular value decomposition; forecasting, energy.

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Tail Behavior of Randomly Weighted Infinite Sums

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Abstract

Let X_1, X_2,…be a sequence of pairwise asymptotically independent and real-valued random variables, and θ_1, θ_2,…be another sequence of nonnegative and nondegenerate at zero r.v.s., independent of X_1, X_2, ... In this paper, we investigate the tail behavior of randomly weighted infinite sums under the assumption that $S_\infty^+ = \sum_{n=1}^{\infty} \theta_n X_n^+$, where $X_n^+ = \max\{X_n, 0\}$, has a consistently varying tail, as well as some moment conditions on θ_1, θ_2,…. Our obtained result is more easily verifiable than some existing ones restricting the conditions on X_1, X_2,….

Keywords

Asymptotic, tail behavior, consistent variation; asymptotic independence, randomly weighted sum.

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CONTRIBUTED TALKS

Actuarial Calculation of Iranian Social Security Organization as of 21 March 2014

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Abstract

To determine whether or not the Iran social security organization is sustainable over the long-term, periodic actuarial reviews are conducted. Also the policies and decisions that are being implemented in a social insurance plan undoubtedly will bring results and consequences in the future.

This article is being conducted at a time when many social security schemes around the world are reforming their systems. Such changes have become necessary to counter the effects of ageing populations, projected cash shortfalls and declining public confidence in these programs.

In IRAN we face similar circumstances - falling birth rates, increasing life expectancy special among the elderly, a contribution rate that is below the average cost of benefits and a pensioner population that is growing at a faster rate than the number of contributors. Along with a review of the Fund's position as of march 21th, 2014, this report includes projections of ISSO income, expenditure and reserves through 2090.

We use ILO-PEN Model of International Labor Organization and real data to calculating actuarial aspects of ISSO.

Keywords

Social security, Pension, Actuarial evaluation in Social Security.

1- Introduction

To determine whether or not the Iran social security organization is sustainable over the long-term, periodic actuarial reviews are conducted. Also the policies and decisions that are being implemented in a social insurance plan undoubtedly will bring results and consequences in the future

This Actuarial Review of The Social Security Organization is being conducted at a time when many social security schemes around the world are reforming their systems. Such changes have become necessary to counter the effects of ageing populations, projected cash shortfalls and declining public confidence in these programs.

Along with a review of the Fund's position as of march 21th, 2014, this report includes projections of ISSO income, expenditure and reserves through 2090. Since the estimation of future experience is uncertain and depends on many demographic and financial assumptions, three scenarios are presented to show the plausible range of likely outcomes.

2- Methodology

A comprehensive methodology developed at the ILO for reviewing the long-term actuarial and financial status of a national pension scheme. The review has been undertaken by modifying the generic version of the ILO modelling tools to fit the specific case of IRAN. These modelling tools include a population model, an economic model, a labor force model, a wage model and a long-term benefits model.

The actuarial valuation begins with a projection of IRAN's future demographic and economic environment. Next, projection factors specifically related to ISSO are determined and used in combination with the demographic/economic framework to estimate future cash flows and reserves. Assumption selection takes into account both recent experience and future expectations with emphasis placed on long-term trends rather than giving undue weight to recent experience.

3- General Population Projection

The official results of the last national census indicate a population of 75151665 persons in 2011, compared with 70495782 in 2006. This increase of more than 4600000 persons exceeds.

Highlights of the population projections are:

- The total population is expected to increase by 35% to 106416556 in 2090.
- The under-16 population will decline from 19,595,685 in 2015 to 20,663,874 in 2090.
- The number of working aged persons will increase for most of the projection period, reaching a maximum of 65,069,770 in 2040 and declining slowly thereafter.
- The number of persons aged 65 and over will increase more than 5 times from just under 4,443,465 in 2015 to over 22,467,480in 2090.
- Because of population ageing the proportion of the population aged 65 and over will increase from 5.6% to 22% in 2090.

Presently, the IRAN's population is relatively young. However, between 2015 and 2090, the number of working aged persons for each person of pension age is projected to decrease from 12.3 to 2.8. For the ISSO, where pension payments to the elderly already represent more than half of benefit payments, and contributions from workers are needed to meet expenditure, population ageing has significant long-term consequences.

4- Economic and Labor Market Projections

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As contribution income is primarily based on the earnings of employed persons, economic and labor market activity directly affect ISSO finances. Projections of the economy and labor force are necessary, therefore, to estimate the number of employed persons and total insurable earnings in each projection year.

During the last 10 years IRAN economy has averaged annual real GDP growth rate to 3.5%.

Unemployment rates have declined in recent years to 11% in 2014, and the rate of inflation has been averaging 19.2% over the past 10 years.

5- ISSO Financial & Demographic Projections

In this section we present and analyze projections of ISSO finances up to 2090. The purpose of these projections is twofold. First, they are used to identify long-term trends for contributions, benefits and the reserve, so that the financial viability of the ISSO may be assessed. Secondly, by using these projections as a base, the sensitivity of the results to changes in the assumptions, and contribution and benefit provisions, may be identified. The projected ageing of the general population is also noticeable in ISSO demographic projections. As shown table 3, the number of contributors is only expected to increase from 13,603,296 to 17,397,561 while the number of pensioners is projected to increase nearly 6.5 times, to 13,103,994.

6- Projected Benefit Costs

The cost of ISSO benefits and administrative expenditure may be viewed from several perspectives. Firstly, each year's total expenditure can be expressed as a percentage of that year's insurable wages. This is often referred to as the pay-as-you-go rate and is the answer to the question "what contribution rate is required to exactly meet that year's expenditure?" Therefore, investment income, and eventually proceeds from the sale of assets, will be required to meet benefit payments and administrative costs. If the Fund becomes depleted, there would be no investment income, and thus contribution rates of almost 25% in 2074 would be required to meet current expenditure.

7- Conclusions

The key results of the projections are:

- The ageing of the general population will have a major impact on the ratio of workers to retirees. It is projected that the number of ISSO will be from 13,603,396   in 2014 to 17,397,561   in 2090.
- For the entire projection period annual expenditure is projected to exceed that year's contribution income.
- Benefit expenditure will increase from 2.4% of GDP in 2014 to 50% of GDP in 2090.
- The pay-as-you-go-rate in 2050, or the rate required to produce just enough contribution income to meet expenditure if there is no Fund, will be 28.5%. This rate will decrease gradually to almost 20.5% in 2090.
- Between 2014 and 2090 the present value of total expenditure is projected to exceed the present value of contributions plus current assets.

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- The contribution rate beginning 2014 that will make the present value of contributions equal to the present value of expenditure through 2090 is 20.51%.

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A Modified Economic Statistical Design of the VSS T^2 Control Chart

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Abstract

An economic statistical design model for a T^2 control chart which uses a variable sample size feature is developed in this paper. The determination of the optimal design requires the computation of the average number of samples (ANS) and the average number of items (ANI) taken when the process is in control or out of control. In this article, these characteristics have been calculated via a new approach. Application of the proposed VSS T^2 control chart design is demonstrated through a numerical example.

Keywords

T^2 control chart; Variable Sample Size (VSS); Economic Statistical Design

1- Introduction

A control chart, which uses this traditional approach to sampling, will be called a fixed ratio sampling (FRS) control chart. Variable sample size (VSS) control charts vary the sample size as a function of the process data. Another approach to control chart design is based on explicitly specifying the various cost and process parameters and then choosing the sampling and chart parameters to minimize the expected cost per hour. This economic design approach was originally developed by Duncan (1956) and a number of papers have since considered this approach. Woodall (1986) has criticized the economic design of control charts, noting that in many economic designs the type I error of the control chart is considerably higher than it usually would be in a statistical design, and that this will lead to more false alarms—an undesirable situation. Saniga (1989) has reported such a study relating to the joint economic statistical design of $\overline{\text{X}}$ and R charts. Saniga uses constraints on type I error, power, and the average time to signal for the charts. Saniga and Shirland (1977) and Chiu and Wetherill (1975) report that very few practitioners have implemented economic models for the design of control charts. This is somewhat surprising, as most quality engineers claim that a major objective in the use of statistical process-control procedures is to reduce costs. Hotelling (1974) did the original work on multivariate quality control using the widely known Hotelling's T^2 control chart. One procedure to improve the statistical performance and the economic benefits of the FRST^2chart is a Variable Sample Size (VSS) scheme that varies the sampling size as a function of prior sample results. The determination of the optimal design requires the computation of the average number of samples (ANS) and the average number of items (ANI)

taken when the process is in control or out of control. In this article, these characteristics have been calculated via a new approach.

2- VSS T^2 control scheme and Markov chain approach

In order to control a process with p correlated characteristics using the T^2 scheme, it is first assumed that the joint probability distribution of the quality characteristics is a p-variate normal distribution with in-control mean vector $\boldsymbol{\acute{\mu}_0} = (\mu_{01}, \dots, \mu_{0p})$ and variance-covariance matrixΣ, denoted as$X \sim N_p(\boldsymbol{\mu_0}, \Sigma)$. Then the subgroups (each of size n) statistics$T_i^2 = n(\overline{X}_i - \boldsymbol{\mu_0})'\Sigma^{-1}(\overline{X}_i - \boldsymbol{\mu_0})$ are plotted in sequential order to form the T^2 control chart. The chart signals as soon as$T_i^2 \geq k$.

In this paper, it is assumed that the process starts in a state of statistical control with mean vector $(\boldsymbol{\mu_0})$ and covariance matrix Σ and then after a while assignable causes occur resulting in a shift in the process mean$(\boldsymbol{\mu_1})$. The magnitude of the shift is measured by

$$d = \sqrt{(\mu_1 - \mu_0)'\Sigma^{-1}(\mu_1 - \mu_0)}$$

One procedure to improve the statistical performance of the $FRST^2$ chart is a Variable Sample Size (VSS) scheme that varies the sampling size as a function of prior sample results. Let N_j be the size of the sample taken at the jth sampling time and let $\overline{X}_j$be the sample mean vector. Then an T^2-chart can be maintained by plotting the $T_j^2 = N_j(\overline{X}_j - \boldsymbol{\mu_0})'\Sigma^{-1}(\overline{X}_j - \boldsymbol{\mu_0})$ at each time j on a chart with control limit k. In $FRST^2$ charts, the sample size is a fixed constant, say n. In the VSS scheme, however, the size N_j depends on the previous sample statisticT_{j-1}^2. Let the interval $(0, k)$ be partitioned into 2 regions $(0, w), (w, k)$ such that

$$N_j = \begin{cases} n_1, & 0 < T_{j-1}^2 < w \\ n_2, & w < T_{j-1}^2 < k \end{cases} \tag{1}$$

where $n_1 \leq n_2$.

2.1 *Performance indicators*

The assignable cause is assumed to occur according to a Poisson process with an intensity of λ occurrences per hour. Let Y be the time interval between sample points just prior to the occurrence of the assignable cause and the occurrence itself. Therefore, given the occurrence of the assignable cause between the jth and $(j+1)$st samples, the expected time of occurrence within this interval is

$$\tau = E(Y) = \frac{\int_{jh}^{(j+1)h} \lambda(t - jh)e^{-\lambda t}dt}{\int_{jh}^{(j+1)h} \lambda e^{-\lambda t}dt} = \frac{1 - (1 + \lambda h)e^{-\lambda h}}{\lambda(1 - e^{-\lambda h})} \tag{2}$$

Let S_o be the number of samples taken when the process is in control. Then

$$ANS_0 = E(S_0) = E\left(\frac{\frac{1}{\lambda} - Y}{h}\right) = \frac{\frac{1}{\lambda} - \tau}{h} = \frac{e^{-\lambda h}}{1 - e^{-\lambda h}} \tag{3}$$

Lemma 2.1. Let I_0 be the number of items taken when the process is in control. Then the following is true:

$$ANI_0 = E(I_0) = \frac{e^{-\lambda h}}{1 - e^{-\lambda h}}\{p_0 e^{-\lambda h}n_1 + (1 - p_0 e^{-\lambda h})n_2\} \tag{4}$$

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where $p_0 = F(w, p, 0)$, and $\alpha = 1 - F(k, p, 0)$.

The expected number of false alarms generated during a cycle is α times the expected number of samples taken before the shift, or

$$ANF = \alpha \frac{e^{-\lambda h}}{1 - e^{-\lambda h}} \tag{5}$$

Lemma 2.2. Let ANS_1 and ANI_1 be the average number of samples and items taken when the process is out of control, respectively. Then the following is true:

$$ANS_1 = \frac{1 + a_2 - a_1 - (b_2 - b_1)(1 - p_0 q)}{(1 - b_2)(1 - a_1) + (1 - b_1)a_2} \tag{6}$$

$$ANI_1 = \frac{[n_2(1 - b_1) - n_1(1 - b_2)]p_0 q + (1 - b_2 + a_2)n_1 + (b_1 - a_1)n_2}{(1 - b_2)(1 - a_1) + (1 - b_1)a_2} \tag{7}$$

Where $q = e^{-\lambda h}, a_1 = F(w, p, \eta_1), a_2 = F(w, p, \eta_2), b_1 = F(k, p, \eta_1), b_2 = F(k, p, \eta_2), p_0 = F(w, p, 0)$.

Furthermore, let $AATS$ be the expected time since occurrence of an assignable cause until an alarm is given. It is clear that, $AATS = h(ANS_1) - \tau$

3- Cost Model Development

The process cycle consists of the following phases: in control, out of control, assignable cause detection, and repair. Therefore, the expected length of a production cycle is given by

$$E(T) = \frac{1}{\lambda} + T_0 ANF + AATS + T_1 \tag{8}$$

Where T_0 is the average amount of time wasted searching for the assignable cause when the process is in control and T_1 is the average time to find and remove the assignable cause. The expected net profit from a production cycle is given by

$$E(C) = V_0\left(1/\lambda\right) + V_1(AATS) - C_0 ANF - C_1 - S \times (ANI_0 + ANI_1) \tag{9}$$

where V_0 is the average profit per hour earned when the process is operating in control, V_1 is the average profit per hour earned when the process is operating out of control, C_0 is the average consequence cost of a false alarm, C_1 is the average cost to detect and remove the assignable cause and S the cost per inspected item. The loss function $E(L)$ is given by

$$E(L) = V_0 - \frac{E(C)}{E(T)} = V_0 - \frac{V_0/\lambda + V_1 AATS - C_0 ANF - C_1 - S \times (ANI_0 + ANI_1)}{\frac{1}{\lambda} + T_0 ANF + AATS + T_1} \tag{10}$$

4- Solution to the cost model

The goal of economic design of the VSST[2] control chart is to find the five chart parameters (k, w, n_1, n_2, h) which minimize (14), given the five process parameters $(p, \lambda, d, T_0, T_1)$ and the five cost parameters (C_0, C_1, V_0, V_1, S). Therefore, the general optimization problem is defined as follows:

$$\begin{aligned} &minE(L) \\ &s.t: \\ &0 \le w < k \\ &1 \le n_1 < n_2 \\ &h \le 8 \\ &n_1, n_2 \in Z^+ \end{aligned} \tag{11}$$

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5- An application of the model

5.1 An example

In this section, the model application is illustrated through an industrial example. Consider a product with two important quality characteristics that should be monitored jointly($p = 2$). The estimated cost associated with the inspected item is \$5($S = 5$). The process mean shift occurs every 20 hours of operation which reasonably can be modeled with an exponential distribution with parameter λ=0.01. The average time to investigate an out-of-control signal and repairing the process is 1 hours($T_1 = 1$), while the time spent to investigate a false alarm is 2 hours($T_0 = 2.5$). The cost of investigating a true out-of-control signal is \$500($C_1 = 500$), while the cost of investigating a false alarm is \$500($C_0 = 500$). The estimated profit per hour earned when the process is operating in control is \$500 per hour($V_0 = 500$). The expected profit per hour earned when the process is operating out of control is \$50 per hour($V_1 = 50$).To obtain an ESD, statistical constraints are added which may be expressed in terms of $ANF \leq 0.5$ and $AATS \leq 2$. Clearly, it is desirable to have small ANF and $AATS$ values for offering the best protection against false alarms and detecting process shifts as quickly as possible.

Table 1 The optimal parameters of ESD of the FRS and VSS schemes for different values of d

d	FRS scheme						VSS scheme								%
	k	h	n	ANF	AATS	E(L)	k	w	h	n_1	n_2	ANF	AATS	E(L)	
0.50	8.36	3.01	59	0.50	2.00	121.61	16.85	9.67	0.88	1	123	0.02	2.00	41.85	65.58
0.75	8.36	3.01	26	0.50	2.00	68.84	17.79	9.58	0.96	1	58	0.01	1.47	29.67	56.90
1.00	**9.48**	**3.06**	**16**	**0.28**	**2.00**	**49.51**	**18.30**	**9.62**	**1.03**	**1**	**34**	**0.01**	**1.18**	**24.52**	**50.47**
1.25	10.41	3.10	11	0.17	2.00	39.63	18.63	9.76	1.11	1	23	0.01	1.03	21.81	44.97
1.5	11.17	3.12	9	0.12	2.00	33.88	18.93	9.97	1.18	1	17	0.01	0.93	20.20	40.38
1.75	11.80	3.16	7	0.09	2.00	30.22	19.21	10.22	1.25	1	13	0.01	0.88	19.17	36.56
2.00	12.35	3.16	5	0.06	2.00	27.75	19.50	10.51	1.30	1	10	0.00	0.84	18.49	33.37
2.25	12.84	3.01	4	0.05	1.90	25.97	19.76	10.82	1.34	1	8	0.00	0.82	18.02	30.63
2.50	13.28	2.75	4	0.05	1.72	24.54	20.01	11.14	1.38	1	7	0.00	0.80	17.68	27.96
2.75	13.67	2.53	3	0.04	1.58	23.36	20.23	11.48	1.40	1	6	0.00	0.79	17.44	25.35
3.00	14.03	2.34	3	0.04	1.46	22.35	20.42	11.84	1.42	1	5	0.00	0.78	17.25	22.83

Table 1 gives the optimal design vector for different values of mean shift $d = 0.5(0.25)3$ with a cost comparison to the corresponding optimal *FRS* and *VSS* schemes. The percent reduction in cost (denoted as %) of the *VSS* chart relative to the *FRS* chart is also given in the table. The percent reductions in cost range from 22.35 to 65.58. Considering the process working 8 hours a day, 6 days a week and 26 days a month; the *VSS* scheme helps practitioners to save more than \$62375 annually for the mean shifts of d = 1, just for one production line.

6- Concluding remarks

This paper has developed a $VSS\ T^2$ control chart in the context of an economic statistical design model. A cost model was derived by the Markov Chain approach, and genetic algorithms were applied to find the optimal design parameters. The design procedure and sensitivity analysis on the process parameters and cost parameters are illustrated through an industrial application. The results indicate that the *VSS* scheme outperform the *FRS* schemes and on the average approximately 50% more savings per hour can be achieved by applying the VSS scheme. While the similar studies on

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economic statistical design of $VSS\ T^2$ control chart had reported this extent of saving to be less than 15%.

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A Simulation Study on Non-homogeneous Bivariate Compound Poisson Model with Short-Term Periodicity

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Abstract

In this paper, we study the nonhomogeneous bivariate compound Poisson process (NHBCPP) which the intensity function of Poisson process is short term periodic. The dependence structure in this process is modeled by an Le´vy copula. Finally, we investigate the behavior of MLE of dependence parameter by a simulation.

Keywords

Le´vy process, short term periodic, nonhomogeneous Poisson process, Clayton Le´vy copula.

1- Introduction

Over last two decades, bivariate compound Poisson process has increased the importance of this process in order to modeling real data. It is used for modeling a business book containing bivariate claim count distribution and bivariate claim severities. Often independence assumption between classes of business is made. However there are many situations that this assumption isn't hold. For example in fire insurance, the loss of content and loss of building in a bivariate class can be dependent, see e.g., $\ddot{O}$zel (2013). But in some situations, non-constant λ may occur because of seasonal effects or trends. For example bad weather conditions will influence on car accidents. For this, more accidents will happen in winter in compared with other seasons. The number of events in this case has a nonhomogeneous Poisson distribution with periodic intensity function λ(t). These are reasonable for natural phenomena such as hurricanes, car accidents and catastrophic events.

Univariate periodic compound Poisson process used for studying an embedded discrete risk model. Lu and Garrido (2005) used a doubly periodic NHPP for modeling Atlantic hurricanes.

Le´vy copulas is extensively used for modeling dependency in multivariate Le´vy process and multivariate compound Poisson process that is theoretically introduced by Cont and Tankov (2004). Esmaeili and Kluppelberg (2010) found the maximum likelihood estimates of parameters in homogeneous bivariate compound Poisson process which the dependence structure between jumps is modeled by Clayton Le´vy copula.

In this paper, we consider a non-homogenous bivariate compound Poisson process (NHBCPP) as below

$$\begin{cases} S_1(t) = \sum_{i=1}^{N(t)} Z_{1i} \\ S_2(t) = \sum_{i=1}^{N(t)} Z_{2i} \end{cases} \quad (1)$$

such that $\{N(t); t \geq 0\}$ is NHPP with short-term periodic intensity function $\{\lambda(t); t \geq 0\}$ and is independent of both Z_1 and Z_2. Z_1 and Z_2 are dependent variables and their dependence structure is modeled by Clayton Le′vy copula. Our objective is to obtain maximum likelihood estimate of the copula parameter. Additionally, we aim to investigate the behavior and accuracy of this estimate.

This paper is organized as follows: Section 2 provide a brief description of the nonhomogeneous bivariate compound Poisson process. Section 3 discuss about finding the likelihood function of NHBCPP. Section 4 presents a numerical study and conclusions.

2- Nonhomogeneous Bivariate Compound Poisson Process (NHBCPP)

Let $\{S(t); t \geq 0\} = \{(S_1(t), S_1(t)); t \geq 0\}$ be an NHBCPP on probability space $(\Omega, \mathcal{F}, P)$ as (1) where $P(Z_{1i} = 0, Z_{2i} = 0) = 0$ for $i = 1, \ldots, N(t)$. We can rewrite $S(t)$ for non zero values of Z_1 and Z_2 as

$$\begin{pmatrix} \sum_{i=1}^{N_1(t)} Y_{1i} \\ \sum_{i=1}^{N_2(t)} Y_{2i} \end{pmatrix}, \quad (2)$$

Where $N_1(t)$ and $N_2(t)$ are the number of non zero values of Z_1 and Z_2 up to time t. $\{N_1(t); t \geq 0\}$ and $\{N_2(t); t \geq 0\}$ are NHPP with intensity functions $\lambda_1(t)$ and $\lambda_2(t)$ respectively. Hence for for $i = 1,2$ $\{S_i(t); t \geq 0\}$ is compound Poisson process with cumulative intensity function $\Lambda_i(t)$.

Considering Esmaeili and Kluppelberg (2010), we can decompose (2) as $\begin{pmatrix} S_1(t) \\ S_2(t) \end{pmatrix} =$

$\begin{pmatrix} S_1^\perp(t) + S_1^\parallel(t) \\ S_2^\perp(t) + S_2^\parallel(t) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{N_1^\perp(t)} Y_{1i}^\perp + \sum_{i=1}^{N^\parallel(t)} Y_{1i}^\parallel \\ \sum_{i=1}^{N_2^\perp(t)} Y_{2i}^\perp + \sum_{i=1}^{N^\parallel(t)} Y_{2i}^\parallel \end{pmatrix}$ where $S_1^\perp(t) = \sum_{i=1}^{N_1^\perp(t)} Y_{1i}^\perp$, $S_2^\perp(t) = \sum_{i=1}^{N_2^\perp(t)} Y_{2i}^\perp$ are

called the independent parts of $S(t)$ and $S^\parallel(t) = \left(S_1^\parallel(t), S_2^\parallel(t)\right) = (\sum_{i=1}^{N^\parallel(t)} Y_{1i}^\parallel, \sum_{i=1}^{N^\parallel(t)} Y_{2i}^\parallel)$ is called the dependent part of $S(t)$. $S_1^\perp(t)$, $S_2^\perp(t)$ and $S^\parallel(t)$ are mutually independent. $N_1^\perp(t)$ and $N_2^\perp(t)$ are the number of jumps occurring only in the first and second components up to time t respectively and $N^\parallel(t)$ is the number of jumps that occurred in both components up to time t. So $N_1(t) = N_1^\perp(t) + N^\parallel(t)$, $N_2(t) = N_2^\perp(t) + N^\parallel(t)$.

$\{S_i^\perp(t); t \geq 0\}$ is compound Poisson process with parameter $\Lambda_i^\perp(t)$ for $i = 1,2$, $\{S^\parallel(t); t \geq 0\}$ is compound Poisson process with parameter $\Lambda^\parallel(t)$ and $\Lambda_i(t) = \Lambda_i^\perp(t) + \Lambda^\parallel(t)$; $i = 1,2$.

Suppose that $\{N_i(t); t \geq 0\}$ is NHPP with short term periodic intensity function $\lambda_i(t)$ for $i = 1,2$ and period $c = 1$. According to Dimitrov et .al (1997) the cumulative intensity function of $\{N_i(t); t \geq 0\}$ is as below:

$$\Lambda_i(t) = \lfloor t \rfloor \Lambda(1) + \Lambda(t - \lfloor t \rfloor). \quad (3)$$

For modeling dependence structure between two processes $S_1(t)$ and $S_2(t)$ above, we use a special case of Le′vy copula called Clayton-Le′vy copula as:

$$C(u, v; \alpha) = (u^{-\alpha} + v^{-\alpha})^{-1/\alpha}; u, v > 0. \quad (4)$$

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This will cause to have a parsimonious model with setting $\Lambda^{\|}(t) = C(\Lambda_1(t), \Lambda_2(t); \alpha)$.

We will estimate the copula parameter α in NHBCPP in next section.

3- Parameter Estimation

Assume that an NHBCPP $\{(S_1(t), S_2(t)); t \geq 0\}$ is observed continuously over time interval $[0, T]$. The jump size distribution functions is denoted by F_1 and F_2 respectively. Let also $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are the parameters vector of F1 and F2 respectively. By using of Le'vy measures $S_1^{\perp}(t)$, $S_2^{\perp}(t)$ and $S^{\|}(t)$, the expressions of dependence structure in NHBCPP in terms of cumulative intensity functions, jump size distributions and Le'vy copula will be obtained as:

$$\begin{cases} \Lambda_1^{\perp}(T)\bar{F}_1^{\perp}(x_1; \boldsymbol{\theta}_1) = \Lambda_1(T)\bar{F}_1(x_1; \boldsymbol{\theta}_1) - C(\Lambda_1(T)\bar{F}_1(x_1; \boldsymbol{\theta}_1), \Lambda_2(T); \alpha) \\ \Lambda_2^{\perp}(T)\bar{F}_2^{\perp}(x_2; \boldsymbol{\theta}_2) = \Lambda_2(T)\bar{F}_2(x_2; \boldsymbol{\theta}_2) - C(\Lambda_1(T), \Lambda_2(T)\bar{F}_2(x_2; \boldsymbol{\theta}_2); \alpha) \\ \Lambda^{\|}(T)\bar{F}^{\|}(x_1, x_2; \boldsymbol{\theta}_1, \boldsymbol{\theta}_2) = C(\Lambda_1(T)\bar{F}_1(x_1; \boldsymbol{\theta}_1), \Lambda_2(T)\bar{F}_2(x_2; \boldsymbol{\theta}_2); \alpha). \end{cases} \quad (5)$$

Denote by n the number of jumps in $[0, T]$, we decompose it as below:

- $n_1^{\perp}$: number of jumps in first independent component,
- $n_1^{\perp}$: number of jumps in second independent component and
- $n^{\|}$: number of jumps occuring in both component.

Denote by $(y_{11}^{\perp}, \ldots, y_{1n_1^{\perp}}^{\perp})$, $(y_{21}^{\perp}, \ldots, y_{2n_2^{\perp}}^{\perp})$ and $((y_{11}^{\|}, y_{21}^{\|}), \ldots, (y_{1n^{\|}}^{\|}, y_{2n^{\|}}^{\|}))$ the jump sizes for the first and second independent components and for the dependent component, respectively. Using the decomposition of NHBCPP, the likelihood function can be written as below:

$$L(\alpha) = (\Lambda_1^{\perp}(T))^{n_1^{\perp}} e^{-\Lambda_1^{\perp}(T).T} \prod_{i=1}^{n_1^{\perp}}[f_1^{\perp}(y_{1i}^{\perp}; \boldsymbol{\theta}_1)] \times (\Lambda_2^{\perp}(T))^{n_2^{\perp}} e^{-\Lambda_2^{\perp}(T).T} \prod_{i=1}^{n_2^{\perp}}[f_2^{\perp}(y_{2i}^{\perp}; \boldsymbol{\theta}_2)] \times$$
$$(\Lambda^{\|}(T))^{n^{\|}} e^{-\Lambda^{\|}(T).T} \prod_{i=1}^{n^{\|}}[f^{\|}((y_{1i}^{\|}, y_{2i}^{\|}); \boldsymbol{\theta}_1, \boldsymbol{\theta}_2)], \quad (6)$$

where $f_1^{\perp}(y_{1i}^{\perp}; \boldsymbol{\theta}_1)$, $f_2^{\perp}(y_{2i}^{\perp}; \boldsymbol{\theta}_2)$ and $f^{\|}((y_{1i}^{\|}, y_{2i}^{\|})$ will be obtained by differentiating of $F_1^{\perp}$, $F_2^{\perp}$ and $F^{\|}$ in (5), respectively.

4- Numerical Study and Conclusion

Consider an NHBCPP with short term periodic $\lambda_1(t) = 100[\sin(2\pi t) + 1]$ and $\lambda_2(t) = 80[\sin(2\pi t) + 1]$ both with $c = 1$. Assume also that jump sizes for first and second compound processes are exponentially distributed with parameters $\theta_1 = 1$ and $\theta_2 = 2$,respectively; and the dependence structure between jumps is Clayton Le'vy copula (4). we simulate 100 trajectories of the NHBCPP according to the following simulation algorithm:

1. Generate $n_1^{\perp}$, $n_2^{\perp}$ and $n^{\|}$ from a Poisson distribution with parameters $\Lambda_1^{\perp}(T)$, $\Lambda_2^{\perp}(T)$ and $\Lambda^{\|}(T)$ respectively.

2. Simulate the first independent part jump sizes $(y_{1i}^{\perp})$ with d.f $F_1^{\perp}$ and the second common with d.f $F_2^{\perp}$ according to (5)

3. For simulating dependent jump sizes, first draw $y_{1i}^{\|}$ from d.f $F_1^{\|}$ according to (5). Now conditioning $y_{1i}^{\|} = x$, we simulate $y_{2i}^{\|}$ from $H_x(y) = \frac{\partial}{\partial u} C(u, v)|_{u=F_1^{\|}(x), v=F_2^{\|}(y)}$.

So, according to (6) the likelihood function is given by:

$$L(\alpha) = (\Lambda_1(T))^{n_1^{\perp}} e^{-\Lambda_1^{\perp}(T).T - \Sigma_{i=1}^{n_1^{\perp}} y_{1i}^{\perp}} . \prod_{i=1}^{n_1^{\perp}} \left[1 - \left(1 + \left(\frac{\Lambda_1(T)}{\Lambda_2(T)} \right)^{\alpha} e^{-\alpha.y_{1i}^{\perp}} \right)^{-\frac{1}{\alpha}-1} \right] (2\Lambda_2(T))^{n_2^{\perp}} e^{-\Lambda_2^{\perp}(T).T - \Sigma_{i=1}^{n_2^{\perp}} y_{2i}^{\perp}}$$

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$$\times \prod_{i=1}^{n_2^{\perp}} \left[1 - \left(1 + \left(\frac{\Lambda_2(T)}{\Lambda_1(T)} \right)^{\alpha} e^{-2\alpha.y_{2i}^{\perp}} \right)^{-\frac{1}{\alpha}-1} \right] \times (2(1+\alpha)(\Lambda_1(T)\Lambda_2(T))^{\alpha+1})^{n^{\parallel}} e^{-\Lambda^{\parallel}(T).T-(1+\alpha)(\sum_{i=1}^{n^{\parallel}} y_{1i}^{\parallel} + 2\sum_{i=1}^{n^{\parallel}} y_{2i}^{\parallel})}$$

$$\times \prod_{i=1}^{n^{\parallel}} (\Lambda_1(T)^{\alpha} e^{-\alpha y_{1i}^{\parallel}} + \Lambda_2(T)^{\alpha} e^{-2\alpha y_{2i}^{\parallel}})^{-\frac{1}{\alpha}-2}.$$

The considered time intervals are [0, 0.05], [0, 0.10] and [0, 0.50]. We calculate biasedness, variance, mean square error (MSE) and mean absolute error (MAE) for dependence parameter estimations. The results are summarized in table (1).

α	Quantity	T=0.05	T=0.10	T=0.50
	Bias	0.063999	0.017943	-0.038071
0.20	Variance	0.000331	9.8978e-05	6.40e-06
	MSE	0.004424	0.000420	0.001456
	MAE	0.063998	0.018205	0.038071
	Bias	0.230525	-0.103948	-0.1517753
0.50	Variance	0.035444	0.001343	0.000115
	MSE	0.088231	0.012135	0.023150
	MAE	0.232903	0.104184	0.151775
	Bias	-0.977265	-1.082887	-2.271679
2.00	Variance	0.285834	0.075096	2.41e-06
	MSE	1.238023	1.24699	5.160528
	MAE	0.978078	1.082887	2.271679

Table1: Statistical investigation of the MLE for dependence parameter of NHBCPP

In all states, the α estimates are biased. For constant α, with increasing T the variance of α estimates are decreased. We observe from the estimated MSE and MAE that the accuracy of estimations is much more for small α than for large.

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Applications of Statistical Machine Learning in Neuroimaging

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Abstract

Functional magnetic resonance imaging (fMRI) is a safe and non-invasive method to recognize brain functions by using signal changes associated with the brain activity. The technique has become a tool in basic, clinical and cognitive neuroscience. In this paper, we want to explain the key role of fMRI data analysis that is able to find the parts of brain which are involved in a mechanism, or to determine the changes that occur in brain activities due to a brain lesion. Hence, we aim to review and discuss the methods of statistical machine learning techniques as one of the most existing recent developments to analyze the fMRI data.

Keywords

FMRI data, Machine learning, Multivoxel Pattern Analysis (MVPA), Statistical Parametric Mapping, Neuroimaging.

1- Introduction

During the last two decades, functional magnetic resonance imaging (fMRI) has become a popular tool for understanding the human brain. Neuroscientists and physicians carry out fMRI experiments to determine precisely which part of brain is active for critical functions such as thinking, speech, motion, sensation and attention (6,8).The effects of stroke, trauma or degenerative diseases (such as Alzheimer's disease) on brain function can be assessed by fMRI. Furthermore, fMRI data analysis can show the growth and the activity of the brain tumor regions for surgical planning, radiotherapy or other surgical treatments of the brain.

After designing an fMRI task design and collecting data from the MRI scanner, various analysis must be applied on the raw data to answer the questions of neuroscientists and physicians about

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activities corresponding to the experiment. The main goal of computer-based analysis is to determine automatically, those parts of the brain which respond to stimuli that presented to the subjects. The fMRI analysis methods are composed of several basic stages: Pre-processing, signal detection and description, and extraction of the brain connectivity. The goal of preprocessing is to eliminate different kinds of artifacts such as systematic noise, motion correction, etc. Pre-processing consist of spatial or temporal filtering of fMRI data and improving the image resolution (1). Signal detection is the second step of analysis that should be done to determine which voxels are activated by the stimulation. The statistical parametric mapping is commonly used to test the difference of signal levels in different stimulus. This step contains model specification, parameter estimation, hypothesis testing and the decision-making stages. The output of this step is an activation map which indicates activated parts of the brain in response to the stimulus. The purpose of signal description is modeling the blood oxygenation level dependent (BOLD) response shape by several parameters and relating these parameters to the description of the stimulation context. Finally, the connectivity analysis tries to estimate brain networks. In this paper, we will review the statistical analysis methods and machine learning classifiers for fMRI data (2).

2- Statistical Machine Learning

Machine learning is the study of computational processes that find patterns and structure in the data. When Statistical techniques and machine learning are combined together, it will be a powerful tool to focus on pattern recognition based on statistical dependencies and consistencies in data. In the last few years there has been growing interest in the use of machine learning techniques for analyzing neuroimaging data. Since the brain function activity is estimated based on a statistical General Linear Model (GLM), most of researchers in the world are motivated to use probabilistic method such as Bayesian network, graphical model and other probabilistic classifiers to detect the brain complexity and network (4,5,7,9) .

The main goal of statistical analysis in Neuroimaging is attempted to determine which voxels are activated by the stimulation. Most fMRI studies are established upon the correlation of hemodynamic response function with stimulation. These methods can categorize into the univariate methods (voxel-wise approaches), and the multivariate methods (Multivoxel pattern recognition).

Univariate Analysis

Univariate analysis is based on change variations of a single voxel. In earlier methods, experts utilized t-test to detect the brain activity. However, in later studies, it is more common to apply GLM which is a mathematical framework of different analysis such as t-test, paired t-test, analysis variance (ANOVA), analysis of covariance (ANCOVA) and the linear regression. This provides a significant computational and statistical improvement in these methods because both parameter estimation and statistical inference are straightforward in GLM. GLM-based methods are utilized for constructing some statistical parametric mapping by considering the hemodynamic response function (HRF), low-frequency noise and the serially correlation of a voxel data. In these approaches, GLM is applied separately to every voxel in the regions of interest (ROI) or the whole brain.

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Multivariate Analysis

Our aim in this subsection is to explain about the multivariate analysis and introduce some common models in fMRI data analysis. Multivariate analysis, unlike univariate which was voxel-wise, considers the whole brain or ROIs to find a network or pattern in the data. On the other hand, these methods describe the brain responses by considering the spatial and temporal correlation of the fMRI data. We discuss some of the multivariate methods in this paper. Two of the most common ones are (functional) principal component analysis (PCA or fPCA) and independent component analysis (ICA). The purpose of these methods is to determine which functional or causal neural networks is significant based on the observed BOLD responses.

Although the purpose of ICA and (f)PCA is similar, assume different structure for the fMRI data. The mechanism of PCA is the best rotation of data to a new coordinate system such that the variance of that direction is maximized and it requires no model fitting process. It should be noted that PCA has no free parameters to estimate from fMRI data. In contrast, ICA is a true model, and it has some parameters that must be estimated from those data. The only assumption of PCA is having a multivariate normal distribution for our data. While ICA has two key fundamental assumptions. Firstly, ICA is supposed to consider some components are statistically independent, although it does not require that they are also orthogonal. Secondly, in contrast with PCA, ICA assumes that components are non-normally distributed.

The connectivity analysis, which is known as a phase of the fMRI data process, is based on correlation, covariance, spectral coherence, or phase locking of BOLD responses. This process is used as the temporal dependency of neuronal activation patterns of anatomically separated brain regions.

A less common approach is to use the Bayesian methods that are potentially more impressive. We are supposed to consider the prior distribution of activation and of all other parameters specified by the model in Bayesian framework. The GLM model can be discussed in a Bayesian approach by consideration of prior distributions for definite parameters.

Among these models, our favorite approach is multivoxel pattern analysis (MVPA). MVPA methods compared to the conventional statistical analysis approaches has the advantage of higher sensitivity in discriminating perceptual and cognitive states in the brain (3). The idea of this method uses the techniques of pattern recognition to categorize the voxels based on their activation. There are five basic steps in a MVPA approach.

First step is the feature extraction. If we consider that the measured brain activity in response to a stimulus is represented as a point in a multidimensional space of voxels (MV), then this multidimensional matrix is transformed into a long vector of features (voxels) of the activities. Second step is the feature selection. The number and quality of the voxels that are given to the classifier plays a key role in pattern recognition, since the performance of this method is dependent on the mentioned factor. This step includes several methods such as t-test, f-score, ANOVA, Voxel activation criteria, SVM (support vector machines) etc.(5) The third step is pattern assembly. It can group the data into separate 'brain patterns' corresponding to the pattern of activities across the chosen voxels at a particular time in the experiment. The brain patterns are labeled according to the experimental condition that generated them (7). The fourth step is the classifier training that is

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related to a multivariate pattern classification algorithm based on the subset of those labeled patterns e.g. support vector machine (SVM), Naïve Bayesian (NB), k-Nearest Neighbor (k-Nearest Neighbor), LDC (linear discriminant classifier) and the decision tree. The last step is the generalization testing which includes a new pattern of brain activity. In this step, we predict the label of the experimental condition that has generated the activity (5).

3- Conclusions

In this paper, we aim to introduce the advantage of machine learning classification methods as one of the most exciting recent developments in fMRI data analysis (7,5). It is worth to mention that the basic idea of these method is that if some brain regions is responding differently to different event types, then it should be possible to find a classification scheme that can look at the responses of all voxels in those regions and correctly identify which event triggered the response. Our goal in this paper was to give a brief intuitive description of these methods and motivate the statisticians to appropriate references that provide more details.

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Application of Skew Laplace Distribution in Risk Measures

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Abstract

Financially, it is widely acceptable that the asset returns exhibit asymmetric features such as skewness and heavy tails. To deal with the asset market data, we consider the skew Laplace (SL) distribution as a new benchmark model. The numerical results show that the new model can outperform many existing model to evaluate risk measures.

Keywords

Risk measurement, Value at risk, Normal Mean-variance mixture model, Skew Laplace distribution.

1- Introduction

Although financial risk management depends on the use of probability distribution and the choice of risk measure, most modern measures of the risk in a portfolio are statistical quantities. Therefore, the choice of probability distribution is more important for investors who are holding a position on portfolios of risky assets than the choice of risk measure. Also, in order to select an appropriate distribution family, two important factors should be considered: 1) The calibration algorithm of parameters must be fast and 2) the used family of distribution should be closed under liner combinations.

Despite the satisfying of the normal and Student-*t* distributions in these conditions, empirical evidence support that the returns of financial assets have non-Gaussian features such as heavy tails and strong skewness and kurtosis. To deal with this deficiency, the most commonly accepted distributions which can be taken as promising alternatives for describing the asymmetric features of asset returns are the skew-normal (SN) [1] and skew-t (ST) [2] distributions. For instance, Eling [3] fitted the SN and ST distributions to compare them with other models and to check how well they are to model insurance claims and asset returns. As expected, the SN distribution theoretically works well when the data is not heavy tails. On the other hand, the ST distribution has some restriction and difficulty in evaluating risk measure. For example, since the moments of this distribution do not exist when the estimated degree of freedom are very low (less than 2).

Another class of skewed and heavy tailed distributions is the normal mean-variance mixture (NMV) distribution. The family of NMV distribution is closed under liner combinations and is fast to calibrate the parameters to data and also has not any restriction in evaluating risk measure. This class also contains symmetric models such as normal and student-*t* distributions, and asymmetric

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models like normal inverse Gaussian (NIG), skew-t, and variance gamma distributions as special cases. The main objective of this paper is to consider skew Laplace distribution [4] as an alternative model for risk measurement. The SL model which is belong to the NMV family of distribution, is heavy tail and unimodal and has a better performance when the data have some peaks. The Dow 30 index is used to illustrate the efficiency of the new model in evaluating most familiar risk measure Value-at-Risk (VaR).

2- The Skew Laplace Distribution

Formally, a random variable (rv) X is said to have a NMV distribution if it admits the stochastic representation as

$$X = \mu + \lambda W + \sqrt{W} Z , \qquad (1)$$

where μ and λ are location and shape parameters, respectively. In representation (1), the rv Z is followed by the normal distribution ($Z \square N(0, \sigma^2)$) and W is a non-negative rv which is independent of Z. If the rv W in the stochastic representation (1) is distributed as an exponential distribution with mean 2 (Exp(0.5)), then the rv X has a SL distribution. As a result, it can be easily shown that the probability density function of the SL distribution is given by

$$f(x; \mu, \sigma^2, \lambda) = \frac{1}{2\tau\sigma} \exp\left\{-\tau \frac{|x-\mu|}{\sigma} + \frac{\lambda(x-\mu)}{\sigma^2}\right\}, \qquad x \in \square$$

where $\tau = \sqrt{1 + (\lambda/\sigma)^2}$.

3- Application to Real Data Set

3-1- Descriptive Statistics and Model Selection

In this section, we use daily return data of "DOW 30" index to model the asset side of companies. For P_t and P_{t-1} corresponding to the adjusted-closed price of the share at time t and t-1, respectively, the return can be computed by $r_t = 100 \times \log(P_t / P_{t-1})$. The period of analysis for the data is from 01/06/2010 to 01/06/2017. We start our study by investigating some properties of data which motivate us to consider them as an appropriate example of fitting assert return. All computational procedure is done in statistical software R and the code is available from the author upon request. In Table 1, we summarize descriptive statistics for the dataset. This table presents the number of observations (n), the first four moments: mean, standard deviation (St.Dev), skewness (γ_x), kurtosis (κ_x), and minimum and maximum of the data. To verify the normality of data, we apply the Jarque-Bera (JB) test and display its statistic (with P-value) in Table 1. From St.Dev as a measure of risk, we can conclude that DOW is a risky return. The values of γ_x and κ_x show that the data are left skewed and have fat tails. The high values of chi-square statistic of the JB test reject the normality assumption for both 10% and 1% confidence levels. These characteristics make sure us that the skewed and heavy tails distributions are more appropriate to fit data.

As a result, the normal distribution and four skewed distributions, namely the SN, ST, NIG and SL distributions were fitted on data. These models are compared against each other based on Akaike information criteria (AIC) and Bayesian information criteria (BIC) (summarized in Table 2). Table

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2 also presents the Kolmogorov-Smirnov (KS) goodness of fit test to study the validity of five fitted model. As a general rule, it could be noted that the model with the lowest values of AIC, BIC and KS

Table 1: Descriptive statistics.

| Index | measure | | | | | | | |
	n	mean	St.Dev	min	max	γ_x	κ_x	JB (P-value)
Dow30	1763	0.041	0.877	-5.706	4.153	-0.400	6.833	1129.7($<2.2\,e^{-16}$)

Table 2: Model comparison criteria of the fitted models.

| Criteria | Model | | | | |
	Normal	SN	ST	NIG	SL
ℓ_{max}	-2267.489	- 2253.747	-2132.119	-2123.159	-2119.356
AIC	4538.978	4513.494	4272.239	4254.318	4244.712
BIC	4549.925	4529.915	4294.133	4276.212	4261.133
KS-test	0.088	0.079	0.028	0.022	0.018

statistics will outperform other competitors. Also, the critical value at 95% confidence level can be calculated by $1.36/\sqrt{1762} = 0.0324$. If the KS statistics remains below this level, we cannot reject the hypothesis that the empirical distribution agrees to the proposed distribution. Table 2 reveals that the SL distribution has the smallest values of AIC, BIC and KS statistics. Therefore, the SL distribution outperforms the other distributions. Figure 1 also displays the histogram of the data along with the fitted benchmark distribution curves in which the outperformance of the SL distribution can be seen. Moreover, the KS statistics of the three heavy tailed skew models, ST, NIG and SL distributions, is below than critical value which shows that the empirical distribution agrees the heavy tailed skew model.

3-2- Application to Estimate VaR

The VaR is one of the most consistently used measure of risk by banking institutions. Theoretically, for the confidence level α in (0,1), the VaR is defined as the negative of the quantile of order $1-\alpha$ of the stock return distribution, that is, $VaR_\alpha = -F^{-1}(\alpha) = -\inf\{x \mid F_X(x) \geq 1-\alpha\}$. The commonly typical used values of α are ranging between 0.95 to 0.99. For instance, let $\alpha = 99\%$ and the time period is one day. Then, the VaR$_{0.99}$ of one million dollars can be interpreted that the probability of incurring loss in excess of one million dollars for the proposed shares over one day is bounded by 0.01.

The estimated 99% VaR values obtained by the fitted SL distribution and four other models along with the empirical VaR calculated from the data are presented in Table 4. It is evident that for this confidence level the SL distribution gives the closest estimate. To assess relative changes on the theoretical prediction, we also calculate the mean absolute relative error (MARE) defined as

$$MARE = \frac{1}{n_q} \sum_{i=1}^{n_q} \left| \frac{VaR_{\alpha_i} - \widehat{VaR}_{\alpha_i}}{VaR_{\alpha_i}} \right|,$$

where n_q represents the number of considered confidence level, VaR_{α_i} and $\widehat{VaR}_{\alpha_i}$ are the empirical and estimated VaR evaluated at confidence level α_i. In Table 4, we summarized the values of MARE.

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Table 4: Empirical and estimated value and tail value at risks.

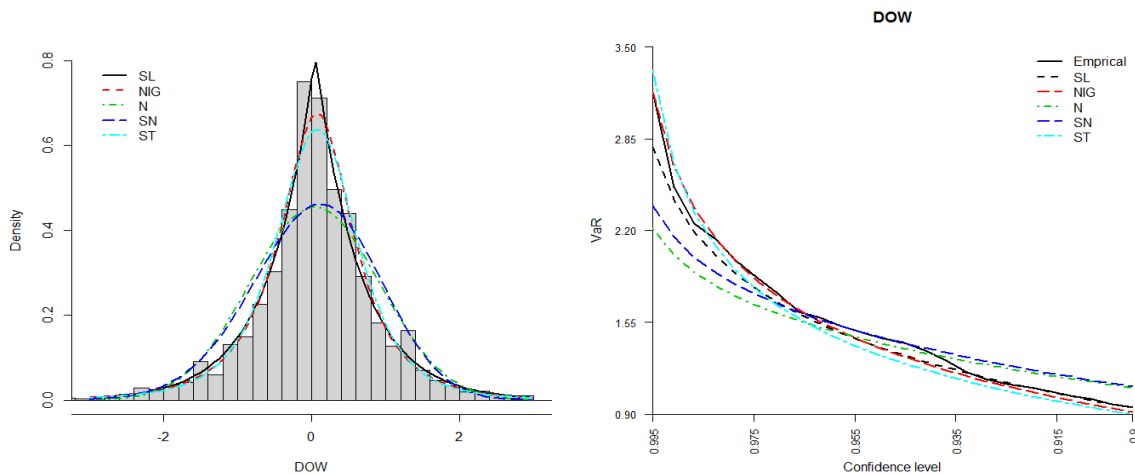|	Empirical	Normal	SN	ST	NIG	SL
$VaR_{0.99}$	2.402	2.001	2.119	2.345	2.569	2.365
MARE	--	9.597	8.283	6.165	3.116	3.010

Figure 1: left) Histogram of the Dow 30 data with overlaid five fitted densities. Right) VaR for varying confidence level.

These result reveal that the SL model has less amount of MARE, indicating that our proposed model is more accurate than other competitors to predict VaR. It can be also seen from the VaR curves plotted in Fig. 1 for varying confidence level that the SL distribution predict VaR better than other models.

4- Conclusion

In this paper, we have proposed the risk measurement based on the skew Laplace distribution. The numerical results suggest that the accuracy of risk measure prediction such as prediction of VaR can be improved by considering the SL distribution. This approach can be extended to implement finite mixture of the SL distributions risk measures.

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An Optimization Portfolio Based on the Comonotonicity Concept

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Abstract

In this paper, using the concept of comonotonicity a multiperiod portfolio selection problem in a Jump diffusion market is investigated. We first consider the portfolio optimization problem of an investor who want to obtain a target capital at a specific time by some investments on the predetermined times. Then, we concern another problem in which a decision maker who invests to be able to achieve a series of future consumptions or payment obligations.

The optimization problem is constructed based on accurate approximations using the concept of comonotonicity. Our analytical approach avoids simulation, and hence reduces the computing effort drastically.

Keywords

Comonotonocity, Portfolio, Jump diffusion market.

Backward Stochastic Volterra Integral Equations in Spaces and Their Application

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Abstract

We consider a backward stochastic Volterra integral equation [BSVIE] in the Banach space $E = L^q(S, \Sigma, \mu)$, where μ is σ -finte measure. The stochastic integral is defined with respect to an infinite dimensional Wiener process. Under appropriate assumptions, the existence and uniqueness of an adapted solution of BSVIE are being proofed by using martingale representation theorem in Banach space E and Banach fixed-point theorem. Some properties of the solution are also discussed. For an application we can consider an optimal control problem in Banach space E where the state process will be defined as forward Ito Volterra stochastic integral equation with respect to a cylindrical Wiener process. There are also applications in finance for example, rate processes in a Heath-Jarrow-Morton model which satisfy a stochastic partial differential equation of first order in Banach space.

Keywords

L^q stochastic integrability, Unconditional martingale difference spaces, Martingale representation theorem, Co-type spaces, Cylindrical Wiener process, Stochastic maximum principle.

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Comparing Two Bonus-Malus Systems in a M-Continuous Years

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Abstract

A solution of moving between two Bonus-Malus systems is considered in this paper. Also it will be shown that a Bonus-Malus system how sensible to change level, claim count distribution and year of Bonus-Malus system in a m-continuous years. In this way, it will be shown which level of Bonus-Malus system1 is most suitable for policyholder wanting to move to Bonus-Malus system2 with exact level. In the new conclusion, roll of Bonus-Malus system will be more sensible. Others calculated suitable level just for one year when Bonus-Malus systems get steady-state, but in the new approach, situation of m continues years are considered. Also the new approach will be obtained for Poisson-gamma, inflated Poisson-gamma, binomial-gamma and inflated binomial-gamma distribution under mean square error loss by using credibility.

Keywords

Bonus-Malus System, Mean square error, Measure of discrepancy.

1- Introduction

This paper wants to answer to the question such as:

1. How sensible is Bonus-Malus System to increase years and levels of Bonus-Malus System?
2. How effect is changing claim count distribution on defined method in Bonus-Malus System?
3. Which BMS is nearer to the exact BMS1 by comparing defined method?
4. Which level of Bonus-Malus System 2 is most suitable for policyholder in Bonus-Malus System 1 with exact level wanting to move?

For answer to these questions, first a new method is introduced that based on summation of (m-l+1)-continuous year of relativity premium $r(L_i)$

$$S_{\ell:m}^{(BMS)} = \sum_{i=\ell}^{m} r(L_i)$$

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Second some calculating is done based on defined summation and above questions will be answered, finally five example will concerned in numerically way.

Effects of Changing Claim Count Distribution, Increasing Year and Levels of Bms

Many cases can be effected on relativity premium paid by policyholders such as number of BMS's levels, number of years that policyholder is in a specific BMS and even changing claim count distribution. In this paper first these three cases are studied by calculating expected and variance values of $S_{\ell:m}^{(BMS)}$.

Moving Between to Two Bonus-Malus Systems

Comparing BMSs are studied by defining new discrepancy simply. Suppose BMS1 has S1 levels and BMS2 has S2 levels, then two BMSs are compared by:

$$
\begin{aligned}
d_{RP}^{(1)} &= \frac{1}{m} \left| E(S_{1:m}^{(BMS1)}) - E(S_{1:m}^{(BMS2)}) \right| \\
d_{RP}^{(2)} &= \frac{1}{m^2} \left| Var(S_{1:m}^{(BMS1)}) - Var(S_{1:m}^{(BMS2)}) \right| \\
d_{RP}^{(3)} &= \left| \frac{\sqrt{Var(S_{1:m}^{(BMS1)})}}{E(S_{1:m}^{(BMS1)})} - \frac{\sqrt{Var(S_{1:m}^{(BMS2)})}}{E(S_{1:m}^{(BMS2)})} \right|
\end{aligned}
$$

Best Level of bms2 for Policyholder in bms1

Best level of BMS2 for policyholder in BMS1 based on average of relativities premium's expected value in m continuous years, so the closest level k in BMS2 from the fixed level of BMS1 is given by new method. Defined measure of discrepancy actually places the policyholder in BMS2 at the level with the closest expected value of relativity premium in continuous m year to the one applicable in the BMS1.

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Collective Risk Model: Bayesian Premium and Operational Risk of the New Generalized Poisson Lindley and Exponential Distributions

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Abstract

This paper developed collective risk model with new generalized Poisson Lindley and exponential distributions. Some properties like generating function, moments are derived. We determine the bayes premium used in actuarial science and also compute the operational risk. The results are illustrated with numerous examples and compared with other approaches proposed in the literature.

Keywords

Bayes premium, collective risk model, operational risk, new generalized Poisson Lindley distribution.

1- Introduction

The collective risk model (crm) is described by a frequency distribution for the number of claims N and a sequence of independent and identically distributed random variables representing the size of single claim X_i . Assume that the frequency and severity are independent. One of the well known actuarial techniques is estimation of the annual loss distribution by modeling the frequency and severity of losses which is used for modeling solvency requirements in the insurance industry. The distribution of aggregate loss that is the sum of the individual calim sizes

$$S = \sum_{i=1}^{N} X_i, \quad N > 0$$

and $S = 0$ if $N = 0$, defines a methodology of premium calculation in the insurance setting that is based on the individual experience of the risk involved. Therefore it is useful for considering both the number and size of the claims, when it is desirable that both must be implemented in the model.

Traditionally, Poisson and Exponential as primary and secondary distributions are used for collective risk model. Kozubowski et, al. (2005) presented some interesting results on the sum of random variables with an exponential distribution and a random number of summands. At the same time, the variance is larger than the average (over dispersion) has derived in the conjunction of so many data sets that is well known difficult. For this reason, for the random variable N , different alternatives have been considered, for example, Karlis, et al. (2005), Nikoloulopoulos, et al. (2008), Nadarajah, et al. (2006).

Poisson Lindley distribution, by assuming the parameter of the Poisson distribution to follow a Lindley distribution is introduced by Sankaran (1970). Ghitany et al. (2008) proposed different methods of estimation for the discrete Poisson Lindley distribution. They have used the zero truncated Poisson-Lindley distribution to model count data when the data structurally excludes zero counts. Mahmoudi et al.(2010) generalized the Poisson Lindley distribution and showed that their generalized distribution has more flexibility in analyzing count data. Hernandez et al. (2011) in the context of insurance have used

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the Poisson-Lindley to study the collective risk model when number of claims and size of a single claim are incorporated into the model. Another form of discrete Lindley distribution was introduced by Gomez et al. (2011). Gomez et al. (2013) analyzed the collective risk model by assuming Erlang distribution to the loss data and generalized Lindley distribution to the claim frequency data. A new generalized Poisson Lindley distribution and these properties are obtained in Bhati (2015). We use the Gaussian hypergeometric function as

$$ _pF_q\left[\{a_1,...,a_p\};\{b_1,...,b_k\};z\right]=\sum_{k=0}^{\infty}\frac{\prod_{j=1}^{p}(a_j)_k \, z^k}{\prod_{j=1}^{q}(b_j)_k \, k!} $$

Where $(\alpha)_j=\alpha(\alpha+1)...(\alpha+j-1)$, for $j\geq 1$, $(\alpha)_0=1$ be the Pochhammer symbol. See Yakubovich et al (2012) for more details.

The rest of the paper is as follows. The collective risk model and its properties are developed in Section 2. Section 3 examines the Bayes premium and the utilization of the crm in operational risk analysis is obtained in Section 4. Some concluding remarks are presented in Section 5.

2- The CRM Model

Suppose that the random variable number of claims N has a new generalized Poisson Lindely distribution (NGPL) with the probability mass function as

$$ P(N=n\,|\,\theta_1,\alpha)=\frac{\theta_1^2}{\theta_1(1-\alpha)+\alpha}(1-\theta_1)^n\left(1+\alpha(n+1)(1-\theta_1)\right),0<\theta_1<1,\alpha\geq 1. \qquad (0.1) $$

The NGPL distribution and its properties proposed in Bhati et al, (2015).

It is well known that this distribution is over dispersed.

Let the random variable (size of single claim) X_i assumed to follow an exponential distribution with parameter $\theta_2>0$.

In practice, heavy tailed distributions such as Lognormal, Pareto distributions have been considered. It is well known the marginal distribution under the exponential and gamma model is a generalized Pareto distribution that is finally considered here for the size of single claim. Following proposition compute the distribution and the moment generating function of S.

Proposition 2.1 (i) the probability density function of the random variable aggregate claim, is given by

$$ f(s\,|\,\theta_1,\theta_2,\alpha)=\frac{\theta_1^2\theta_2(1-\theta_1)}{\theta_1(1-\alpha)+\alpha}e^{-\theta_1\theta_2 s}\left[1+\alpha(1-\theta_1)(\theta_2 s(1-\theta_1)+2)\right],s>0 $$

While $f(0\,|\,\theta_1,\theta_2,\alpha)=P(N=0\,|\,\theta_1,\alpha)$.

(ii) The moment generating function of S is given by

$$ M(t;\theta_1,\theta_2,\alpha)=\frac{\theta_1^2}{\theta_1(1-\alpha)+\alpha}\frac{t^2(1+\alpha(1-\theta_1))-\theta_2 t(1+\theta_1+2\alpha(1-\theta_1))+\theta_2^2(\theta_1+\alpha(1-\theta_1))}{(\theta_1\theta_2-t)^2} $$

In a Bayesian analysis the marginal a prior distribution for the variables θ_1,θ_2 are beta distribution with parameters $a>0,b>0$ and gamma distribution with parameters $c>0,d>0$. It is assumed that these variables are independent.

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In the following proposition we find the marginal distribution of S, The posterior distribution of θ_1, θ_2.

Proposition 2.2 (i) The marginal distribution of S is given by

$$m(s) = \frac{c}{\alpha d B(a,b)} F_1(a+2,1,c+1,a+b+3,\frac{\alpha-1}{\alpha},-\frac{s}{d}) + \frac{c(c+1)s}{d^2 B(a,b)} F_1(a+2,1,c+2,a+b+5,\frac{\alpha-1}{\alpha},-\frac{s}{d})$$

$$+ \frac{2c}{d B(a,b)} F_1(a+2,1,c+1,a+b+4,\frac{\alpha-1}{\alpha},-\frac{s}{d}),$$

where $s > 0$ and $F_1(a,b_1,b_2,c,x,y)$ is Appell function.

(ii) The posterior distribution of θ_1, θ_2 is

$$\pi(\theta_1,\theta_2 \mid s) = \frac{d^c}{B(a,b)\Gamma(c)m(s)} \frac{\theta_1^{1+a}\theta_2^c (1-\theta_1)^b}{\theta_1(1-\alpha)+\alpha} e^{-\theta_1\theta_2 s - \theta_2 d} \left[1 + \alpha(1-\theta_1)(\theta_2 s(1-\theta_1)+2)\right], s > 0.$$

3- Concluding Remarks

This paper studied the Bayesian analysis and Bayes premiums in the actuarial science. Problem of determining the regulatory capital in operational risk analysis is developed,

Furthermore, a numerical analysis was carried out, comparing the latter with the use of the aggregate loss distribution. Remarkable differences between the two approaches are pointed out.

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Characterization of Beta-Pareto Distribution

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Abstract

A generalized type of Pareto distribution is defined as beta-pareto distribution. It is a four-parameter distribution that have been studied various mathematical and statistical properties of it. In this work, is provided some characterizations for beta-pareto distribution based on truncated moments.

Keywords

Pareto distribution; beta-pareto distribution; characterization; truncated moment.

1- Introduction

A random variable X is said to have Pareto distribution with parameters α and β, if its density function is as follows

$$f(x) = \frac{\alpha \, \beta^{\alpha}}{x^{\alpha+1}} \, . \qquad x \geq \beta \, \cdot$$

So far, several types of Pareto distribution are introduced, such as Pareto types I-IV.

Pareto type II is known as lomax distribution that is used in business, economic, actuarial science, queueninig and internet traffic modelling. In general, the Pareto diatribution is a heavy tail and skewed distribution and it can describe phenomena's distribution such as income, population and so on. In the area of Pareto distribution application, the Pareto principle or 80-20 rule says that, for many events, roughly 80% of the effects come from of the 20% of the causes corresponds to Pareto distribution with particular value of α that can be used specially in statistical quality control.

There are some relations between Pareto distribution with other distributions such as the exponential distribution, the power function distribution, the logistic and the chi-square distribution through some transformations. One can see more details in Newman (2005) and Zwillinger and Kokoska (2000).

Because of versatility of Pareto distribution in modelling many types of data with long tail, in the last decade, new generalization of Pareto distribution based on the inverse cumulative distribution function to a beta distributed random variable (see Eugene et al. (2002)) are obtained.

In this among, the beta-Pareto distribution is introduced by Akinsete et al. (2008). They studied a lot of statistical properties of this distribution such as expressions for the mean, variance, skewness, kurtosis, entropy and the method of maximum likelihood to estimate the parameters.

A random variable X is said to have beta-Pareto distribution , if its density function and cumulative distribution function are as follows, respectively

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$$g(x) = \frac{k}{\theta \, B(\alpha.\beta)} \{1 - \left(\frac{x}{\theta}\right)^{-k}\}^{\alpha-1} \left(\frac{x}{\theta}\right)^{-k(\beta-1)}. \quad x \geq \theta \quad .\alpha.\beta.\theta.k > 0.$$

Where $B(\alpha.\beta) = \frac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ and

$$G(x) = \int_x^\infty \frac{k}{\theta \, B(\alpha.\beta)} \left\{1 - \left(\frac{t}{\theta}\right)^{-k}\right\}^{\alpha-1} \left(\frac{t}{\theta}\right)^{-k(\beta-1)} dt. \quad t \geq \theta.$$

In the sequel, is provided some characterization results based on truncated moments from different types of data.

2- Characterization Results

Let $X_1 .X_2. \ldots$ be a sequence of independent and identically distributed (iid) random variables with continuous distribution function F and density function f. The corresponding order statistics are the $X_i's$ arranged in non-decreasing order, denoted by $X_{1:n} \leq X_{2:n} \leq \ldots \leq X_{n:n}$. The density function of the rth order statistics is as follows

$$f_{r:n}(x) = \frac{n!}{(r-1)!(n-r)!} \{F(x)\}^{r-1} \{1 - F(x)\}^{n-r} f(x).$$

An another ordered data is called upper record value that is defined as follows

$$T_1 = 1. \quad T_n = \min\{j \mid j > T_{n-1}.X_j > X_{T_{n-1}}. \quad n \geq 1,$$

Where $\{T_n\}_{n=1}^\infty$ indicates a sequence of upper record time and X_{T_n} is the nth upper record value. The density function of the nth upper record value is as follows

$$f_n(x) = \frac{\{-\log \bar{F}(x)\}^{n-1}}{(n-1)!} f(x).$$

Recently, many researchers are interested in characterizing a distribution that will be useful for investigators who need to know a model if their model verify the special requirement of underlying distribution.

Theorem. Let $X_1 .X_2. \ldots$ be a sequence of iid continuous random variables. Then $X_i's$ have beta-Pareto distribution if and only if one of the following conditions holds.

a) $E(X_1|X_1 > x) = \frac{1}{G(x)} \int_x^\infty \frac{k}{B(\alpha.\beta)} \{1 - \left(\frac{y}{\theta}\right)^{-k}\}^{\alpha-1} \left(\frac{y}{\theta}\right)^{-k(\beta-1)+1} dy.$

b) $E(X_{1:n}|X_{1:n} > x) = \frac{1}{G_{1:n}(x)} \int_x^\infty \frac{nk}{B(\alpha.\beta)} \{1 - \left(\frac{y}{\theta}\right)^{-k}\}^{\alpha-1} \left(\frac{y}{\theta}\right)^{-k(\beta-1)+1} \{\bar{G}_{1:n}(x)\}^{n-1} dy.$

c) $E(X_{T_n}|X_{T_n} > x) = \frac{1}{G_n(x)} \int_x^\infty \frac{k}{B(\alpha.\beta)} \{1 - \left(\frac{y}{\theta}\right)^{-k}\}^{\alpha-1} \left(\frac{y}{\theta}\right)^{-k(\beta-1)+1} \frac{\{-\log \overline{G(y)}\}^{n-1}}{(n-1)!} dy .$

Proof. a) The necessity is obvious. By the definition of conditional expectation is concluded that

$$E(X_1|X_1 > x) = \frac{1}{G(x)} \int_x^\infty y \, g(y) dy . \quad (1)$$

By equating (1) with assumption (a), is derived

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$$\int_x^\infty y\, g(y)dy = \int_x^\infty \frac{k}{B(\alpha.\beta)}\{1 - (\tfrac{y}{\theta})^{-k}\}^{\alpha-1} (\tfrac{y}{\theta})^{-k(\beta-1)+1}\, dy\ .$$

Differentiating both sides of the above equation with respect to x, is obtained

$$x\, g(x) = \frac{k}{B(\alpha.\beta)}\{1 - (\tfrac{x}{\theta})^{-k}\}^{\alpha-1} (\tfrac{x}{\theta})^{-k(\beta-1)+1}.\ (2)$$

Equation (2) completes the proof a. Other parts can be proved by similar method.

Remark. The same results as the above theorem can be derived based on right truncated moment instead of left truncated moment.

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Copula Function and Measures of Association

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Abstract

Copulas allow expressing multivariate distributions in terms of their marginal distributions and multivariate dependence structure. Copulas are a very popular tool and are widely applied. In this paper, we use the copula function for computing some measures of association between two random variables from various copulas. Then we compare these measures numerically and graphically. Finally, by examining a real data, the role of a copula function in estimating measures of association has been investigated.

Keywords

Copula Function, Spearman's rho, Kendall's tau, Blomqvist beta.

1- Introduction

Copulas provide a useful way to model different types of dependence structures explicitly. Instead of having one correlation number that encapsulates everything known about the dependence between two variable, copulas capture information on the level of dependence as well as whether the two variables exhibit other types of dependence.

Let (X, Y) be a random vector with density function $f(x, y)$, distribution function $F(x, y)$ and marginals $F_X(x)$ and $F_Y(y)$. The copula function $C(u, v)$ is a bivariate distribution function with uniform marginals on $[0,1]$, such that $F(x, y) = C_F(F_X(x), F_Y(y))$. By Sklar's Theorem (Sklar, 1959), this copula exists and is unique if $F_X(x)$ and $F_Y(y)$ are continuous. Also, the copula C_F is given by

$$C(u, v) = F(x, y) = F(F_X^{-1}(u), F_Y^{-1}(v)), \ \forall u, v \in [0,1],$$

where, F_X^{-1} and F_Y^{-1} are quasi-inverses of F_X and F_Y respectively, (Nelsen, 2006). The partial derivatives $\frac{\partial C(u,v)}{\partial u}$ and $\frac{\partial C(u,v)}{\partial v}$ exist and density function of $C(u, v)$ is defined as

$$c(u, v) = \frac{\partial^2 C(u, v)}{\partial u \partial v} = \frac{\partial^2 f(x, y)}{f_X(x) f_Y(y)}.$$

In this paper, we review three measures of association and calculate these measures for some common Archimedean copula. Then we compare these measures numerically and graphically.

2- Some Measures of Association

In this section, we review some measures of association for two random variables from copula function $C(u, v)$. These measures include: Spearman's rank correlation coefficient, Kendall's tau

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correlation and Blomqvist's beta medial coefficient compute directly by using copula function and are calculated as follows:

$$\rho_s = 12\int_0^1\int_0^1 uv\, dC(u,v) - 3 = 12\int_0^1\int_0^1\big[C(u,v) - uv\big]dudv = 12\int_0^1\int_0^1 C(u,v)dudv - 3 = E\big[C(U,V)\big] - 1,$$

$$\tau = 1 - 4\int_0^1\int_0^1\left[\frac{\partial C}{\partial u}\cdot\frac{\partial C}{\partial v}\right]dudv = 4\int_0^1\int_0^1 C(u,v)dC(u,v) - 1 = 4\,E_C\big[C(U,V)\big] - 1,$$

$$\beta = 4C\left(\frac{1}{2},\frac{1}{2}\right) - 1,$$

where U and V are two i.i.d. $U(0,1)$ random variables.

3- Archimedean Copulas

In this section, some one-parameter families of copulas were considered. These are three of the most common Archimedean copula models, namely:

(a) The Clayton family, also known in the survival analysis literature as the gamma frailty model;

(b) The Frank family;

(c) The Gumbel family originally considered by Gumbel in the context of extreme-value theory.

The Archimedean models are commonly used in practice, particularly in survival analysis, because of their interpretation as mixture models and the natural extension they provide for Cox's proportional hazards model; these copulas also used in clinical, actuarial, economic and engineering studies; for example Klugman and Parsa (1999) provides a strategy for fitting bivariate models and for checking goodness-of-fit and Wang and Wells (2000) propose model selection procedures for bivariate survival models for censored data generated by the Archimedean copula family.

Definition 3.1. A 2-dimensional copula C is called an Archimedean copula if and only if there exists a continuous, strictly decreasing, convex function φ (called generator) from $[0,1]$ to $[0,\infty]$ such that $\varphi(1) = 0$ and $C(u,v) = \varphi^{[-1]}(\varphi(u) + \varphi(v))$.

In Table 1, three copula functions mentioned along with their generator function. In these copula functions, θ is dependence parameter; also range of dependence parameter θ displayed in Table 1 for various copula functions.

Remark. Let (X,Y) be random vectors with an Archimedean copula C generated by φ. The Kendall's tau for X and Y is given by

$$\tau = 1 + 4\int_0^1\frac{\varphi(t)}{\varphi'(t)}dt.$$

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Table 1: Archimedean copulas and measures of association

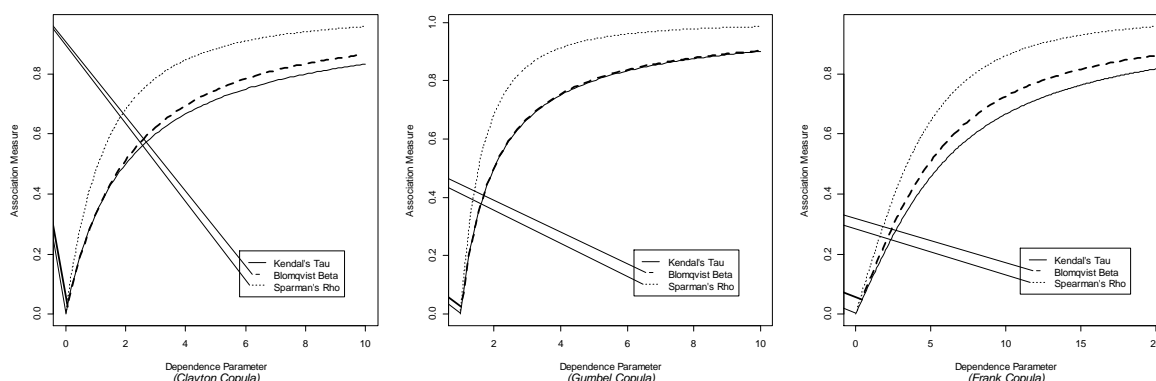| Copula	Clayton	Frank	Gumbel
C(u, v)	$(u^{-\theta}+v^{-\theta}-1)^{-1/\theta}$	$-\dfrac{1}{\theta}\log\left[1+\dfrac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right]$	$exp\left[-\{(-\log u)^{\theta}+(-\log v)^{\theta}\}^{1/\theta}\right]$
Generator $\varphi(t)$	$\dfrac{t^{-\theta}-1}{\theta}$	$-\log(\dfrac{e^{-\theta t}-1}{e^{-\theta}-1})$	$(-\log t)^{\theta}$
Range of θ	$[-1,\infty)\setminus\{0\}$	$(-\infty,\infty)\setminus\{0\}$	$[1,\infty)$
Spearman's rho	No closed form	$1-\dfrac{12}{\theta}[D_1(\theta)-D_2(\theta)]$	No closed form
Kendall's tau	$\dfrac{\theta}{\theta+2}$	$1-\dfrac{4}{\theta}[1-D_1(\theta)]$	$\dfrac{\theta-1}{\theta}$
Blomqvist beta	$4\left[2^{\theta+1}-1\right]^{-1/\theta}-1$	$-\dfrac{4}{\theta}\log\left[\dfrac{2e^{-\theta/2}}{e^{-\theta/2}+1}\right]-1$	$4exp\{-\left[2^{1/\theta}\log 2\right]\}-1$

Figure 1: Comparing plots of measures of association for some Archimedean copulas

The measures of association mentioned in Section 2 for selected Archimedean copulas are given in Table 1, where D_k is the Debye function of order k is defined as:

$$D_k(x)=\frac{k}{x^k}\int_0^x\frac{t^k}{e^t-1}dt.$$

4- Numerical Results

It is easy to see that the measures of association are increasing functions from dependence parameter θ. In Table 2, we compute three measures of association for some Archimedean copulas. For simplicity, we consider the positive values of dependence parameter θ. As noted in Table 1, in Clayton and Gumbel copulas, spearman's rho don't have closed form; thus we calculate it numerically. Also, we draw comparing plots of these measures for three Archimedean copulas in Figure 1. According to the plots, it follows that spearman's rho is larger than the other two measures; also for small values of dependence parameter θ Blomqvist beta is smaller than Kendall's tau, but for other values of θ Blomqvist beta is greater than Kendall's tau.

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Table 2: The Measures of Association for some Archimedean copulas

Dependence parameter θ		0	0.5	1	1.5	2	3	4	5	6	7	8	9	10	20
Clayton Copula	Spearman's rho	0	0.29	0.48	0.60	0.68	0.79	0.85	0.88	0.91	0.93	0.94	0.95	0.96	0.99
	Kendall's tau	0	0.20	0.33	0.428	0.50	0.60	0.67	0.71	0.75	0.78	0.80	0.82	0.83	0.91
	Blomqvist beta	0	0.196	0.33	0.434	0.51	0.62	0.69	0.75	0.78	0.81	0.83	0.85	0.87	0.93
Gumbel Copula	Spearman's rho	----	----	0	0.48	0.68	0.85	0.91	0.94	0.96	0.97	0.977	0.98	0.99	0.996
	Kendall's tau	----	----	0	0.333	0.50	0.667	0.75	0.80	0.83	0.86	0.87	0.89	0.90	0.95
	Blomqvist beta	----	----	0	0.331	0.50	0.67	0.75	0.80	0.84	0.86	0.88	0.89	0.90	0.95
Frank Copula	Spearman's rho	0	0.08	0.16	0.24	0.32	0.45	0.56	0.64	0.71	0.76	0.80	0.83	0.86	0.96
	Kendall's tau	0	0.05	0.11	0.16	0.21	0.31	0.39	0.46	0.51	0.56	0.60	0.64	0.66	0.82
	Blomqvist beta	0	0.06	0.12	0.18	0.24	0.34	0.43	0.51	0.57	0.62	0.66	0.70	0.72	0.86

5- Modeling Example: Insurance Data

Insurance data are given in data frame '*Insurance*' in R software consist of the numbers of policyholders of an insurance company who were exposed to risk, and the numbers of car insurance claims made by those policyholders in the third quarter of 1973. In this data set '*Holders*' is numbers of policyholders and '*Claims*' is numbers of claims. In this section, by using the goodness of fit tests, we have fitted a copula function for two variables '*Holders*' and '*Claims*' in insurance data. For this purpose, we have used 'gofCopula' package in R software and we obtained following results by using two statistics 'RosenblattSnB' and 'RosenblattSnC' (These statistics are presented in Genest et al. (2009)). Then, by using the estimated value for the dependence parameter θ and the formulas in Table 1, we have estimated three measures of association. The results are shown in Table 3. According to the p-values of tests, the null hypothesis is not rejected in any of the cases, but by comparing the p-values of tests, it follows that the two copulas Frank and Gumbel have a good fit to the data.

Table 3: Goodness of fit test for Insurance data

Copula Function	P-value		$\hat{\theta}$	$\hat{\tau}$	$\hat{\rho}_s$	$\hat{\beta}$
Clayton	0.551	0.599	5.183	0.721	0.890	0.754
Frank	0.585	0.988	25.393	0.853	0.973	0.891
Gumbel	0.551	0.827	6.941	0.856	0.970	0.859

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On the other hand, by using the raw data, three measures of association for the two variables '*Holders*' and '*Claims*' are as follows: $\hat{\tau} = 0.863, \hat{\rho}_s = 0.973, \hat{\beta} = 0.969$. After comparing these values with those obtained using the goodness of fit test in Table 3, we see the role of the copula function in the estimate of the values of the measures of association.

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Data Mining Techniques on Educational Databases: A Case Study, Iranian Administration System Dataset

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Abstract

Data mining techniques can be used to extract meaningful knowledge and useful information from the data; consequently assisting educational institutions in better allocating resources and staff. The main goal of this study is to use data mining techniques in analyzing the effects of different aspects of family background, and the socioeconomic status of parents on educational performance of applicants in Wide Entrance Examination (WEE): university admission system in Iran. The Iranian university and college admission system involves prospective students listing up to one hundred majors in order of their preference in the application. This research deals with specific WEE data created and collected over 10 years by NOET (National Organization of Educational Testing). In order to investigate the effects of family background factors and other related variables on educational performance (entrance examination grades) we apply various statistical methods and data mining. We do so as we consider the complete data set which consisting of roughly six million WEE applicants' information from three different main testing groups (mathematics and physics, empirical sciences, and human sciences). The results of this analysis show that on one hand parental education, parental occupation and the socioeconomic status of family have a large effect on entrance examination grades and as a result on university and college acceptance. On the hand the number of family members and age of applicants have a negative effect. Moreover, the proportion of acceptance in universities for the high social class families is more than the proportion of candidates in the other categories. In other words, applicants who come from higher social classes have a significantly better chance of getting admission at a university. We believe that this study provides a better understanding of the factors that could have significant impact on the applicant's admission, and as a result on the higher education achievement. Such insights can assist decision makers about conditions and quotas applied for future exams.

Keywords

Educational Data Mining, Educational Performance, Prediction model, Classification & Regression Tree, Artificial Neural Network, Entrance Examination Grades.

Detecting Bubbles in Tehran Stock Exchange

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Abstract

In the last 15 years there has been new researches in mathematical models for financial bubbles. Recent advances suggest the use of strict local martingales as models for bubbles. In this article, we have applied these methods to the stock prices in Tehran Stock Exchange to test the existence of possible bubbles.

Keywords

Bubble, Strict local martingale, Stochastic differential equation.

1- Introduction

Financial bubbles have caused financial crisis since the seventeenth century. Some historical examples are the Dutch tulipmania in 1634-1637, Mississippi bubble in 1719-1720 and the South Sea bubble in 1720.

Briefly speaking, a bubble is said to exist in the price of an asset if that asset is traded with a price very greater than it's intrinsic value. Usually bubbles consist of two phases. First, a run-up phase where the price strongly increases and second, a decline phase in which the price decreases strongly. More recent examples of bubbles in financial markets are the great depression of 1929 in United States and, the housing bubble of 1970-1989 in Japan, the Dot-Com bubble of 1997-2002 and the housing bubble of 2008-2011 in United States.

Its clear that predicting a bubble or detecting it in early stages is very important. This problem has been studied by the economists since long ago but it is just in the last 15 years where financial mathematics has gained some advances in this subject. In financial mathematics, bubbles are closely related to strict local martingales (i.e local martingales which are not true martingales). This idea first appeared in Loewenstein and Willard [1]. In Cox and Hobson [2] it is shown that a prices contains bubble if its price process is a strict local martingale under the risk neutral measure. This method is the basis of seminal works of Jarrow, Protter and shimbo [1] and [5]. In section 2, we give a precise definition for a bubble in terms of the risk neutral measure. Then review the relation between the bubbles and strict local martingales. In section 3, we model the price with a stochastic differential equation and review some conditions for its solution to be a strict local martingale.

The main part of this article is based on the method developed in [5]. We review this method in sections 4 where an estimator of the volatility is provided based on the discretization of the price

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process. Section 5 is the main contribution of this article. In that section we apply the introduced method on the data from Iran's stock prices.

2- Bubbles as Strict Local Martingales

Let (Ω, F, P) be a complete probability space and $F = (F_t)$ be a filtration which satisfies the usual condition. Let $r = (r_t)$ be a progressively measurable process indicating the instantaneous interest rate and let

$$B_t = \exp(\int_0^t r_u \, du)$$

be the time t value of a money market account. We work on a fixed time interval $[0, t]$ where T is a fixed finite time or is ∞.

Assume there is one risky asset (e.g. a stock) with a lifetime τ which is a stopping time and τ<T. Let $D = (D_t)$ be the dividend process which we assume to be a semi-martingale. The asset has a market price which is denoted by $S = (S_t)$ and is again a semi-martingale. The wealth process is defined to be

$$W_t = 1_{\{t < \tau\}} S_t + B_t \int_0^{t \wedge \tau} \frac{1}{B_u} dD_u + \frac{B_t}{B_\tau} \Delta 1_{\{t \leq \tau\}}$$

where Δ is the terminal payoff of the asset at time τ.

In financial mathematics, the no arbitrage assumption is shown to be equivalent to the existence of an equivalent probability measure under which the price process is a local martingale. These measures are called risk-neutral measures. Moreover, if the market is complete, this measure is unique and the price of any derivative equals the expectation of its expected discounted future cash flow with respect to this measure. This price is called the fundamental value. Hence we define the fundamental value process by

$$S_t^* = E_Q \left(\int_{t \wedge \tau}^\infty \infty \frac{1}{B_u} dD_u + \frac{\Delta}{B_\tau} 1_{\{t \leq \tau\}} \mid F_t \right) B_t$$

Definition. The bubble process is $\beta_t = S_t - S_t^*$

One can show that always $\beta_t \geq 0$ (see [6], Theorem 1). We say that an asset price has bubble if β_t is not identically zero. This is the main theorem of this section.

Theorem([6], Corollary 1.(

Asset price has bubble if and only if S is a strict local martingale.

3- Necessary and Sufficient Condition for Strict Local Martingales

From now on, we assume that the price of the asset satisfies a stochastic differential equation

$$dS_t = \sigma(S_t) dB_t + \mu_t dt$$

where μ_t is the drift term and $\sigma(x)$ is the volatility function. Since under the risk-neutral measure the drift vanishes, we can assume that S_t satisfies

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$$dS_t = \sigma(S_t)dB_t \tag{1}$$

The following theorem provides a necessary and sufficient condition for S_t to be a strict local martingale.

Theorem (see [7]](

The solution of (1) is a strict local martingale if and only if, for some $\grave{o} > 0$,

$$\int_{\grave{o}}^{\infty} \frac{x}{\sigma(x)^2} dx < \infty$$

4- Estimating the Volatility

We saw in the previous section that the existence of bubble is equivalent to $\int_{\grave{o}}^{\infty} \frac{x}{\sigma(x)^2} dx < \infty$. This condition depends on the asymptotic behavior of the function $x \mapsto \sigma(x)$ for large x. In this section, our goal is to estimate the asymptotic behavior of the $\sigma(x)$ given the asset prices in a finite time interval. We use the method described in [5]. In this method, first we estimate $\sigma(x)$ for values that are attained by X_t and then using curve fitting, we approximate $\sigma(x)$ for large x. Let h_n be a sequence with $h_n > 0$ and $h_n \to 0$. Let $t_i = \frac{iT}{n}$ and define

$$\sigma_n^2(x) = \frac{\sum_{i=1}^{n} 1_{\{|S_{t_i} - x| < h_n\}} n(S_{t_{i+1}} - S_{t_i})^2}{\sum_{i=1}^{n} 1_{\{|S_{t_i} - x| < h_n\}}} \tag{2}$$

The following theorem shows that $\sigma_n^2(x)$ is a consistent estimator for $\sigma^2(x)$.

Theorem (see [5]).

Assume that $\sigma(x)$ is a C^3 function and S never vanishes and $nh_n^4 \to 0$ and $nh_n \to \infty$. Then $\sigma_n^2(x)$ converges in probability to $\sigma^2(x)$ on the interval $D = [\min S, \max S]$.

Unfortunately, the above theorem does not provide the asymptotic behavior of $\sigma(x)$ since it only estimate it on a finite domain. Therefore, we use curve fitting to approximate σ by the best power function x^α and then according to whether $\alpha > 1$ or $\alpha \le 1$, we can say that the bubble exists or does not exist.

5- An Example in Tehran Stock Exchange

In this section we apply the methods of previous sections on some stocks which are in TSE (Tehran Stock Exchange) to test that whether they contain bubbles or not. In the estimate (2), we take

$$h_n = \frac{1}{\sqrt{n}}.$$

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The stocks under study are the Esfahan Mobarakeh Steel Co. (EMSC), Civil Pension Fund Investment Co. (CPFIC), SAIPA Group (SAIPA), Bahman Group (BHMN) and Kharazmi Investment Group (KHIG). The time interval of study is 2012-2017. The data has been downloaded from website of Tehran Securities Exchange Technology Management Co. The test recognizes the existence of bubble in some of these stocks.

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Determining the Bias value in Comparison of Economic Statistical Design of T^2-FRS and $T^2-VSICL$ Control Schemes using Genetic Algorithm

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Abstract

The Hotelling's T^2 control chart, a direct analog of the univariate Shewhart $\overline{\mathrm{X}}$ chart, is perhaps the most commonly used tool in industry for simultaneous monitoring of several quality characteristics. Recent studies have shown that using variable sampling intervals and control limits (VSICL) schemes results in charts with more statistical power when detecting small to moderate shifts in the process mean vector when it is compared to the fixed ratio sampling (FRS) T^2 control scheme. Moreover, it is shown that the VSICL scheme is more economical than the FRS scheme. It is applied the cost model proposed by Lorenzen and vance (1986). Furthermore, it is assumed that the length of time that the process remains in control is exponentially distributed which allows applied the Markov chain approach for developing the cost model. It is applied genetic algorithm to determine the optimal values of model parameters by minimizing the cost function. This paper studies unbias comparison between Economic Statistical design T^2-FRS and $T^2-VSICL$ control charts with respect to the expected cost per unit time.

Keywords

T^2 **Control Chart, Economic Statistical Design (ESD), Fixed Ratio Sampling (FRS), Variable Sampling Intervals and control limits (VSICL), Unbias, Genetic Algorithm(GA).**

1- Introduction

The processes are characterized by several, usually correlated, variables indicating the need for the use of a multivariate control chart such as that due to Hotelling [1]. Lowry and Montgomery [2] mentioned the popularity of the Hotelling's T^2 chart in industrial applications leading to the development of control chart software for its application. One procedure to improve the statistical performance of FRS control schemes is a variable sampling intervals and control limits scheme that the sampling sizes and control limits are different between successive samples as a function of prior sample results. Aparisi and Haro [3] presented a T^2-VSI chart in which they adopted the simplifying assumption that the process starts form an out of control state corresponding to the specific amount of shift in the process mean. Faraz and Moghadam [4] extended their work to statistically design the T^2-VSI control scheme when the shift in the process mean does not occur at the beginning but at some random time in the future. Thus they developed a model involving a prior distribution for the amount of time the process remains in control. Further, they assumed that the

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occurrence time of the shift is an exponentially distributed random variable. As Woodall [5] mentioned, the main drawback of the ED's is that they typically have a high Type I error probability, which can lead to unnecessary process adjustments or a loss of trust in the control system. Saniga [6] remedied this shortcoming by developing a design method he called Economic Statistical Design (ESD); Here statistical constraints are placed upon the cost model of ED. Taylor [7] noted that economic control charts using FRS schemes are non-optimal designs. Chen [8] proposed another ED $T^2 - VSI$ control scheme method based on Duncan's model. Torabian et al. [9] presented $T^2 - VSICL$ base on the Costa and Rahim's model.

This paper is organized as follows: In section 2, $\mathrm{T}^2 - VSICL$ control scheme are reviewed. In section 3 unbias comparison between ESD $\mathrm{T}^2 - FRS$ and $\mathrm{T}^2 - VSICL$ is discussed. In section 4, we have industrial example and final section provides some concluding remarks.

2- The $\mathrm{T}^2 - VSICL$ control scheme and markov chain approach

Consider a process in which p correlated characteristics are being measured simultaneously and is jointly. It is assumed that the joint probability distribution of the p quality characteristics is a p −variate normal distribution with in-control mean vector $\mu_0' = (\mu_{01},...,\mu_{0p})$ and variance-covariance matrix Σ. The T^2 control chart requires computing the sampling means for each of the p quality characteristics from a sample of size n. Then the subgroup statistic $T_i^2 = n(\overline{x}_i - \mu_0)'\Sigma^{-1}(\overline{x}_i - \mu_0)$ is plotted on a control chart in sequential order. The chart signals as soon as $T_i^2 \geq k$. We assume the parameters μ_0 and Σ are known. In this case k is given by the upper α percentage point of a chi-square variable with p degrees of freedom.i.e., $k = \chi_\alpha^2(p)$. The following function summarizes the control scheme of the $\mathrm{T}^2 - VSICL$ control chart:

$$(h_i, k_i) = \begin{cases} (h_1, k_1) & if \quad 0 \leq T_{i-1}^2 < w \\ (h_2, k_2) & if \quad w \leq T_{i-1}^2 < k \end{cases} \tag{1}$$

The *AATS* is the average time from the process mean shift until the chart produces a signal. If the assignable cause occurs according to an exponential distribution with parameter λ then the expected time interval that the process remains in-control is $\frac{1}{\lambda}$. Therefore, $AATS = ATC - \frac{1}{\lambda}$. The control chart produces a signal when $T^2 \geq k$. If the current state is 3, the signal is a false alarm and absorbing state (state 6) is reached when the true alarm occurs. Therefore, the transition probability matrix is given by

$$p = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ 0 & 0 & 0 & p_{44} & p_{45} & p_{46} \\ 0 & 0 & 0 & p_{54} & p_{55} & p_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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where p_{ij} denotes the transition probability that i is the prior state and j is the current state. In what follows, $F(x, p, \eta)$ will denote the cumulative probability distribution function of a non-control chi-square distribution with p degrees of freedom and non-centrality parameter, $\eta = nd^2$, where $d^2 = (\mu_1 - \mu_0)\Sigma^{-1}(\mu_1 - \mu_0)$. In the at state, the expected number of trials is each state to reach the absorbing state can be obtained from $b'(I - Q)^{-1}$ where Q is the 5×5 matrix obtained from p on deleting the elements corresponding to the absorbing state, I is the identity matrix of order 5 and $b' = (p_1, p_2, p_3, 0, 0)$ is a vector of initial probabilities, with $\sum_{i=1}^{3} p_i = 1$. Hence, $ATC = b'(I - Q)^{-1}h$

Where $h' = (h_1, h_2, h_2, h_1, h_2)$ is the vector of sampling time intervals. In this paper assumed $b' = (0, 1, 0, 0, 0)$. In-control period$= \frac{1}{\lambda} + (1 - \gamma_1)T_0 \times ANF$ Where $\gamma_1 = 1$ if the process is not shut down during false alarms and 0 otherwise. T_0 stands for the expected time spent searching for a false alarm and ANF is the expected number of false alarms in each quality cycle and is calculated as follows $ANF = b'(I - Q)^{-1}f$ Where in that $f' = (0, 0, 1, 0, 0)$.

The expected total cycle time is given as the sum of the in control and out of control cycle times as follows

$$E[T] = \frac{1}{\lambda} + (1 - \gamma_1)T_0 ANF + AATS + nE + T_1 + T_2 = ATC + (1 - \gamma_1)T_0 ANF + nE + T_1 + T_2 \qquad (2)$$

The expected time to discover the assignable cause is given as T_1 and the expected time to repair the assignable cause is given as T_2. The expected cost of producing non-conformities while the process is in control and out of control is given by the following $\frac{C_0}{\lambda} + C_1[AATS + nE + \gamma_1 T_1 + \gamma_2 T_2]$ where γ_2 is an indicator function for if production continues during the repair of the process, C_0 and C_1 stand for the cost of producing non-conformities while the process is in control and out of control, respectively. The expected cost of evaluating false alarms and repairing the process is given by $a_3' ANF + a_3$ where a_3' is the cost of investigating false alarms and a_3 stands for the cost of locating and repairing an assignable cause. The expected sampling cost per cycle is given by $(a_1 ANS + a_2 ANI) + \frac{(a_1 + a_2 n_2)(nE + \gamma_1 T_1 + \gamma_2 T_2)}{h}$ where a_1 and a_2 are the fixed and variable cost components of sampling and testing, respectively. the ANI and ANS stand for the expected number of inspected items and samples taken during the quality cycle and are calculated using follow equations. $ANI = b'(I - Q)^{-1}n$, $ANS = b'(I - Q)^{-1}1$ where $n' = (n, n, n, n, n)$ and $1' = (1, 1, 1, 1, 1)$. Now the expected cost per cycle is

$$E(C) = \frac{C_0}{\lambda} + C_1[AATS + nE + \gamma_1 T_1 + \gamma_2 T_2] + a_3' ANF + a_3 + (a_1 ANS + a_2 ANI) + \frac{(a_1 + a_2 n_2)(nE + \gamma_1 T_1 + \gamma_2 T_2)}{h} \qquad (3)$$

Due to the renewal reward assumption, the expected cost per hour is just as follows

$$E(A) = \frac{E(C)}{E(T)} \qquad (4)$$

Therefore, the general optimization problem is defined as follows:

$$\min E(A)$$
$$s.t : 0 \leq w < k_2 \leq k_1, \ 0.1 \leq h_2 \leq h_1 \leq 8, n \in Z^+, \ ANF \leq ANF_0 \ and \ / \ or \ AATS \leq AATS_1 \tag{5}$$

3- An unbias comparison between ESD $\mathrm{T}^2 - FRS$ and $\mathrm{T}^2 - VSICL$ scheme

In order to make a unbias comparison between ESD $\mathrm{T}^2 - FRS$ and $\mathrm{T}^2 - VSICL$ scheme, it should be noted that both economic FRS and VSS schemes should have the same in-control time and cost to guarantee a meaningful comparison between these two schemes. Therefore, the two charts then require the same ANF, ANS and ANI values to be inspected during the in-control period.

4- An industrial example

In this section the proposed approach to the ESD of the $\mathrm{T}^2 - VSICL$ control chart is illustrated through on industrial example concerning the GM casting operation as presented by Lorenzen and Vance [10]. 5. It´s solved the optimization problem (5) with the constraint $ANF \leq 0.5$ to obtain the ESD of the $\mathrm{T}^2 - VSICL$ control chart scheme.

5- Concluding remarks

In this paper we have presented unbias comparison between Economic Statistical design $T^2 - FRS$ and $\mathrm{T}^2 - VSICL$ control charts when the in control process mean vector and process covariance matrix are known. The cost model adopted in the presented study in that of Lorenzen and Vance (1986) and derived by the markov chain approach. We applied the genetic algorithm to find the optimal chart parameters. The numerical comparison between the both ESD FRS and VSICL schemes has shown that when we use unbias design , results show that mean percentage decrease cost per unit time in $\mathrm{T}^2 - VSICL$ scheme with respect to $T^2 - FRS$ is 0.10, while the if we use bias design, it is 0.12, so this will lead to 2 percent error.

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Estimation of Risk Premium in the Pareto Distribution with the Presence of Outliers

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Abstract

The moment and credibility estimators of the Pareto parameter α are derived in the presence of outliers and the estimators of the risk premium are considered. At the end, it is shown that the risk premium based on maximum likelihood estimator of parameter α is the best. Finally, an actual example is considered to obtain the risk premium.

Keywords

Pareto distribution, Maximum likelihood, Moment method, Credibility, Outliers, Risk premium, Insurance.

1- Introduction

This paper presents a model based on the Pareto distribution to use in specific situations whether one or more observations are likely vary greatly from the rest of the data. Also, the tick tail of the Pareto distribution is an important topic for producing observations far from the rest of the data. In this paper, the most observations are from Pareto (α, θ) model with a certain lower bound of θ, but that outliers are generated from another Pareto model with a higher lower bound, say $\beta\theta$ when $\beta > 1$.

We know that an application of the Pareto distribution include insurance where it is used to model claims where the minimum claim is also the modal value, but where there is no set maximum. The minimum value of the claim amounts always are known for the insurance companies. Also, according to Benktander (1963), the Pareto distribution is useful for the automobile insurance problems.

Dixit and Jabbari Nooghabi (2011) considered a Pareto model with the presence of outliers. They assumed that a set of n random variables $(X_1, X_2, \ldots, X_n)$ represent claim amounts of a motor insurance company and assumed that k of them $(k \geq 1)$ are more expensive (because of expensive/severe damaged vehicle) such that their claims are β times higher than the normal vehicles. They considered that the claim amounts of these vehicles are distributed with the following Pareto (α, θ) probability density function (pdf)

$$f(x, \alpha, \theta) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}} \qquad 0 < \theta \leq x \ , \alpha > 0,$$

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and the others have the Pareto $(\alpha, \beta\theta)$ pdf

$$f(x, \alpha, \beta, \theta) = \frac{\alpha(\beta\theta)^\alpha}{x^{\alpha+1}} \qquad 0 < \beta\theta \leq x \quad, \alpha > 0 \quad, \beta > 1 \quad, \theta > 0.$$

Dixit and Jabbari Nooghabi (2011) assumed that β, θ and k are known and α is unknown. In this paper, the maximum likelihood and uniformly minimum variance unbiased estimator of α are introduced. Afterwards the moment and credibility estimators of the Pareto parameter α are derived and the estimators of the risk premium are considered. Finally, an actual example from Rytgaard (1990) is considered to obtain the risk premium. This example is presented information about all single loss amounts (from ground up) exceeding 1.5 million for the five years. At the end by using this example, it is shown that the risk premium based on maximum likelihood estimator of the parameter α is the best.

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Estimation of a Hazard Rate Function for Life Insurance Claim Data

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Abstract

Estimation of a hazard rate function can be beneficial for insurance company since it informs them about the instantaneous failure rates of specific ages. A feature that may arise in life insurance claim data is censoring. We extended the maximum likelihood estimator of a hazard function under a new extended Generalized Weibull distribution for both cases of uncensored and right censored life insurance claim data. Our numerical studies reveal that the proposed approach outperforms other parametric counterparts.

Keywords

Hazard Rae Function; Life Insurance Claims Data; Right Censored Data; Uncensored Data.

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Health Insurance Fraud Detection Through Data Mining

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Abstract

One of the challenges that insurers are facing is fraud. Fraud in health insurance involves intentional deception or misrepresentation intended for gaining some benefit. Data mining tools and techniques are applied to detect fraud in big data. In this paper, we present the concept of data mining and current techniques used in health insurance fraud detection, using supervised and unsupervised data mining approaches. This article provides a comprehensive review of different data mining techniques for detecting fraud in health insurance.

Keywords

Health insurance, data mining, fraud detection.

Inferences on the Linear Combination Mean of Several Heterogeneous Log-Normal Distributions

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Abstract

We consider the problem of making inferences on the linear combination mean of several heterogeneous log-normal distributions. We apply the generalized confidence interval (GCI) approach and the method of variance estimate recovery (MOVER) to construct confidence intervals for the linear combination of log-normal means. We then compare the performances of the proposed confidence intervals via a simulation study and a real data example. Simulation results show that our proposed MOVER and GCI confidence intervals can be recommended generally for different sample sizes and number of groups.

Keywords

Coverage probability; Log-normal distribution; Monte Carlo simulation; method of variance estimate recovery; generalized confidence interval.

1- Introduction

In practice such as biology, economy, insurance, medicine and pharmacology, a very applicable distribution for positive right-skewed data is the log-normal distribution; see [3, 6, 8]. As an example, [2] and [10] consider the medical charge data from analyzed this data set and concluded that the African American group (119 patients) and the white group (106 patients) have equal mean charge and both data are distributed as a log-normal distribution [section 5]. Also, [1] studied a possible link between the epidemiologic patterns of esophageal cancer (EC) and the anomalous concentration of some ions and elements in the drinking water sources in the four regions: Gonbad, Dashlibroon, Maravetappeh and Gorgan all in North of Iran. Primary analysis showed that the log-transformed data are normally distributed in all regions. The log-normal distribution is especially used in biomedical research. As noted by [5], the growth rates of soft tissue metastases of breast cancer can be modeled by a log-normal distribution and the transcript levels of five different genes in individual cells from mouse pancreatic islets are log-normally distributed. For more examples and applications of log-normal distribution in epidemiological and health related studies see [6, 8].

One of the problem of interest is to make inferences on the linear combination mean of populations. It is worth mentioning that several researchers have considered the problem of making inferences on the linear combination mean of several normal populations; among others, see [4] and the references therein. However, there are situations that the log-transformed observations have a normal distribution instead of the original observations; i.e. the observations have a log-normal distribution.

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Although by the logarithm transformation the problem backs to the normal case, a research may interest in making inference about the mean response in the original scales since the mean of log-transformed data may not have a practical interpretation; see [7]. For example, in the example of [1], they considered Gorgan as a case–control point due to its comparatively low incidence rate of EC and Gonbad, Dashlibroon and Maravetappeh as high esophageal cancer areas with a common mean. It is of interest in making inferences about the anomalous concentration of some ions in the drinking water sources in the three regions with comparison of Gorgan region.

In this paper, we apply the generalized pivotal quantity (GPQ) approach to construct a confidence intervals for the linear combination mean of several heterogeneous log-normal distributions. Note that a test for the linear combination mean can be performed by inverting the confidence interval. We also apply the MOVER to construct a closed-form approximate confidence. We then compare the proposed confidence intervals via a simulation study in terms of coverage probability (CP) and average length (AL). Simulation results show that our proposed MOVER confidence interval are preferred generally for different sample sizes and number of populations.

This article is organized as follows. Some necessary background and notation are presented in Section 2. The new confidence intervals are discussed in Section 3. The performances of confidence intervals are studied in Section 4. In Section 5, the proposed methods are illustrated with a real examples. Section 6 contains our conclusion.

2- Notation and Necessary Background

Suppose that $X_{i1}, \dots, X_{in_i}$, $i = 1, \dots, k$, is a set of random samples from the i-th population which has a log-normal distribution with parameters μ_i and σ_i^2, hereafter $LN(\mu_i, \sigma_i^2)$. Let $\eta_i = E(X_{ij}) = \exp(\eta_i^*) = \exp(\mu_i + \sigma_i^2/2)$ denote the mean of the i-th population. We are interested in making inferences about $\eta = \sum_{i=1}^{k} c_i \eta_i$ such as testing hypothesis $H_0: \eta = \eta_0$ and constructing confidence interval for η where c_i's are known values.

Let $Y_{ij} = \log(X_{ij})$, $i = 1, \dots, k$ and $j = 1, \dots, n_i$. It is well known that the Y_{ij} is distributed as a normal distribution with mean μ_i and variance σ_i^2, henceforth $N(\mu_i, \sigma_i^2)$. The unbiased maximum likelihood estimators and uniformly minimum variance unbiased estimators (UMVUEs) of μ_i and σ_i^2 can be obtained by the following equations $\bar{Y}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} Y_{ij}$, and $S_i^2 = \frac{1}{n_i-1}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y}_i)^2$. It is well-known that $\bar{Y}_i$ and S_i^2 are independent and $\bar{Y}_i \sim N(\mu_i, \sigma_i^2/n_i)$, and $(n_i - 1)S_i^2/\sigma_i^2 \sim \chi^2_{(n_i-1)}$, $i = 1, \dots, k$, where $\chi^2_{(r)}$ denotes a chi-square distribution with r degrees of freedom.

3- Proposed Confidence Intervals

In this section, we first provide a version of the MOVER confidence interval for the log-normal linear combination mean. Then, we propose a generalized confidence intervals for it.

a. The Mover Confidence Interval

The MOVER is a useful technique to find a closed-form approximate confidence interval for a linear combination of parameters based on the confidence intervals of the individual parameters. To describe the MOVER technique, consider a linear combination $\sum_{i=1}^{k} w_i \theta_i$ of parameters $\theta_1, \dots, \theta_k$

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where w_i are known constants. Let $\hat{\theta}_i$'s be independent unbiased estimates of θ_i's. In addition, let (L_i, U_i) denote the $100(1 - \alpha)\%$ confidence interval for θ_i, $i = 1, \dots, k$. Then, the $100(1 - \alpha)\%$ MOVER confidence interval for $\sum_{i=1}^{k} w_i \theta_i$ is (L, U) where

$$L = \sum_{i=1}^{k} w_i \hat{\theta}_i - \sqrt{\sum_{i=1}^{k} w_i^2 (\hat{\theta}_i - L_i^*)^2}, \text{ with } L_i^* = L_i \text{ if } w_i > 0 \text{ and } L_i^* = U_i \text{ if } w_i < 0, \text{ and}$$

$$U = \sum_{i=1}^{k} w_i \hat{\theta}_i - \sqrt{\sum_{i=1}^{k} w_i^2 (\hat{\theta}_i - U_i^*)^2}, \text{ with } U_i^* = U_i \text{ if } w_i > 0 \text{ and } U_i^* = L_i \text{ if } w_i < 0.$$

In our problem, let $\eta_i^* = \mu_i + \sigma_i^2/2$ and $\hat{\eta}_i^* = \bar{Y}_i + S_i^2/2$ be an unbiased estimate of η_i^*, $i = 1, \dots, k$. Therefore, a confidence interval can be obtained for the log-normal mean η_i^* by combining the individual confidence intervals for μ_i and σ_i^2. In this regard, notice that the exact confidence intervals for μ_i and σ_i^2 are $(\bar{Y}_i \pm t_{1-\alpha/2}(n_i - 1) S_i/\sqrt{n_i})$ and $\left(\frac{(n_i-1)S_i^2}{\chi^2_{1-\alpha/2}(n_i-1)}, \frac{(n_i-1)S_i^2}{\chi^2_{\alpha/2}(n_i-1)} \right)$, where $t_\gamma(n)$ and $\chi^2_\gamma(n)$ denote the γ-th percentile of t-distribution and chi-square distribution with n degrees of freedom. Using the MOVER, an approximate confidence interval for η_i^* can be (L_i^*, U_i^*) where $L_i^* = $

$$\hat{\eta}_i^* - S_i \sqrt{ \frac{t_{\alpha/2}^2(n_i-1)}{n_i} + \frac{S_i^2}{4} \left(1 - \frac{n_i-1}{\chi^2_{1-\frac{\alpha}{2}}(n_i-1)} \right)^2 } \text{ and } U_i^* = \hat{\eta}_i^* + S_i \sqrt{ \frac{t_{\alpha/2}^2(n_i-1)}{n_i} + \frac{S_i^2}{4} \left(1 - \frac{n_i-1}{\chi^2_{\frac{\alpha}{2}}(n_i-1)} \right)^2 }, \text{ see}$$

[12]. Therefore, the $100(1 - \alpha)\%$ approximate confidence interval for the ith population mean, i.e. η_i, is (L_i, U_i) where $L_i = \exp(L_i^*)$ and $U_i = \exp(U_i^*)$. Also, the $100(1 - \alpha)\%$ approximate confidence interval for the η can be (L, U) where

$$L = \sum_{i=1}^{k} c_i \exp(\hat{\eta}_i^*) - \sqrt{\sum_{i=1}^{k} c_i^2 (\exp(\hat{\eta}_i^*) - L_i^{**})^2}, \quad L_i^{**} = \begin{cases} L_i & \text{if } c_i \geq 0 \\ U_i & \text{if } c_i < 0 \end{cases},$$

and

$$U = \sum_{i=1}^{k} c_i \exp(\hat{\eta}_i^*) + \sqrt{\sum_{i=1}^{k} c_i^2 (\exp(\hat{\eta}_i^*) - U_i^{**})^2}, \quad U_i^{**} = \begin{cases} U_i & \text{if } c_i \geq 0 \\ L_i & \text{if } c_i < 0 \end{cases}.$$

$$(1)$$

Remark 1: The null hypothesis $H_0 : \eta = \eta_0$ at level α is rejected when $(L - \eta_0) \times (U - \eta_0) > 0$.

b. The Generalized Confidence Interval

The concepts of generalized test variable and generalized pivotal quantity (GPQ) are introduced by Tsui and Weerahandi[9] and Weerahandi[11], respectively, and are successfully applied to different problems. Here, we use these concept of generalized pivotal quantity for constructing a generalized confidence interval. Based on $\bar{x}_i$ and s_i as observed values of $\bar{X}_i$ and S_i, GPQ of η_i^* is given by

$$R_{\eta_i^*} = \bar{x}_i + Z_i s_i \sqrt{ \frac{n_i - 1}{n_i U_i} + \frac{n_i - 1}{2n_i U_i} }, \tag{2}$$

for $i = 1, \dots, k$, where Z_i and U_i are independently distributed as $N(0, 1)$ and $\chi^2_{(n_i-1)}$. So, the generalized pivotal quantity related to η_i is $R_i = \exp(R_{\eta_i^*})$ for $i = 1, \dots, k$. Therefore, the generalized pivotal quantity $\eta = \sum_{i=1}^{k} c_i \eta_i$ is $R = \sum_{i=1}^{k} c_i R_i$. The generalized confidence interval at $(1 - \alpha)$ confidence level for η is

$$\left(R_{\alpha/2}, R_{1-\alpha/2} \right), \tag{8}$$

where R_β is a 100βth percentile of R_{12}. It can be estimated as well as algorithm 1.

Remark 2: Equality hypothesis is rejected at α level when the generalized p-value

$$p_G = 2\min\big(P(R > \eta_0), P(R_{12} \le \eta_0)\big),$$

is less than α.

4- Simulation Study

In this section, we present the results of our simulation study in which we compare the performances of our proposed generalized confidence interval and the method of variance estimate recovery confidence interval to compare the coverage probability (CP) and average length (AL) of the these proposed approaches. We considered different values and scenarios for σ and n. The results of simulation for the different values of σ, n, and $1 - \alpha = 0.95$ are done. Note that estimated CPs are approximately normal variables with mean 0.95 and variance $0.95 \times 0.05/10000$. Therefore, a 95% lower bound for estimated CPs is 0.946. If the estimated CP of a method is less than 0.946, we can conclude that the method is liberal. In Tables 1, the bold face values show that the estimated CPs are significantly less than 0.95. We note that a method is preferred to other methods when its estimated CP is not significantly less than 0.95 and it has the smallest AL among the methods that are not liberal. The simulation results show that our proposed MOVER and GCI confidence intervals can be recommended generally for different sample sizes and number of groups.

5. Numerical Example

Based on the medical charge data, the sample mean (sample variance) of the log-transformed data are 9.067 (1.824) and 8.693 (2.693), respectively. The 95% two-sided GCI, and MOVER confidence intervals for the different linear combination are shown in Table 1. We also presented the lengths of confidence intervals in this table.

Table 1. The 95% confidence intervals for the different linear combination mean charges.

c_1, c_2	Methods	Confidence interval	Length
1, -1	GCI	(-19108.77, 11259.35)	30368.13
	MOVER	(-15597.67, 11204.45)	26802.12
2, -1	GCI	(-769.31, 41250.05)	42019.38
	MOVER	(1766.93, 38819.41)	37052.48
-1, 2	GCI	(5693.73, 59564.70)	53870.97
	MOVER	(-1722.03, 45274.84)	46996.87
3, -2	GCI	(-19267.57, 52478.14)	71745.71
	MOVER	(-13397.37, 49600.15)	62997.51
-2, 3	GCI	(-5949.07, 77810.31)	83759.39
	MOVER	(-14383.33, 58824.56)	73207.89

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Insurance Against Losses from Natural Disasters. Empirical Evidence from Natural Disasters in the History of Iran

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Abstract

Natural disasters cause significant financial pressures, with pecuniary short-term impacts and long-term development implications. Reallocation is essentially fiscal response to disaster. Panel data is essentially cross-sectional data that is pooled together for a certain period of time with equal intervals. Panel data analysis is applied to series of data collected from the years 2000 to 2011 for 8 different natural disasters in Iran which are collected from different sources. The aim of this paper is to examine the relationship between the economic growth and natural disasters and to explain the pricing the net premium and offering applicable plan for insurance policies in different fields like fire insurance and life insurance to recover the losses. Each natural disaster has its impact on economic growth based on its severity. It is concluded that the impact of disasters could be covered by the insurance industry and government budget. The government must provide budget prospects for future natural disasters to afford the losses.

Keywords

Natural disasters, financial pressures, insurance industry, panel data, economic growth.
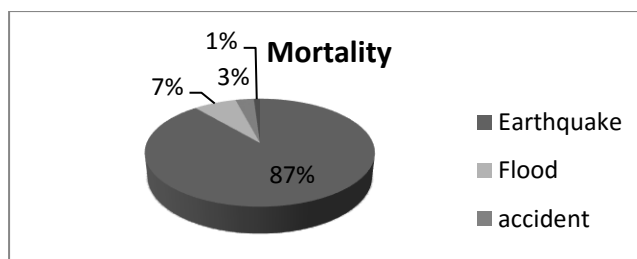
1- Introduction:

Natural disasters such as earthquake, flood, storm, landslide, thunderstorm, drought and hurricane expose serious damages and so seem to be terrible for the economy. For firms, natural disasters destroy touchable assets such as buildings and equipment – as well as human capital – and by that decline their production capacity. These negative impacts may sometimes be fatal to the firms and result in them being forced to close down. While most natural disasters are fairly local in their impact, the worst can change the world economy. The economic impact on the world can be strong and serious. Extensive natural disasters have serious negative economic impacts. Disasters also appear to have negative long-term consequences for economic growth and poverty reduction. But, negative impacts are not unavoidable. The vulnerability is shifting rapidly, especially in countries experiencing economic revolution, urbanization, and related technical changes.

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The impact of natural hazards - weather instability, climate extremes, and geophysical events - on economic well-being and human sufferings have increased seriously. More than three-quarters of recent losses can be attributed to windstorms, floods, droughts, thunder storm and other climate-related hazards. Beyond the headline costs are the potential chain reactions of negative economic impacts to countries around the world. We live in a truly global economy, and we are regularly reminded of this fact whenever faced with a significant natural hazard.
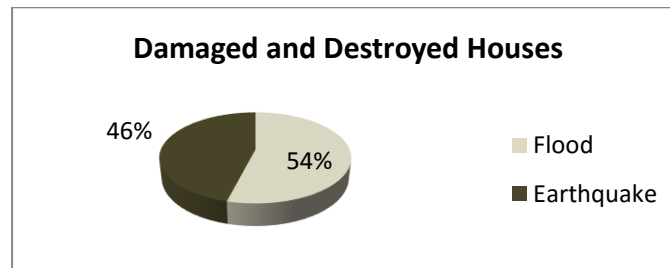
Iran is located in south-west Asia and covers an area of 1,648, 000 square kilometers. Located on the world dry belt, 60 percent of Iran is covered with mountains and the remaining part is desert and arid lands. According to its location, Iran is a disaster prone country. Among the 40 different types of natural disasters observable in different parts of the world, 31 types have been identified in Iran. Major natural disasters include frequent serious earthquakes, floods, droughts, landslides, desertification, deforestation, storms and the like. Earthquakes take a heavy toll. Iran is part of the Alp-Himalaya orogenic belt and is known as part of the youngest and last orogenic regions of the world. As a result, Iran suffers severe economic and social damages resulting from seismic activities within its territory. Earthquakes have killed more than 180 000 people during the last 90 years. Many cities including Tehran, Tabriz, Rudbar, Manjil, Tabas, Lar, Qazvin, Zanjan, Hamedan, Kermanshah, and Fars have sustained substantial damages due to high magnitude earthquake activities. Review of the historical seismic data shows that almost all parts of the country are affected by the physical, social and economic problems associated with earthquakes. It is worth mentioning that due to the political, social and economic stability of our country in the region, Iran has been the largest refugee host country for more than a decade and thus Iran regularly deals with complex human emergencies on top of all the natural disasters. (National report of Islamic Republic of Iran, 2005)

The panel data for 11 years starting from 2000 to 2011 on annual basis and 8 Different catastrophic disasters in Iran are collected from different sources. These 8 disasters are the most significant disasters that are included; accident, earthquake, fire, flood, storm, landslide, thunder storm and drought. Regarding the natural disasters in Iran, the 1990 Manjil earthquake occurred and it was estimated 40.000 loss of life with 75.000 injured. The Blizzard of February 1972 resulted in the deaths of more or less 4,000 people. The Iran drought on 2000 affected 37.000.000 person's life and the north of Iran's flood on Aug 2001 affected on 1.200.200 life. The 2003 earthquake was particularly destructive in Bam, with the death of at least 26,274 people and injuring up to 30,000. These are part of the natural disasters that have occurred in Iran.

The category of mortality due to natural disasters in Iran is shown as below:

And the classification of damaged and destroyed houses by natural disasters is shown as:

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Damaged and Destroyed Houses

46% 54%

■ Flood
■ Earthquake

According to statistics, the number of deaths in earthquakes is more than the other disasters and the damaged or destroyed houses in flood risk are highest among others. There are number of studies which explore the effect of natural disasters on economic growth. In reducing economic damages caused by disasters, not only income rather a strong financial sector is also important. If finances are readily available, it facilitates the speedy recovery which in turn enhances the development and regaining of the pre-disaster economic growth. Countries with higher levels of domestic credit better able to withstand and endure natural disasters without affecting their economic output much Governments need convenient risk management strategies for future probable disasters that include financial planning for years.

Natural disasters often have an adverse impact on insurance companies such as on the insured. They cause considerable losses and persuade insurers to take various measures to balance their technical performance. Cover possible damages of natural disasters by providing an idea about the future and get a low premium to the future supply is our suggestion.

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Inference on Selecting Optimal Copula Function Based on the Tracking Interval with Application in Insurance

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Abstract

One of the important issues in order to model dependency structure between interested variables is finding the proper copula function. Extensive studies has been done based on some criterions. The previous methods of selecting copula functions when the sample size is too small is not satisfactory. Therefore, our method is based on tracking interval for the semi-parametric copula function which is obtained using expected Kullback-Leibler risk between the two proposed non-nested semi-parametric copula model. It can find optimal semi-paramteric copula between proposed copula functions in a good level of significance. At the end, efficiency and capability of the presented method for Insurance data using simulation has been shown.

Keywords

Copula, Tracking Interval, expected Kullback-Leibler risk.

1- Introduction

The connection between a multivariate distribution function and its margins is consequential for statistical modelling in some fields of science such as Insurance. Since long ago, many statisticians have been interested and examined on this subject. Under reasonable conditions, we can uniquely determine a joint distribution for n random variables by specifying the univariate distribution for each variable and, in addition, specifying the copula (Sklar, 1959). In the semi-parametric approach the copula function is parametric, while marginal distributions are non-parametric. To estimate unknown parameters in semi-parametric copula function and their asymptotic behavior introduced some approaches in two steps. One of the challenges in this area is how between different existed semi-parametric copula functions, which have different characteristics dependence, select optimal copula function. The previous methods of selecting copula functions when the sample size is too small is not satisfactory. In this paper, we intend to provide another tool for selecting the optimal copula function based on the tracking interval which provides equivalent models or closer model to the true model of data in a good level of significance among the set of proposed models. Since each confidence interval is a set of accepted hypotheses under the null hypothesis, so the obtained interval can be used to select the appropriate model. In fact, we offer another criteria using tracking interval to select the appropriate copula function.

In the following, we first consider the behavior of estimators of the multivariate semi-parametric copula functions when the proposed model is badly described. In such a way that these estimators minimize the Kullback-Leibler criteria. Then, based on the Vuong's test(Vuong, 1989), we examine the statistical assumptions for choosing the appropriate copula function. We will also introduce the tracking interval which at a predetermined level of significance can determine the optimal model among non-nested proposed models. Then, with simulation study, we examine the tracking interval for the proposed models. At the end, the application of the proposed inference for the insurance data is analyzed as the stated theory is confirmed.

2- Copula Density Estimation

Let $X_1 = (X_{11}, \dots X_{d1})', \dots, X_n = (X_{1n}, \dots X_{dn})'$ be a random sample from the unknown multivariate distribution function $F_1(x_1, \dots, x_d)$. Following Sklar's theorem (Sklar, 1959) the joint distribution function F can be written as $F(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d))$, where C is a copula distribution function, and $F_1, \dots, F_d$ are the marginal distribution functions.

Let $\{c_i(F_1(X_1), \dots, F_d(X_d)), \theta_i \in A_i \subset R^{a_i}\}$ for $i = 1,2$ is a class of semi-parametric multivariate copula density functions. θ_i, A_i and a_i are the dependency parameter, the parameter space, and the number of parameters of the i-th copula function, respectively. we consider $F_j(X_{jt}) = U_{jt}$ for $t = 1, \dots, n$ and $j = 1, \dots, d$. The pseudo log likelihood function of the proposed multivariate semi-parametric model is , $l_1(U_{1t}, \dots, U_{dt}; \theta_1) = \sum_{t=1}^n \log c_1(U_{1t}, \dots, U_{dt}; \theta_1)$, the parameter θ_1 and $U_{1t}, \dots, U_{dt}$ are unknown, which should be estimated .(see Chen et al, 2005).

3- Test Statistic to Select Non-Nested Copula Function

Vuong(1989) proposed , based on the second sentence, a hypothesis test the equivalence or closeness of one the proposed models. The two copula $c_1(u_{1t}, \dots, u_{dt}; \theta_1)$ and $c_2(u_{1t}, \dots, u_{dt}; \theta_2)$ are non-nested if $c_1(u_{1t}, \dots, u_{dt}; \theta_1) \cap c_2(u_{1t}, \dots, u_{dt}; \theta_2) = \emptyset$. following Vuong(1989) consider hypotheses as:

$H_0 : E^0[\log c_1(F_1^0, \dots, F_d^0; \theta_1)] = E^0[\log c_2(F_1^0, \dots, F_d^0; \theta_2)]$

$H_0' : E^0[\log c_1(F_1^0, \dots, F_d^0; \theta_1)] > E^0[\log c_2(F_1^0, \dots, F_d^0; \theta_2)]$

$H_0'' : E^0[\log c_1(F_1^0, \dots, F_d^0; \theta_1)] < E^0[\log c_2(F_1^0, \dots, F_d^0; \theta_2)]$

In this sense, the non-rejection of the null hypothesis states that the two proposed models for a semi-parametric multivariate copula function are equivalent. The accept H_0' shows the superiority of the c_1 model to the c_2 model. Also, the accept H_0'' states that c_2 is closer than c_1 to an unknown data model.

In order to construct the test statistic, we will display the difference between the pseudo-log likelihood functions of copula with the symbol LR_n , and we have $LR_n(\hat{\theta}_2, \hat{\theta}_1) = \frac{1}{n} \sum_{t=1}^n \log \frac{c_2(u_{1t}, \dots, u_{dt}; \hat{\theta}_2)}{c_1(u_{1t}, \dots, u_{dt}; \hat{\theta}_1)}$

Therefore, we will have convergence is distribution as follows: (see Chen et al. 2005)

$$\sqrt{n}(LR_n(\hat{\theta}_2, \hat{\theta}_1) - E^0\left[\log \frac{c_2(F_1^0, \dots, F_d^0; \theta^*_2)}{c_1(F_1^0, \dots, F_d^0; \theta^*_1)}\right]) \xrightarrow{L} N(0, V^2)$$

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4- Obtaining Tracking Interval

The expected Kullback-Leibler risk for the proposed copula function $c_1(u_{1t}, \ldots, u_{dt}; \hat{\theta}_1) = c_1(.; \hat{\theta}_1)$ will be obtained as follow:

$$EKL\left(c^0, c_1(.; \hat{\theta}_1)\right) = E^0[\log c^0] - E^0\left[\frac{1}{n} l_1(.; \hat{\theta}_1)\right] + \frac{1}{n} trac(A_1 B_1^{-1}) + o_p(n^{-1})$$

Similarly, for the proposed copula function $c_2(u_{1t}, \ldots, u_{dt}; \theta_2)$,

Therefore, difference of the Kulback-Leibler risk expectation is shown for the two proposed multivariate semi-parametric copula functions c_1 and c_2 with the symbol $D(\hat{\theta}_2, \hat{\theta}_1)$, and we have

$$D\left(\hat{\theta}_2, \hat{\theta}_1\right) = -E^0[n^{-1}\{LR_n(\hat{\theta}_2, \hat{\theta}_1) - \{trac(A_2 B_2^{-1}) - trac(A_1 B_1^{-1})\}\}]$$

Using Akiake approximation $trac(A_i B_i^{-1}) \approx a_i$, we obtain a simple estimator of

$$d\left(\hat{\theta}_2, \hat{\theta}_1\right) = -n^{-1}\{LR_n(\hat{\theta}_2, \hat{\theta}_1) - (a_2 - a_1)\}$$

Theorem. Under the general conditions of Vuong (1989), the tracking interval with the cinfidence coefficient $\%(1 - \alpha)$ for $D(\hat{\theta}_2, \hat{\theta}_1)$ is (L_n, U_n) where $L_n = d\left(\hat{\theta}_2, \hat{\theta}_1\right) - Z_{\alpha/2} n^{-1/2} \hat{v}$ and $U_n = d\left(\hat{\theta}_2, \hat{\theta}_1\right) + Z_{\alpha/2} n^{-1/2} \hat{v}$.

5- Application

In this section, we apply our presented tracking interval to some real dataset in actuarial science. Consider the data of Frees et al. (1998), which were later reanalyzed by Chatrabgoun et al. (2016) The data consists of 1500 general liability claims randomly chosen from late settlement lags and were supplied by Insurance Services Office, Inc. For each claim, the indemnity payment (LOSS, being the random variable X) and the allocated loss adjustment expense (ALAE, being the random variable Y) were recorded. Here, ALAE are types of insurance company expenses that are specifically attributable to the settlement of individual claims such as lawyers fees and claims investigation expenses. Frees et al. (1998) fit the copula using the parametric method. They checked many families of copulas and identified the final copula, Gumbel copula with dependency parameter $\theta_2 = 1.453$. Our ultimate goal is to study the dependence structure of losses and expenses from nonnested copula function using the tracking interval presented in good level of significance to show the superiority of our approach. Now, using interested data we estimate suggested copula parameter (t-student), and then we obtain tracking interval. In Table 1 by taking into account the true parameter value, the interval range consist of negative values which show that Gumbel copula with the corresponding parameter is better than the proposed t-student copula. In other words, the data tend to have Gumbel distribution with parameter close to the true parameter (i.e., $\theta_2 = 1.453$). By getting away from true parameter tracking interval includes zero value, and lower and upper interval will be negative and positive, respectively. This shows that the proposed copula models are equivalent. On the other hand by getting away from the true parameter, the interval range will be increased. In fact, by increasing the interval range which includes zero the equivalent of two models is confirmed, while we're getting away from the true parameter of the interested data.

parameters	Tracking interval	Distance
1.453	(-0.7501, -0.6677)	0.0802
1.930	(-0.7163, -0.6333)	0.0830

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2.450	(-0.6111, -0.5190)	0.1532
3.50	(-0.1317, 0.0267)	0.1584
3.54	(-0.1098, 0.0541)	0.1639

Table 1: Tracking interval to select the optimal copula (Gumbel and t-Student) function

6- Simulation Study

We first generate n random samples from semi-parametric Clayton copula (c^0) with the parameter dependence 2, and we consider it as a true model of data. Then, we introduce two proposed model Clayton (c_2) and t-student (c_1). In order to obtain tracking interval at significance level % 95, different sample size 50, 100 and 200 from the true model (Clayton parameter 2.00) is generated. Now, Using presented estimated method in this paper the parameters of proposed t-sudent copula is estimated, and the values of $d(\hat{\theta}_2, \hat{\theta}_1)$ and $\hat{v}$ is calculated. when the parameters of the proposed model is close to the true model, negative values are included in tracking interval. It states that Clayton model is better than the t-student model. In fact, a well described model is the best model. By getting away from the true model parameters, tracking interval includes zero. Finally, it states that two proposed models are equivalent.

7- Conclusions

In this paper tracking interval based on the differences in expected Kullback-Leibler risk was introduced which in order to find the optimum model from non-nested copula functions offers a useful idea. Model selection criteria based on log maximum likelihood functions such as AIC and CIC which there are for model selection of density functions and copulas function are constant by transforming any random variables. Second, it depends on the sample size and for small sample volume not well treated. On the other hand, they measure distance between the copula functions. But the expressed idea in this paper is based tracking interval which act accurately for small sample size and does not have the defects of the previously cited measures. This method can be used to select the proper copula function in various disciplines such as economics, insurance and medical which have numerous applications.

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Modeling and Forecasting Mortality Rate for Rasht Population

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Abstract

Over the past decades, a number of approaches have been applied for forecasting mortality. Lee and Carter [1] published a new method for long-run forecast of the level and age pattern of mortality. Many authors welcomed this method, and extended it through a wider class of generalized parametric and nonlinear model. In this paper, we investigate the feasibility of the classic Lee-Carter model and two of the main extensions of the model known as age-cohort (AC) model and age-period-cohort (M) model. We applied these three models on Rasht mortality in order to find the best model. A suitable ARIMA process is also used to forecast future Rasht morality rate by extrapolation.

Keywords

ARIMA process, Cohort, Lee-Carter model, Mortality rate, Time series.

1- Introduction

The projection of mortality trends is an important issue in a wide number of areas: for examples, planning for social security, providing health care systems, funding of retirement income systems, and in actuarial applications, pricing and reserving of annuity portfolios (Renshaw and Haberman, [2]). The Lee-Carter method was published about three decades ago with an application to US mortality data, 1900-1989, Lee-Carter [1]. The model assumes that the dynamics of death rates over time are driven by a single time-varying parameter and that mortality forecasting relies on the extrapolation of this index using appropriate statistical time-series method. There are many ways to forecast mortality, but the Lee–Carter method is extrapolative and makes no effort to incorporate knowledge about medical, behavioral, or social influences on mortality changes, see, e.g. Lee [3], Lee and Miller [4] and Rajendra et al. [5] for more details. Renshaw and Haberman [2] have extended the Lee-Carter model to produce the APC (age-period-cohort) model and applied it for England and Wales mortality.

1.1. The classic Lee-Carter Model

Let the random variable D_{xt} denote the number of deaths in a population at age x and time t and (d_{xt}, e_{xt}) represent a rectangular array of data, where d_{xt} is the actual number of deaths and e_{xt} is the matching exposure to the risk of death. Also, the force of mortality and the empirical mortality rates are denoted by μ_{xt} and m_{xt}, respectively, where $m_{xt} = \dfrac{d_{xt}}{e_{xt}}$.

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The structure of the interesting alternative method for forecasting mortality was proposed in 1992 by Lee and Carter as,

$$\ln(m_{xt}) = \alpha_x + \beta_x k_t + \varepsilon_{xt}$$

This method is a powerful approach to mortality projections which describes the log of a time series of age-specific death rates m_{xt} as the sum of an age-specific component, α_x, the time index parameter κ_t, and an age-specific component β_x. The error term ε_{xt} reflects particular age-specific historical influences not captured by the model.

1.2. Lee-Carter Model Incorporating Cohort Effect

Year of birth or cohort year in time t and for aged grouped x is equal to $z = t - x$. Renshaw and Haberman [2] have extended the Lee-Carter model to produce age-period-cohort. Based on the Lee-Carter model we have

$$\mu_{xt} = \exp(\alpha_x + \beta_x \kappa_t). \qquad (1.1)$$

In order to contain mortality reduction factor, Eq. (1.1) may rewrite as

$$\mu_{xt} = \exp(\alpha_x + \log F(x,t)),$$

where

$$F(x,t) = \exp(\beta_x^{(0)} \iota_{t-x} + \beta_x^{(1)} \kappa_t).$$

We shall name this model as 'M'. As it is clear, the new bilinear term $\beta_x^{(0)} \iota_{t-x}$ indicates additional cohort effects. Model M is a general form of two other models LC (Lee-Carter) and AC (age-cohort), it means:

$$LC : \beta_x^{(0)} = 0 \quad (\beta_x^{(1)} \equiv \beta_x), \qquad AC : \beta_x^{(1)} = 0 \quad (\beta_x^{(0)} \equiv \beta_x)$$

1.3. Parameters Estimation

One usual way for estimating parameters is using SVD method, but two alternative model-fitting procedures presented here in order to apply instead of SVD method. They are named method (A) and method (B). Method (A) is based on an unpublished technical report by Wilmoth [6] and method (B) is based on a method of mortality analysis incorporating age-year interaction with applications to medical statistics by James and Segal [7].

2. Application

In this section, the feasibility of using the Lee-Carter methodology to construct mortality forecasts for the Rasht's population is investigated. The classic Lee-Carter model (LC) and its extensions, AC (age-cohort) and M (age-period-cohort) models are fitted to the matrix of Rasht's death rates from 1380 to 1389, reported by Rasht cemeteries organization. These models are fitted based on Poisson error structure. We follow the iterative algorithms to estimate the parameters. Then, by using the ARIMA framework, a time-varying index of mortality will be forecasted. The estimation of the parameters for each gender will be reported to find the best model between three models LC, AC and M. A direct comparison is made between them from residual plots. Then by using the chosen model, mortality rates of Rasht's population are predicted for the next years. The MSE of the models LC, AC and M was calculated to compare the flexibility of the models for estimating the Rasht's

mortality rate. Table 1 shows that model M is more appropriate for forecasting. Forecast of mortality rate of Rash population for six aged group is plotted in Figure 1.

Table 1. Residual mean square error based on model LC, AC and M.

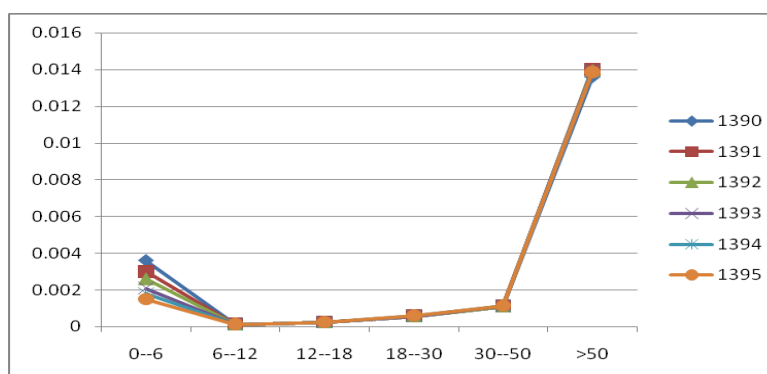| Model	Total	Male	Female
LC	5676.229	2274.533	368.734
AC	705.065	207.076	318.164
M	104.479	161.520	111.168

Figure 1. Forecast of mortality rate of Rash population in 1390-1395.

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Modeling Hospital Costs Using the Generalized Gamma Additive Model

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Abstract

The Generalized Gamma (GG) is a flexible distribution in statistical literature with the special cases of exponential, gamma, Weibull and lognormal distributions. This paper proposes the GG additive model for modeling hospital claim costs. Compares to other models, the GG is more flexible and has a better performance in modeling positive skewed data. The proposed model is fitted to the hospital costs data from the Nationwide Inpatient Sample of the Healthcare Cost and Utilization Project (NIS-HCUP), a nationwide survey of hospital costs conducted by the U.S. Agency for Healthcare Research and Quality (AHRQ). The results show that the claim coasts affected by the given explanatory variables and based on the AIC and BIC criterion, The GG has a better fit for the given data compared to the Gamma, Weibull and log-normal models.

Keywords

Generalized gamma, Generalized additive models, GLM, Insurance data.

1- Introduction

The regression is a method to investigate the relationship between a response variable and one or more explanatory variables which can be expressed through a mathematical formula.

The generalized linear models (GLMs), is an extension of the linear regression model that the response variable departs from the normal to an exponential distribution. The response variable may have a discrete distribution or follows a continuous distribution with right skewed shape.

2- Data

The data (Hospital Costs) used in this paper is a standard dataset in health care insurance. The

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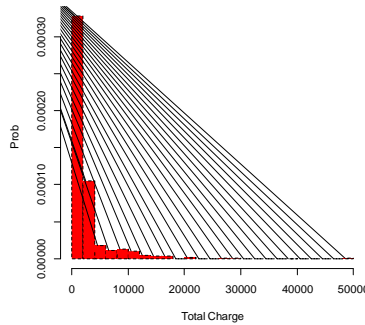Pricing and Industrial Statistics
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Fig. 1. Distribution of Total Charge in the Hospital Costs data

dataset is from the Nationwide Inpatient Sample of the Healthcare Cost which considered by Frees *et al*. (2014). The histogram of the total charges in figure 1shows that the hospital costs is right skewed.The mean and the standard deviation of the variables are given in table 1.

Table. 1. Mean and standard deviation of the variables

Variable	Type	Mean	Std.deviation
Total Charge	Continuous	2774.388	3888.407
Age	Continuous	5.086	6.95
Length of Stay	Discrete	2.82	3.363
Gender(Female=1)	Categorical	0.512	0.500

3- Statistical Models

Generalized Gamma Distribution

In the statistical literature the GG distribution with the parameters α, τ and λ denoted by $GG(\alpha, \tau, \lambda)$ has the mean $E(Y) = \lambda \Gamma(\frac{1}{\tau} + \alpha) / \Gamma(\alpha)$. The GG distribution reduces to an exponential ($\alpha = \tau = 1$), gamma ($\tau = 1$) and the Weibull ($\alpha = 1$) distribution. The log-normal distribution also is a limiting case of GG when $\alpha \to \infty$.

To insert the GG distribution in the generalized linear model we adjust the parameters of distribution in such a way that $E(Y) = \mu$. The re-parameterization which apply for this aim is $\tau = v$, $\alpha = \frac{1}{\sigma^2 v^2}$, $\lambda = \mu(\sigma^2 v^2)^{\frac{1}{v}}$. With this re-parameterization the probability density function of GG becomes as bellow,

$$g(y \mid \mu, \sigma, v) = \frac{v\mu^{-1}}{\Gamma(\frac{1}{\sigma^2 v^2})(\sigma^2 v^2)^{\frac{1}{\sigma^2 v^2}}} \left(\frac{y}{\mu}\right)^{\frac{1}{\sigma^2 v}-1} e^{-\frac{(y\mu^{-1})^v}{\sigma^2 v^2}} \qquad y > 0 \qquad (1)$$

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with $\mu > 0, \sigma > 0$ and $\nu \in R$. The mean and the variance of the new functional form are given respectively as $E(Y) = \mu$, $Var(Y) = \mu^2 \sigma^2$.

Generalized Gamma Regression Models

Now let us assume the random variable y_i affected by covariates x_i. As an example, in the insurance literature, the claim coast is affected by age, gender, occupation and so on.

The GG contains three parameters including location parameter (μ), scale or dispersion parameter (σ) and shape (ν). The whole or some of these parameters can be incorporated in the regression model via a suitable link function.

The suitable link function for the μ and σ is a log link function while an identity function can be used for the ν parameter. So, the generalized additive regression model can be represented as

$$\log(\mu) = X_1^T \beta + \sum_{j=1}^{m_1} h_{j1}(x_{j1}), \ \log(\sigma) = X_2^T \gamma + \sum_{j=1}^{m_2} h_{j2}(x_{j2}), \ \nu = X_3^T \delta + \sum_{j=1}^{m_3} h_{j3}(x_{j3}) \quad (2)$$

Where β, γ and δ are the vector of regression coefficients corresponding to μ, σ and ν respectively and $h_{jk}(x_{jk})$ are the penalized spline functions as the non-parametric smoothing terms. The regression parameters can be estimated using maximum likelihood procedure.

The penalized spline function $h_{jk}(x_{jk})$ is modeled using penalized B-spline which allows the estimation of the smoothing parameters using a local maximum likelihoodminimizing Akaike Information Criterion (AIC) which is defined as $AIC = -2 \log L + 2P$ with L being the log of penalized likelihood and P the number of parameters in the fitted model.

4- Results and Discussion

The GG and its nested models are fitted to hospital costs data using the Generalized Additive Models for Location, Scale and Shape (GAMLSS package) in R.3.3.1 (Rigby & Stasinopoulos, 2005). The fitted GG regression model can be compared using several measures such as (AIC) and (BIC). The smaller AIC or BIC results in better fitting. Table 2 provides the log-like, the AIC and BIC of the fitted models. It can be seen that the GG model has a better performance compared to its nested models based on all criterions. The likelihood ratio can be employed to assess the adequacy of the nested models. Here, the likelihood ration tests are implemented for testing the adequacy of GG model over the Gamma, Weibull or Log-normal models since all of the models reduce to GG distribution. In Table 3 the results of fitting the GG model on the hospital costs data are given.

Table. 2. Likelihood ratio, AIC and BIC

Test/Criteria	GG	Gamma	Weibull	Log-normal
-2 log L	7585.29	7985.81	8207.9	7867.74
Likelihood ratio	-	400.52	622.61	282.45
AIC	7683.24	8049.39	8268.39	7934.03
BIC	7890.27	8183.38	8395.87	8073.73

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Table. 3. The estimation of coefficients of GG regression model

Model Parameters	Estimates	SE	*P*-value
$\log(\mu)$			
Intercept	7.006	0.0074	<0.001
pb(AGE)	0.0044	0.0011	<0.001
pb(LOS)	0.1304	0.0001	<0.001
FEMALE=1	-0.0729	0.0026	<0.001
$\log(\sigma)$			
Intercept	-1.5221	0.0450	<0.001
pb(AGE)	0.0368	0.0037	<0.001
pb(LOS)	-0.1153	0.0085	<0.001
FEMALE=1	-0.2464	0.0525	<0.001
ν			
Intercept	-5.4757	1.1136	<0.001
pb(AGE)	0.0834	0.0664	0.2095
pb(LOS)	-2.4011	0.4031	<0.001

pb() is the penalized beta spline (P-spline)

5- Conclusion

This study proposes the generalized gamma regression model referred to as the GG that parametrically nests the gamma, Weibull and log-normal models for modeling hospital costs. The GG and its nested models are fitted and compared on hospital costs as the response versus some covariates. Based on the results, all models produce the similar estimates for the regression coefficients. However the likelihood ratio tests given in table 2 conclude that the GG model is more flexible and has a better performance versus the alternatives.

The fitted GG includes the additive components using log-link for both of the mean and dispersion and an identity link for the shape parameters. These components can be fitted with the same or different covariates. For fitting models, the beta spline method was used. The advantage of the spline method over the others is that it considers the non-linearity between the response and covariates. Therefore the bias of the estimated parameters will be reduced which results in decreasing the deviance of the model.

More things the effected plots given in Fig. 4-6 for the covariates within each parameter show that the relationship between the covariates and given link functions are not linear and hence the beta spline method is a suitable choice for fitting the model.

The diagnostic plots in Fig. 7 also confirm the adequacy of the fitted GG regression on Hospital costs.

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On Discrete Time Spatial Renewal Processes

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Abstract

In this article, we introduce a certain class of discrete time stochastic processes based on a spatial renewal process. We provide certain key renewal results, then using reward theory we give an application for our model in modeling of the experience of teams in the Primer League.

Keywords

Renewal Fields, Renewal Stochastic Processes, Reward Processes.

1- Introduction

Renewal theory is a celebrated fundamental and rich topic in stochastic processes and their applications. Spatial renewal processes, renewal fields, in contrast to the classical renewal processes, that have received substantial developments, are gradually advancing. These days, spatial processes are becoming important in different areas of stochastic processes and time series. In this article, we introduce a certain class of discrete time stochastic processes based on a spatial renewal process. We provide certain key renewal results, then using reward theory we give an application for our model in quantization of the experience of teams in the Primer League.

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On the Finite Time Ruin Probability in the Dependence Collective Insurance Risk Model

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Abstract

In the risk theory, work concerning the financial surplus of insurance companies in continuous time has been proceeding for nearly a century. In the present paper, we consider the classical compound Poisson risk model with dependence between claim sizes and claim inter-arrival time. The dependence assumption between the interclaim times and the claim sizes is well suited for insurance contracts during extreme and catastrophic events. We attempt to analyze the approximation of finite time ruin probability for such model as the initial capital increases.

Keywords

Collective insurance risk model, Dependent claims, Finite time ruin probability, Insurance Company.

1- Introduction

In the literature of risk theory, two risk models have emerged to have been studied extensively. These are the classical compound Poisson risk model and the continuous time renewal risk model which is also referred to as Sparre-Andersen model. Insurance companies are in the business of risks. In its simplest form, when certain events occur, an insurance contract will provide the policyholder the right to claim all or a portion of the loss. (See e.g., Asmussen, 2001). We denote by R_t the amount of surplus of an insurance portfolio at time t:

$$R_t = u + ct - \sum_{k=1}^{N(t)} Y_k, \qquad t \geq 0 \tag{1.1}$$

where $u \geq 0$ is the amount of initial reserves, $c > 0$ is the premium income rate and $S(t) = \sum_{k=1}^{N(t)} Y_k$ is the total amount up to time t which is a compound Poisson process. $\{N(t), t \geq 0\}$ is a Poisson process, with parameter λ, which counts the claim occurrences until time t, and $\{Y_k, k = 1, 2, \ldots\}$ are successive claim amounts which are represented by non-negative independent identically distributed random variables. In this paper we shall assume throughout that R_t given in (1.1) is a dependent Sparre Andersen risk process. Ruin occurs as soon as the surplus becomes negative or null, i.e. at time $T(u) = \inf\{t > 0 : R_t \leq 0\}$ $(T(u) = \infty$, if $R_t \geq 0$ for all $t > 0$, i.e. ruin does not

occur). The process $N(t)$ is independent of the claim sizes. Let $\phi(u,t)$ be the probability of ruin before time t, $t \geq 0$:

$$\phi(u,t) = P\left(\exists s \in [0,t], R_s < 0 \middle| R_0 = u\right) \qquad u,t \geq 0. \tag{1.2}$$

As $t \to \infty$, (1.2) becomes the ultimate non-ruin probability $\phi(u) = P(T(u) = \infty)$. Much research has been devoted to the evaluation of ruin probabilities, over finite or infinite horizon, when some independence and stationarity assumptions of the model are relaxed. Let $\{T_k, k = 1, 2, \ldots\}$ be the inter-arrival times of successive claims. Moreover, the inter-occurrence times are assumed to be independent of the claim amounts. Typically, this arises in the case of natural phenomenal like fire or natural disasters like earthquake. The occurrence of an earthquake often increases the probability of by-claims in a near future. If the last earthquake occurred a long time ago, the next earthquake is likely to be more severe.

2- Literature review

The classical theory emerged as a compound Poisson risk model, which embodies an initial capital plus collected premiums less paid claims. Since the pioneering works of Lundberg and Cramer, it has been the object of a number of theoretical studies and practical applications. In the classical collective risk insurance model, the independence among inter-claim times is one of the crucial assumptions. Introducing dependent structure to risk models has captured more and more researchers' attention in recent years, and it provides a new perspective for the ruin probabilities theory. For the risk model with dependent inter-claim times, the reader can look at some asymptotic results via adopting theories developed for general regenerative processes in Palmowski and Zwart (2010). Albrecher and Teugels (2006), sought to introduce an arbitrary dependence structure among the interclaim time and the claim size prior to deriving the asymptotic finite time and infinite time ruin probabilities. Cossette et al. (2010), proposed a dependence structure through the (generalized) Farlie-Gumbel-Morgenstern copula and studied the discounted penalty function.

3- Ruin Probability in the Classical Compound Poisson Risk Model

Suppose that k_t^γ be that random variable which gives the number of spacings of the Poisson process $\{N(s), 0 \leq s \leq t\}$ which are larger than γ and $k_t^\gamma(n)$ be that random variable which counts the number during the interval $(0,t)$, of Poisson spacings which are larger than γ, given that $N(t) = n$, $n \geq 1$ and $0 < \gamma < t$. Note that $P(k_t^\gamma(n) = j) = 0$ for $j > \frac{t}{\gamma}$.

Theorem. For $t > 0$ and large u, if $\alpha < \beta$, then the ruin probability is as

$$\phi(u,t) \sim \left(\sum_{n=1}^{\infty} P(N(t) = n) \sum_{j=1}^{\min\left(n, \left\lfloor \frac{t}{\gamma} \right\rfloor\right)} P(k_t^\gamma(n) = j) e_j^\alpha \right) \overline{F}(u + ct), \tag{3.1}$$

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while if $\alpha > \beta$ for $t > 0$ and large u,

$$\phi(u,t) \sim \left(\sum_{n=1}^{\infty} P(N(t) = n) \sum_{j=\max\left(1, n-\left\lfloor \frac{t}{\gamma} \right\rfloor\right)}^{n} P\left(k_t^\gamma(n) = n - j\right) e_j^\beta \right) \overline{G}(u + ct), \tag{3.2}$$

with, for $0 \le j \le n$, the p.m.f. of the random variable $k_t^\gamma(n)$ is given by

$$P\left(k_t^\gamma(n) = j\right) = \sum_{k=j}^{n} (-1)^{k-j} \binom{n}{j} \binom{n-j}{k-j} P\left(T_1^* > \frac{\gamma}{t}, \ldots, T_k^* > \frac{\gamma}{t}\right), \tag{3.3}$$

where

$$P\left(T_1^* > d, \ldots, T_k^* > d\right) = \begin{cases} 1 & d \le 0 \\ (1 - kd)^n & 0 < d < \dfrac{1}{k} \\ 0 & \dfrac{1}{k} \le d < 1. \end{cases} \tag{3.4}$$

4- Plots of ruin probabilities with dependence structure

In this section, the ruin probabilities are plotted with dependence structure for fixed values of α and t. As we said the parameter p varies from 0 to 1 with step 0.01. When the dependence parameter equals 0 we have the independence case, and when it equals 1 we obtain the Fréchet upper bound case. The ruin probabilities are investigated when $\alpha < \beta$. First, these probabilities are plotted for fixed threshold value associated to the claim inter-arrival time and then those plotted for fixed dependence parameter in Nelsen (2006) copula. For $t = 10$ and for both $\alpha = 0.5$ and $\alpha = 3$, two cases are plotted, one with $\gamma = 1$ and the other with $\gamma = 2$. Each case is plotted first separately and then in a single graph. When $\alpha = 0.5$ (Figure 1) the asymptotic ruin probability is a decreasing function of the dependence parameter p. But the asymptotic ruin probability is an increasing function of the dependence parameter p, when $\alpha = 3$ (Figure 2). Also, for $t = 10$ and for $\alpha = 3$, the values $p = 0.4$ and $p = 0.8$ are considered. The plotted are given in Figure 3. It shows that the asymptotic ruin probability is a decreasing function of γ.

Figure 1. The ruin probability as a function of dependence parameter in Nelsen (2006) copula when $\alpha = 0.5$ and $t = 10$.
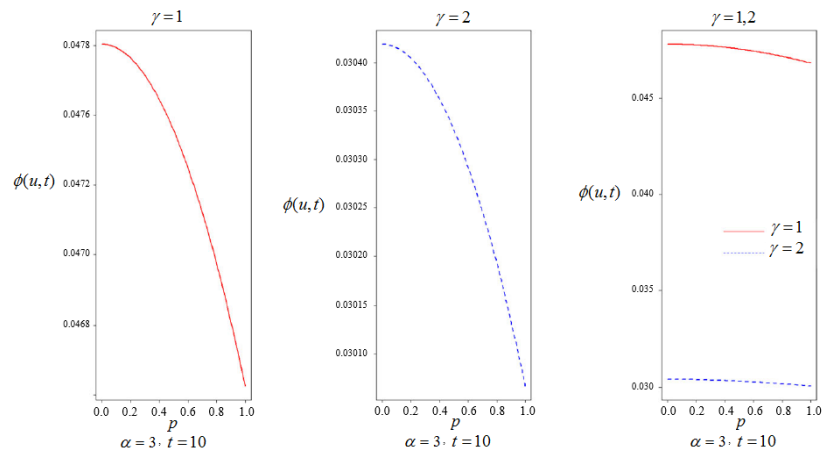

Figure 2. The ruin probability as a function of dependence parameter in Nelsen (2006) copula when $\alpha = 3$ and $t = 10$.
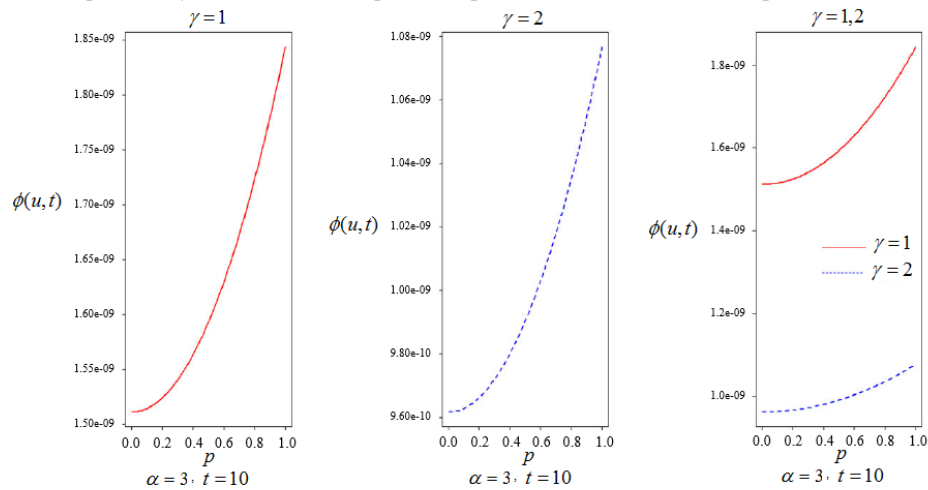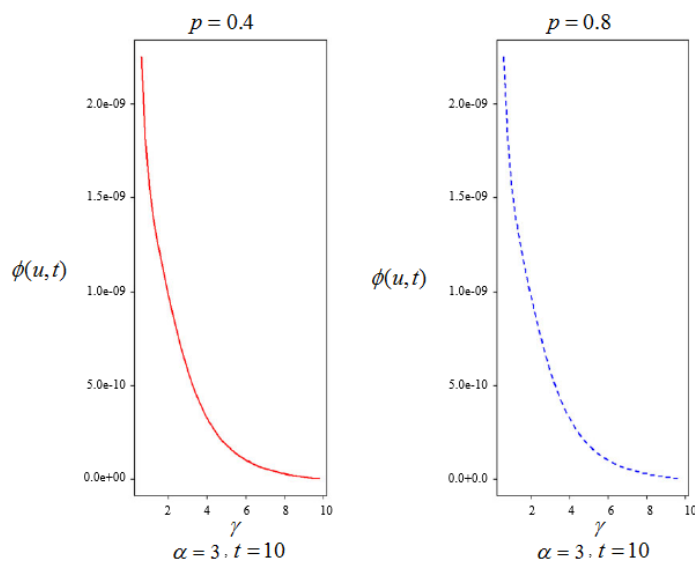

Figure 3. The ruin probability as a function of γ when $\alpha = 3$ and $t = 10$.

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5- Concluding remarks

In this paper, we considered the classical compound Poisson risk model in which the interclaim time arrivals and the claim sizes are structurally dependent. The approximation of finite time ruin probability obtained for and finite time ruin probabilities plotted. It showed that the asymptotic ruin probability is a decreasing function of threshold value associated to the claim inter-arrival time.

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Ordering Properties of the Smallest Claim Amounts from Two Heterogeneous Portfolios and Their Applications in Insurance

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Abstract

Suppose $X_1, \ldots, X_n$ is a set of non-negative random variables with X_i having Exponentiated Weibull distribution for $i = 1, \ldots, n$ and $I_1, \ldots, I_n$ are independent Bernoulli random variables, independent of the X_i's, respectively, with $E(I_i) = p_i$, $i = 1, \ldots, n$. Let $Y_i = I_i X_i$, for $i = 1, \ldots, n$. In applications, actuarial science thus Y_i corresponds to claim amount in a portfolio of risks. In this paper, We obtain the usual stochastic order between the smallest claim amounts when the matrix of parameters (α, λ) changes to another matrix in a mathematical sense. We also show that, under some conditions on the common copula, the usual stochastic order of smallest claim amounts with heterogeneous claims is smaller than the smallest claim amounts with homogeneous claims having a common survival function, which is equal to the average of the survival functions of the heterogeneous claims. The results established here extend some well-known results in the literature.

Keywords

Smallest Claim Amounts; Exponentiated Weibull Distribution; Matrix Majorization; Schur-Convexity; Schur-Concavity.

1- Introduction

The Exponentiated Weibull distribution, as a generalization of the Weibull distribution, has been discussed in reliability theory, life-testing and actuarial science. A random variable X is said to have the Exponentiated Weibull distribution with shape parameters $\alpha > 0$ and $\beta > 0$ and scale parameter $\lambda > 0$ (denoted by $X_i \sim EW(\alpha, \beta, \lambda)$) if its cumulative distribution and probability density functions are given by

$$F(x, \alpha, \beta, \lambda) = \left(1 - e^{-(\lambda x)^\beta}\right)^\alpha \qquad x > 0$$

and

$$f(x, \alpha, \beta, \lambda) = \alpha \beta \lambda^\beta x^{\beta-1} e^{-(\lambda x)^\beta} \left(1 - e^{-(\lambda x)^\beta}\right)^{\alpha-1} \qquad x > 0$$

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respectively. The Exponentiated Weibull distribution includes several well-known distributions, such as Weibull, generalized exponential (GE), exponential, Rayleigh and Burr type X, all as special cases.

Suppose X_i denotes the total of random claims that can be made in an insurance period and I_i denotes a Bernoulli random variable associated with X_i defined as follows: $I_i = 1$ whenever the i −th policyholder makes random claim X_i and $I_i = 0$ otherwise. Then, $Y_i = I_i X_i$ corresponds to the claim amount in a portfolio of risks.

It is of interest to note that large values of $\varrho[X]$ show that X is dangerous. Specifically, if X is a possible loss of some financial portfolio over a time horizon, we interpret $\varrho[X]$ as the amount of capital that should be added as a buffer to this portfolio so that it becomes acceptable to an internal or external risk controller. In such a case, $\varrho[X]$ is the risk capital of the portfolio. Such risk measures are used for determining provisions and capital requirements in order to avoid insolvency.

The Value-at-Risk, denoted by VaR, which is defined based on quantiles of a random variable plays a critical role in risk measurement. Two random risks X and Y can be compared by means of their VaRs. We may have two probability levels p_0 and p_1 such that

$VaR[X; p_0] \leq VaR[X; p_0]$ and $VaR[X; p_1] \leq VaR[X; p_1]$. So, it is reasonable to consider a situation under which $VaR[X; p] \leq VaR[X; p]$ for all probability level $p \in (0,1)$. For a risk $X,$ the Value-at-Risk (VaR) at level p is defined as

$$VaR[X; p] = F^{-1}(p) = inf\{u: F_X(u) \geq p.\}$$

A single VaR at a predetermined level p does not give any information about the thickness of the upper tail of the distribution function. This is a considerable shortcoming since in practice a regulator is not only concerned with the frequency of default, but also with the severity of default. Therefore, one often uses another risk measure, which is called the tail value-at-risk (TVaR)

$$TVaR[X; p] = \frac{1}{1-p} \int_p^1 VaR[X; \xi]\, d\xi \qquad 0 < p < 1.$$

Stop loss is a non-proportional type of reinsurance and works similar to excess-of-loss reinsurance. While excess-of-loss is related to single loss amounts, either per risk or per event, stop-loss covers are related to the total amount of claims X in a year net of underlying excess-of-loss contracts and/or proportional reinsurance. The reinsurer pays the part of X that exceeds a certain amount, say d. Reinsurance treaties usually do not cover the risk fully. Stop-loss (re)insurance covers the top part and is defined as follows: for a loss X, assumed non-negative, the payment is $(X - d)_+ = max\{X - d, 0\}$. The insurer retains a risk d (retention) and lets the reinsurer pay the remainder, and so the insurer's loss stops at d. In the reinsurance practice, the retention equals the maximum amount to be paid out for every single claim and d is called the priority. The function $\pi_X(d) =$

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$E\{(X - d)_+\}$, called the stop-loss transform of X, is a continuous convex function that is strictly decreasing in the retention d. If X is non-negative, then it is evident that $\pi_X(0) = E\{X\}$.

The problem of comparing the numbers of claims and aggregate claim amounts with respect to some well-known stochastic orders is of interest on both theoretical and practical viewpoints. Several authors have worked on this problem. Recently, Barmalzan et al. (2015) have presented a complete version of the results of Khaledi and Ahmadi (2008) for aggregate claim amounts and also extended to a more general case. Barmalzan et al. (2016) have also presented ordering properties of the smallest and largest claim amounts in a general scale model.

The rest of this paper is organized as follows. In Section 2, we introduce some definitions and notations pertinent to stochastic orders, vector majorization, Matrix Majorization and related orders. In Section 3, we establish some ordering results relating to stochastic comparisons of the smallest claim amounts via matrix majorization. The results established here extend some well-known results in the literature.

2- Preliminaries

Suppose X and Y are two non-negative continuous random variables with distribution functions $F(t) = P(X \leq t)$ and $G(t) = P(Y \leq t)$, survival functions $\bar{F}(t) = 1 - F(t)$, $\bar{G}(t) = 1 - G(t)$ and stop-loss transformations $\pi_X(d)$ and $\pi_Y(d)$, respectively.

Definition 1: (i) X is said to be larger than Y in the usual stochastic order (denoted by $X \succeq_{st} Y$) if $\bar{F}_X(x) \geq \bar{F}_Y(x)$ for all $x \in \mathbb{R}^+$.

(ii) X is said to be larger than Y in the stop-loss order, or equivalently the increasing convex order (denoted by $X \succeq_{st} Y$), if $\pi_X(d) \geq \pi_Y(d)$ for all $d > 0$.

For a comprehensive discussion on various stochastic orders and their applications, we refer the readers to Shaked and Shanthikumar (2007).

Definition 2: For two vectors $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_n)$ and $\boldsymbol{\lambda}^* = (\lambda_1^*, \dots, \lambda_n^*)$, suppose $\lambda_{(1)} \leq \dots \leq \lambda_{(n)}$ and $\lambda_{(1)}^* \leq \dots \leq \lambda_{(n)}^*$ denote the increasing arrangements of their components, respectively. Then, we say that the vector $\boldsymbol{\lambda}$ is larger than the vector $\boldsymbol{\lambda}^*$ in the majorization order (denoted by $\boldsymbol{\lambda} \succeq^m \boldsymbol{\lambda}^*$) if $\sum_{j=1}^n \lambda_{(j)} = \sum_{j=1}^n \lambda_{(j)}^*$ and $\sum_{j=1}^i \lambda_{(j)} \leq \sum_{j=1}^i \lambda_{(j)}^*$ for $i = 1, \dots, n-1$.

Lemma 1: (Marshall et al. (2011), p. 84) Consider the real-valued continuously differentiable function φ on J^n where $J \subseteq \mathbb{R}$ is an open interval. Then, φ is Schur-convex on J^n if and only if

    (i) φ is symmetric on J^n, and

    (ii)   for all $i \neq j$ and all $\boldsymbol{u}$ in J^n

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$$(u_i - u_j)\left\{\frac{\partial}{\partial u_i}\varphi(\boldsymbol{u}) - \frac{\partial}{\partial u_j}\varphi(\boldsymbol{u})\right\} \geq 0,$$

where $\frac{\partial}{\partial u_i}\varphi(\boldsymbol{u})$ denotes the partial derivative of φ with respect to its i-th argument. If the sign of the inequality is reversed, then φ is said to be Schur-concave.

3- Main Results

Let us set

$$\mathcal{F}_n = \left\{\begin{bmatrix} v_1 & \dots & v_n \\ \tau_1 & \dots & \tau_n \end{bmatrix}, v_i \geq 1, \tau_j > 0 \quad and \quad (v_i - v_j)(\tau_i - \tau_j) \leq 0, i = 1,2,\dots,n\right\}.$$

Theorem 1: Suppose $X_1, X_2(X_1^*, X_2^*)$ are independent non-negative random variables with $X_i \sim EW(\alpha_i, \beta, \lambda_i)$ $\left(X_i^* \sim EW(\alpha_i^*, \beta, \lambda_i^*)\right)$, $i = 1,2$. Further, suppose $I_1, I_2(I_1^*, I_2^*)$ is a set of independent Bernoulli random variables, independent of X_i's (X_i^*)'s, with $E(I_i) = p_i$ $(\mathrm{E}(I_i^*) = p_i^*)$. Then, for $\alpha \geq 1, \beta \geq 1$ and $p_1 p_2 \leq p_1^* p_2^*$, we have

$$\begin{bmatrix} \alpha_1 & \alpha_2 \\ \lambda_1 & \lambda_2 \end{bmatrix} \gg \begin{bmatrix} \alpha_1^* & \alpha_2^* \\ \lambda_1^* & \lambda_2^* \end{bmatrix} \Rightarrow Y_{1:2}^* \succcurlyeq_{st} (\succcurlyeq_{sl}) Y_{1:2}.$$

Some generalizations of the result in Theorem 1 to the case when the number of underlying random variables is arbitrary are now presented.

Theorem 2: Suppose $X_1, \dots, X_n(X_1^*, \dots, X_n^*)$ are independent non-negative random variables with $X_i \sim EW(\alpha_i, \beta, \lambda_i)$ $\left(X_i^* \sim EW(\alpha_i^*, \beta, \lambda_i^*)\right)$, $i = 1,2,\dots,n$. Further, suppose $I_1, \dots, I_n(I_1^*, \dots, I_n^*)$ is a set of independent Bernoulli random variables, independent of X_i's (X_i^*)'s, with $E(I_i) = p_i$ $(\mathrm{E}(I_i^*) = p_i^*)$, $i = 1,2,\dots,n$. Then, for $\alpha \geq 1, \beta \geq 1$ and $p_1 \dots p_n \leq p_1^* \dots p_n^*$, we have

$$\begin{bmatrix} \alpha_1^* & \dots & \alpha_n^* \\ \lambda_1^* & \dots & \lambda_n^* \end{bmatrix} = \begin{bmatrix} \alpha_1 & \dots & \alpha_n \\ \lambda_1 & \dots & \lambda_n \end{bmatrix} T \Rightarrow Y_{1:n}^* \succcurlyeq_{st} (\succcurlyeq_{sl}) Y_{1:n}.$$

Theorem 3: Under the assumption of Theorem 2 if the T-transform matrices $T_1, \dots, T_k$ have the same structure, then for $(\boldsymbol{\alpha}, \boldsymbol{\lambda}) \in \mathcal{F}_n$, we have

$$\begin{bmatrix} \alpha_1^* & \dots & \alpha_n^* \\ \lambda_1^* & \dots & \lambda_n^* \end{bmatrix} = \begin{bmatrix} \alpha_1 & \dots & \alpha_n \\ \lambda_1 & \dots & \lambda_n \end{bmatrix} T_1 \dots T_k \Rightarrow Y_{1:n}^* \succcurlyeq_{st} (\succcurlyeq_{sl}) Y_{1:n}.$$

Theorem 4: Under the assumption of Theorem 2 assume that $(\boldsymbol{\alpha}, \boldsymbol{\lambda}) \in \mathcal{F}_n$ and $(\boldsymbol{\alpha}, \boldsymbol{\lambda})T_1 \dots T_i \in \mathcal{F}_n$ for $i = 1, \dots, k-1$, where $k \geq 2$. Then for $\alpha \geq 1, \beta \geq 1$, we have

$$\begin{bmatrix} \alpha_1^* & \dots & \alpha_n^* \\ \lambda_1^* & \dots & \lambda_n^* \end{bmatrix} = \begin{bmatrix} \alpha_1 & \dots & \alpha_n \\ \lambda_1 & \dots & \lambda_n \end{bmatrix} T_1 \dots T_k \Rightarrow Y_{1:n}^* \succcurlyeq_{st} (\succcurlyeq_{sl}) Y_{1:n}.$$

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On the Largest and the Smallest of the Claims under the Layer Coverage

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Abstract

Let $X_1, \cdots, X_n$ be n risks covered by an insurance layer coverage with deductibles and limits amounts $(d_1, l_1), \cdots, (d_n, l_n)$, respectively. In this paper, we investigate the allocation of insurance layers from the viewpoint of the policyholder. We consider minimizing the expected utility of the largest and the smallest retained loss. Orderings of the optimal allocations are obtained.

Keywords

Deductible, Increasing utility function, Maximum, Increasing convex, Policy limit.

1- Introduction

Let us introduce the risk X as a loss faced by a policyholder which is a non-negative random variable. One of the insurance agreements, which insures only a part of the risk, is insurance layer given by the pay-off

$$X(d, d + l] = \begin{cases} 0, & 0 < X \le d \\ X - d, & d < X \le d + l = (X - d)_+ \wedge l \\ l, & d + l < X \end{cases}$$

where d and l are pre-specified values called deductible and limit, respectively, and $x_+ = \max\{x, 0\}$ and $x \wedge y = \min\{x, y\}$ (Wang, 1996). The insurance layer policy has particular cases known as the deductible policy and the limit policy. It is obvious that when $d = 0$, the layer policy is equivalent to the policy limit and if $l = \infty$, it is reduced to the deductible policy. (cf. Klugman et al., 2004).

Consider a situation that a policyholder is facing by n risks $X_1, \cdots, X_n$ which are insured under an insurance layer coverage. Assume that the total deductible and the total policy limit amounts corresponding to all risks are respectively d and l. Let $d_1, \cdots, d_n$ and $l_1, \cdots, l_n$ be non-negative values such that $\sum_{i=1}^n d_i = d$ and $\sum_{i=1}^n l_i = l$. In some particular situations, the policyholder has the

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right to assign d_i and $l_i, i = 1, \cdots, n$, as the deductible and the limit corresponding to the risk X_i. With these observations, $\sum_{i=1}^{n}(X_i - d_i)_+ \wedge l_i$ is the covered amount by the insurer and the retained risk to the policyholder is given by $\sum_{i=1}^{n}[X_i - (X_i - d_i)_+ \wedge l_i]$. Let w be the initial wealth of the policyholder after paying the required premium which is assumed to be independent of the choice of $(d_1, \cdots, d_n)$ and $(l_1, \cdots, l_n)$. Thepolicyholder is interested in determining the optimal vectors $\boldsymbol{d}^*$and $\boldsymbol{l}^*$in the set

$$s_n(d, l) = \left\{(d_1, \cdots, d_n), (l_1, \cdots, l_n); \sum_{i=1}^{n} d_i = d \text{ and } \sum_{i=1}^{n} l_i = l\right\}$$

such that him/his wealth after paying loss, $w - \sum_{i=1}^{n}[X_i - (X_i - d_i)_+ \wedge l_i]$ is maximized with respect to an optimization criterion. Using the notion of majorization and various types of stochastic orderings, Cheung (2007) and Fathi Manesh et al. (2016) derived an interesting results for the particular cases of the limit policy ($d_i = 0, i = 1, \cdots, n$) and the deductible policy ($l_i = \infty, i = 1, \cdots, n$).

It is known that the policyholder paid amount to an annual basis to cover the cost of the insurance policy that called annual premium. Moreover, the type of insurance effects on the primary cost of transferring the risk to the insurer. Therefore, it is natural to try to relate the annual premium to useful information determined by the smallest and the largest claim amounts. In this paper, we are interested to find the sufficient conditions on the risks, the deductibles and the limits under which the maximum and the minimum of claims as a function of the allocation are minimized. We end this section by recalling some definitions which we use later.

Definition 1.1. The random variable X is said to be smaller than the random variable Y in

(i)   usual stochastic order (denoted as $X \leq_{st} Y$), if $\bar{F}(t) \leq \bar{G}(t)$ for all t;

(ii) increasing concave (convex) order (denoted as $X \leq_{icx} Y$), if $E(\phi(X)) \leq E(\phi(Y))$ for all increasing concave (convex) function ϕ, provided the expectations exist;

(iii) hazard rate order (denoted as $X \leq_{hr} Y$), $\frac{\bar{G}(t)}{\bar{F}(t)}$ is increasing in $t \in (-\infty, \max(u_X, u_Y))$;

(iv) likelihood ratio order (denoted as $X \leq_{lr} Y$), $\frac{g(t)}{f(t)}$ is increasing in $t \in (-\infty, \max(u_X, u_Y))$;

The following lemma is needed to prove the main results of this paper.

Lemma 1.2. (Muller and Stoyan, 2002) Let X and Y be independent random variables.

(i) If $X \leq_{lr} Y$, then $g(X, Y) \leq_{st} g(Y, X)$ for all $g \in G_{lr} = \{g, g(x, y) \geq g(y, x), \forall x \geq y\}$.

(ii) If $X \leq_{hr} Y$, then $g(X, Y) \leq_{icx} g(Y, X)$ for all $g \in G_{hr} = \{g, g(x, y) \text{ and } g(x, y) - g(y, x) \text{ is increasing in } x, \forall x \geq y\}$.

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2- Maximum claim of layer policies

In this section we derive an interesting results for the maximum claim of the layer policy in two cases when $d_1 = d_2 = \cdots = d_n = d'$ and $l_1 = l_2 = \cdots = l_n = l'$. We consider the following problem,

$$\min_{(l,d)\in s_n(d,l)} E\left[u\left(\max_{\{1\leq i\leq n\}}\{X_i \wedge d_i + (X_i - (d_i + l_i))_+\}\right)\right] \qquad (2.1)$$

when u is increasing (convex) utility function.

Lemma 2.1. Suppose that $g_2(x_1, x_2) = max\left\{x_1 \wedge d' + \left(x_1 - (d' + l_1)\right)_+, x_2 \wedge d' + \left(x_2 - (d' + l_2)\right)_+\right\}$, then, whenever $l_1 \leq l_2$,

(i) $g_2(x_1, x_2) \in G_{lr}$, (ii) $g_2(x_1, x_2) \in G_{hr}$.

Theorem 2.2. Let $(X_1, X_2, \cdots, X_n)$ be independent random variables and $X_1 \leq_{lr} X_2 \leq_{lr} \cdots \leq_{lr} X_n$. Then the optimal l^* for the problem (2.1) (when utility function is increasing) satisfies $l_i^* \leq l_j^*$ for any $1 \leq i < j \leq n$ when d_i's are held fixed.

Proof. Using Lemma 2.1(i), for any $1 \leq i < j \leq n$, $g_2(x_i, x_j) \in G_{lr}$, whenever $x_i \geq x_j$ and $l_i \leq l_j$. Now, from Lemma 1.2(i), we have

$$max\left\{X_i \wedge d' + \left(X_i - (d' + l_i)\right)_+, X_j \wedge d' + \left(X_j - (d' + l_j)\right)_+\right\}$$
$$\leq_{st} max\left\{X_i \wedge d' + \left(X_i - (d' + l_j)\right)_+, X_j \wedge d' + \left(X_j - (d' + l_i)\right)_+\right\}. \qquad (2.2)$$

Combining (2.2) and Corollary 4.A.16 of Shaked and Shanthikumar (2007), the required result follows.

Theorem 2.3. Let $(X_1, X_2, \cdots, X_n)$ be independent random variables and $X_1 \leq_{hr} X_2 \leq_{hr} \cdots \leq_{hr} X_n$. Then the optimal l^* for the problem (2.1) (when the utility function is increasing convex) satisfies $l_i^* \leq l_j^*$ for any $1 \leq i < j \leq n$ when d_i's are held fixed and equal to d'.

Proof. Using Lemma 2.1(ii), Lemma 1.2 (ii), the proof is similar to that of Theorem 2.2.

Lemma 2.4. Suppose that $g_3(x_1, x_2) = max\left\{x_1 \wedge d_1 + \left(x_1 - (d_1 + l')\right)_+, x_2 \wedge d_2 + \left(x_2 - (d_2 + l')\right)_+\right\}$, then, $-g_3(x_1, x_2) \in G_{lr}$, whenever $d_1 \leq d_2$.

Theorem 2.5. Let $(X_1, X_2, \cdots, X_n)$ be independent random variables and $X_1 \leq_{lr} X_2 \leq_{lr} \cdots \leq_{lr} X_n$. Then the optimal d^* for the problem (2.1) (when utility function is increasing) satisfies $d_i^* \geq d_j^*$ for any $1 \leq i < j \leq n$ when l_i's are held fixed.

Proof. Using Lemma 2.4 and Lemma 1.2(i), the proof is similar to that of Theorem 2.2.

For the problem (2.1), from Theorem 2.2 and Theorem 2.3, we conclude that, when the deductibles assigned to the risks are equal, if the size of a certain risk is large, then we should allocate a larger limit to it. Theorem 2.5 means that, when the limits assigned to the risks are equal, if the size of a certain risk is large, then we should allocate a smaller deductible to it.

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3- Minimum claim of layer policies

This section is devoted to study of the minimum claim of the layer policy, and our main results are stated and proved in two cases when $d_1 = d_2 = \cdots = d_n = d'$ and $l_1 = l_2 = \cdots = l_n = l'$.

We consider the following problem,

$$\min_{(l,d) \in s_n(d,l)} E\left[u\left(\min_{\{1 \le i \le n\}}\{X_i \wedge d_i + (X_i - (d_i + l_i))_+\}\right)\right] \tag{3.1}$$

when u is increasing (concave) utility function.

Lemma 3.1. Suppose that $g_4(x_1, x_2) = min\left\{x_1 \wedge d' + (x_1 - (d' + l_1))_+, x_2 \wedge d' + (x_2 - (d' + l_2))_+\right\}$, then, whenever $l_1 \le l_2$,

(i) $-g_4(x_1, x_2) \in G_{lr}$, (ii) $-g_4(x_1, x_2) \in G_{hr}$.

Theorem 3.2. Let $(X_1, X_2, \cdots, X_n)$ be independent random variables and $X_1 \le_{lr} X_2 \le_{lr} \cdots \le_{lr} X_n$. Then the optimal $\boldsymbol{l}^*$ for the problem (3.1) (when the utility function is increasing) satisfies $l_i^* \ge l_j^*$ for any $1 \le i < j \le n$ when d_i's are held fixed.

Proof. Using Lemma 3.1(i) and Lemma 1.2(i), the proof is similar to that of Theorem 2.2.

Theorem 3.3. Let $(X_1, X_2, \cdots, X_n)$ be independent random variables and $X_1 \le_{hr} X_2 \le_{hr} \cdots \le_{hr} X_n$. Then the optimal $\boldsymbol{l}^*$ for the problem (3.1) (when utility function is increasing concave) satisfies $l_i^* \ge l_j^*$ for any $1 \le i < j \le n$ when d_i's are held fixed.

Proof. Using Lemma 3.1(ii) and Lemma 1.2(ii), the proof is similar to that of Theorem 2.2.

Lemma 3.4. Suppose that $g_5(x_1, x_2) = min\left\{x_1 \wedge d_1 + (x_1 - (d_1 + l'))_+, x_2 \wedge d_2 + (x_2 - (d_2 + l'))_+\right\}$, then, whenever $d_1 \le d_2$, $g_5(x_1, x_2) \in G_{lr}$.

Theorem 3.5. Let $(X_1, X_2, \cdots, X_n)$ be independent random variables and $X_1 \le_{lr} X_2 \le_{lr} \cdots \le_{lr} X_n$. Then the optimal $\boldsymbol{d}^*$ for the problem (3.1) (when the utility function is increasing) satisfies $d_i^* \le d_j^*$ for any $1 \le i < j \le n$ when l_i's are held fixed.

Proof. Using Lemma 3.4 and Lemma 1.2(i), the proof is similar to that of Theorem 2.2.

For the problem (3.1), Theorem 3.2 and Theorem 3.3 say that, when the deductibles assigned to the risks are equal, if the size of a certain risk is large, then we should allocate a smaller limit to it. Theorem 3.5 means that, when the limits assigned to the risks are equal, if the size of a certain risk is large, then we should allocate a larger deductible to it.

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On the Nonparametric Estimation of Conditional Tail Expectation

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Abstract

One of the most important actuarial risk measures is the conditional tail expectation (CTE) which is the average amount of loss given that the loss exceeds a specified quantile. This paper focuses on the nonparametric estimation of CTE risk measure based on kernel method. Asymptotic properties of the proposed estimator are studied. A simulation study is conducted to compare the performances of the proposed estimator with its empirical competitor.

Keywords

Asymptotic properties, Conditional tail expectation, Kernel method, Risk measure.

1- Introduction

In financial mathematics, a risk measure is used to determine the amount of an asset or set of assets (traditionally currency) to be kept in reserve. A risk measure is defined as a mapping from a loss or profit random variables to the real numbers. This set of random variables represents portfolio returns. One of the most important actuarial risk measures is the conditional tail expectation (CTE) (see, e.g., Denuit et al., 2005), which is the average amount of loss given that the loss exceeds a specified quantile. It is also quite intuitive and is already widely used in actuarial science (see Hardy, 2003 and the references therein, and also Landsman and Valdez, 2003, Bilodeau, 2004, Cai and Li, 2005 and Dhaene et al., 2003). The CTE risk measure (also called Tail-VaR or expected shortfall) is coherent. The term "coherent" risk measure is reserved for risk measures that satisfy a specific set of properties (cf. e.g., Artzner et al., 1997, 1999). In the following, we present basic notation and definitions.

Let X be the loss random variable with cumulative distribution function (CDF) F and quantile function Q_X. Assume that F is defined on the entire real line, with negative loss interpreted as gain. The CTE of the risk or loss X is then defined by

$$\mathrm{CTE}_X(t) = E\big(X \big| X > Q_X(t)\big), \quad t \in (0,1). \qquad (1)$$

If F is a continuous function, it can be shown easily that the right hand side of (1), denoted by C_X, is

$$C_X(t) = \frac{1}{1-t} \int_t^1 Q_X(u)\,du, \qquad t \in (0,1). \quad (2)$$

Estimation of the CTE for company assets and liabilities is becoming an important actuarial exercise, and the size and complexity of these liabilities make inference procedures with good small sample

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performance very desirable. Suppose that we have independent and identically distributed random variables $X_1, X_2, \dots$ with common CDF F, and let $X_{(1)} < \cdots < X_{(n)}$ denote the order statistics of $X_1, \dots, X_n$. It is natural to define an empirical estimator of $CTE_X(t)$ by

$$\hat{C}_n(t) = \frac{1}{1-t} \int_t^1 Q_n(u)du, \qquad t \in (0,1), \qquad (3)$$

where $Q_n(u)$ is the empirical quantile function, which is equal to the ith order statistic, $X_{(i)}$, for all $u \in (\frac{i-1}{n}, \frac{i}{n}]$ and for all $i = 1, \dots, n$.

Statistical estimation of CTE is studied by Jones and Zitikis (2003). The study of the bias and variance of the empirical CTE under a specified set of assumptions were argued by Manistre and Hancock (2005), and Kim and Hardy (2007). Brazauskas et al. (2008) explored parametric and nonparametric approaches to estimate CTE function. Also, they construct confidence intervals and bands for this function. Ko et al. (2009) investigated this fact that an unbiased nonparametric estimator of the CTE does not exist. In addition, they studied asymptotic behavior of the bias of the empirical CTE and derived a closed form expression for its first order term. Necir et al. (2010) suggested a new CTE estimator, which is applicable when losses have finite means but infinite variances. Ahn and Shyamalkumar (2011) established the convergence of the normalized CTE estimator of a continuous random variable under importance sampling to a normal distribution.

The empirical estimator defined in (3), is constructed from sample quantiles. The main drawback to sample quantiles is that they experience a substantial lack of efficiency, caused by the variability of individual order statistics. Averaging over all order statistics leads to kernel type estimators

$$\hat{Q}_n(t) = \frac{1}{h_n} \int_0^1 K\left(\frac{u-t}{h_n}\right) Q_n(u)du, \qquad (4)$$

for an appropriate choice of kernel K and a sequence of bandwidth $h_n > 0$.

According to equation (4), the following estimator is defined for C_X function

$$C_n(t) = \frac{1}{1-t} \int_t^1 \hat{Q}_n(u)du, \qquad t \in (0,1). \qquad (5)$$

The main aim of this paper is to investigate strong consistency and asymptotic normality of $C_n(t)$. Based on the results, we then construct asymptotic confidence intervals for this function. The performance of the proposed estimator will be studied by Monte Carlo simulation. The following assumptions are used to establish the main results:

1. The kernel function K, is a probability density function with finite support.
2. $\int_{-\infty}^{\infty} t \, K(t)dt = 0$.
3. F has a density function f, which is continuous in some neighborhood of $Q_X(t)$.
4. The first derivative of density function i.e. f' exists and is continuous at $Q_X(t)$.
5. $\inf_{0<t<1} f\big(Q(t)\big) > 0$.

2- Main Results

Theorem 1. Suppose that assumption 1-5 are satisfied. If $n \to \infty$ and $h_n \to 0$, in such a way that $n^{\frac{1}{4}}h_n \to 0$, then $C_n(t)$ is a strongly consistent estimator of $C_X(t)$ for each fixed $t \in [0,1]$.

Theorem 2. Under stated assumption in Theorem 1, for each fixed $t \in (0,1)$

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$$\sqrt{n}\left(\hat{C}_{X.n}(t) - C_X(t)\right) \xrightarrow{d} N\left(0, \sigma_X^2(t)\right), \qquad (6)$$

where

$$\sigma_X^2(t) = \frac{1}{(1-t)^2}\int_{Q_X(t)}^{\infty}\int_{Q_X(t)}^{\infty} F(x \wedge y) - F(x)F(y)dx\,dy.$$

Corollary 1. Under stated assumption of Theorem 2,

$$\sigma_n^2(t) = \frac{1}{(1-t)^2}\int_{Q_n(t)}^{\infty}\int_{Q_n(t)}^{\infty} F_n(x \wedge y) - F_n(x)F_n(y)dx\,dy$$

is a consistent estimator of $\sigma_X^2(t)$. Hence, $100(1-\alpha)\%$ asymptotic confidence interval for the CTE, $C_X(t)$ is

$$C_n(t) \pm \frac{\sigma_X^2(t)Z_{\frac{\alpha}{2}}}{\sqrt{n}},$$

where $Z_{\frac{\alpha}{2}}$ is the $100\left(1 - \frac{\alpha}{2}\right)\%$ percentile of the standard normal distribution.

Theorem 3. Suppose that the same conditions of Theorem 1 are satisfied. Then, we have

$$\sqrt{n}\left(C_n(t) - C_X(t)\right) \xrightarrow{d} \frac{1}{1-t}\int_{Q_X(t)}^{\infty} B(F(x))dx, \qquad (7)$$

where $B = \{B(u), 0 < u < 1\}$ is the Brownian bridge.

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Optimal Relativity Premium for a Bonus-Malus System Using Non-asymptotic Approach

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Abstract

An optimal Bonus-malus system can be determined via its relativity premium. Borgan et al., (1981) introduced a non-asymptotic approach for Bonus-malus system. Their method considered an integer-valued random variable that represents the age of the policy in the portfolio and considers the transient behavior of a Bonus-malus system. This article employs the maximum entropy method along with a Bayesian estimation method to derive an optimal linear relativity premium using non-asymptotic approach. Application of our findings has been given for Bonus-malus systems in some country.

Keywords

Non-asymptotic approach; Bayesian relativity; Bonus-malus System; Optimal linear relativity; Maximum entropy method.

1- Introduction

In many countries insurers use Bonus -malus systems (BMS), in auto insurance in order to provide fair premium amounts based on policyholders claim experience. Such systems penalize insured drivers who claim at least one accident (malus) and rewards claim-free drivers (bonuses). In practice, a BMS consists of a finite number of levels, numbered from 1 to s; as policyholders risk classification. A policyholder's premium whose corresponding level at a given BMS is k; can be determined by multiplying relative premium by base premium; see Denuit et al, (2007) for more details.

Norberg (1976) established that a Bayes estimator for relativity premium r_k under the quadratic loss function is

$$r_k = E(\Theta \mid K = k) = \frac{\int_0^{+\infty} \theta \pi_k(\lambda\theta) d\pi_\Theta(\theta)}{\int_0^{+\infty} \pi_k(\lambda\theta) d\pi_\Theta(\theta)} , \quad k = 1, 2, ..., s. \quad (1)$$

where λ is a priori expected claim frequency, Θ is the random relative risk parameter of a randomly-chosen policyholder with distribution function F, and $\pi(\lambda\theta) = (\pi_1(\lambda\theta), ..., \pi_s(\lambda\theta))$ is the stationary probability vector for the policyholder with mean frequency. This approach is well-known as an asymptotic approach.

Several studies have attempted to design an optimal BMS using the asymptotic approach. They employed Bayesian estimator, Eq. (1), as a best estimator for the relativity premium. Unfortunately, the Bayesian estimator, given by Eq. (1), is computationally very expensive. Moreover, one cannot

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be confident that such Bayes estimator will satisfy the necessary condition $r_1^{Bays} < r_2^{Bays} < \cdots < r_s^{Bays}$.

Gild and sundt (1989) suggested the following linear class to estimate relativity premium

$$r_l^{Lin} = \alpha + \beta l; \qquad l = 1,...,s \ (2)$$

They computed α and β by minimizing average quadratic distance between relativity premium and risk parameter. Tan (2015,2016) revisited the determination of optimal relativities under the linear form of relativities because of this form is more viable in designing a commercial BMS.

In reality, the time span of a policy is not infinite. Therefore, it is reasonable to consider a finite number N as the age of a given policy in computations. This approach introduced by Borgan et al., (1981), which is a method that considered an integer-valued random variable for age of the policy in the portfolio and named non-asymptotic approach. In this paper a simple and equitable optimal linear relativity non-asymptotic approach will be presented. The method considered an integer-valued random variable, N, and considers the transient behavior of a BMS.

2- Preliminaries

Mahmoodvand et al., (2013) compared the Iranian BMS with four other systems studied by Lemaire and Zi (1994)). In the present work we compare the Iranian BMS with those employed in Brazil, Denmark, Kenya and Hong Kong.

The maximum-entropy method was initially used in finance and actuarial sciences by Abbas (2002 , 2006) . The maximum-entropy method employs an entropy measure under some restrictions that decreases uncertainty in events.

As far as we know, no publication is available for comparing BMS using non-asymptotic approach. Now, utilizing the Loimaranta efficiency (Loimaranta, (1972)), we introduce an efficiency index for non-asymptotic situation.

Definition: Suppose $\bar{\bar{R}}(\vartheta)$ denotes the average relativity once stationary has been reached for a policyholder with annual expected claim frequency ϑ, i.e., $\bar{\bar{R}}(\vartheta) = \sum_{n=1}^{s} p_n \sum_{k=1}^{s} \bar{r}_k \, p(K_n = k \mid \vartheta)$, if the system is stable after n^* years. Without loss of generality, the above relativity can be restated as

$$\bar{\bar{R}}(\vartheta) = \sum_{n=1}^{n^*-1} p_n \sum_{k=1}^{s} \bar{r}_k \, p(K_n = k \mid \vartheta) + p(n \geq n^*) \sum_{k=1}^{s} \bar{r}_k \pi(\vartheta).$$

Now, an efficiency index $\bar{\bar{\Xi}}$ of the expected ϑ under non-asymptotic situation can be defined as

$$\bar{\bar{\Xi}}(\vartheta) = \frac{d \ln(\bar{\bar{R}}(\vartheta))}{d \ln(\vartheta)}. \qquad (3)$$

3- An optimal linear relativity for BMS systems

In the following we will introduce an estimator which is not only non-asymptotic and efficient but also requires very little computations. It is a simple and practical approach to linear relativity of the form $\bar{r}_k^{Lin} = \alpha + \beta k$, $k = 1, \cdots, s$. The following Theorem provides optimal linear relativity using non-asymptotic approach. Let $\bar{M}_k(t), k = 1,...,s$ be the number of policyholders in the level k at

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time t and u_0 be the initial capital. Hence, $\bar{U}(t) = \Pi \sum_{k=1}^{s} r_k \bar{M}_k(t)$ is the sum of the premium income in a BMS at time t. By considering the sum of premium incomes, the optimal linear relativity is determined here as the sum of premiums with linear relativity, is as close as possible to the sum of premiums in Bayesian relativity. In the following α and β in Eq. (2) will be determined when the average information is maximized.

Theorem. Suppose the BMS has s levels, numbered from 1 to s. The integer-valued random variable N represents the age of the policy in the portfolio and the probability mass function is $P(N = n) = p_n$, $n = 1, 2, \ldots$ Let $\bar{r}_k^{Bays}$ and $\bar{r}_k^{Lin} = \bar{\alpha} + \bar{\beta}k$ for $k = 1, \ldots, s$ be the Bayesian and linear relativity premiums, respectively. In the class of linear relativities, an optimal relativity that (i) minimizes the mean squared-error between the sum of premiums of the linear relativity and Bayesian relativity and (ii) maximizes Shannon's entropy is $\bar{r}_k^{Lin} = \bar{\alpha}_{opt} + \bar{\beta}_{opt}k$ for $k = 1, \cdots, s$ where

$$\bar{\alpha}_{opt} = \frac{E(\bar{U}^{Bay}\bar{M})}{E(\bar{M}^2)}(1-\varpi), \qquad \bar{\beta}_{opt} = \frac{E(\bar{U}^{Bay}\bar{M})}{E(\bar{M}^{*2})}\varpi, \qquad (4)$$

$$\varpi = \frac{E(\bar{U}^{Bays}\bar{M})E(\overline{MM^*})^2 - E(\bar{M}^2)E(\bar{U}^{Bays}M^*)E(\overline{MM^*})}{E(\bar{U}^{Bays}\bar{M})E(\overline{MM^*})^2 - E(\bar{U}^{Bays}\bar{M})E(\bar{M}^2)E(\bar{M}^{*2})}$$

and

$$E(\bar{M}^2) = \sum_{n=1} p_n E(M^2(n)) = \sum_{n=1} p_n (Var(M(n)) + E(M(n))^2), \qquad E(\overline{MM^*}) = \sum_{n=1} p_n E(M(n)M^*(n))$$

$$E(K) = \sum_{n=1} p_n \sum_{k=1}^{s} k\, p(K_n = k), \qquad E(\bar{U}^{Bays}\bar{M}) = \sum_{n=1} p_n E(U^{Bays}(n)M(n)),$$

subject to $E(\bar{U}^{Bays}\bar{M}^*(t))E(\overline{MM^*}) < E(\bar{U}^{Bays}\bar{M})E(\bar{M}^{*2})$.

In the non-asymptotic approach, let the transient distribution be $P(L_n = k \mid \lambda\theta) = p_k^{(n)}(\lambda\theta)$ for $n = 1, 2, \cdots$. The distribution of Θ is given in Table 3 (Norberg, 1976).

Table3: The distribution of Θ quoted from Norberg (1976).

θ	0.033	0.067	0.1	0.73	0.167	0.2	0.233	0.267	0.3	0.333
$p(\Theta = \theta)$	0.143	0.172	0.143	0.114	0.1	0.086	0.072	0.057	0.043	0.029
θ	0.367	0.4	0.433	0.467	0.5	0.533	0.567	0.6	0.633	0.667
$p(\Theta = \theta)$	0.014	0.007	0.006	0.004	0.003	0.003	0.001	0.001	0.001	0.001

The integer-valued random variable N is uniformly distributed on the integers . Bayesian relativity is computed using Eq.(1) and optimal linear relativity is computed using Theorem. We have computed the efficiency of relativities from Eq. (3) for $\lambda \in [0,1]$ and have plotted them for each country. In the beginning $M_0 = 1000$ policyholders are entered into the system. A multinomial distribution was used for the number of policyholders in period n.

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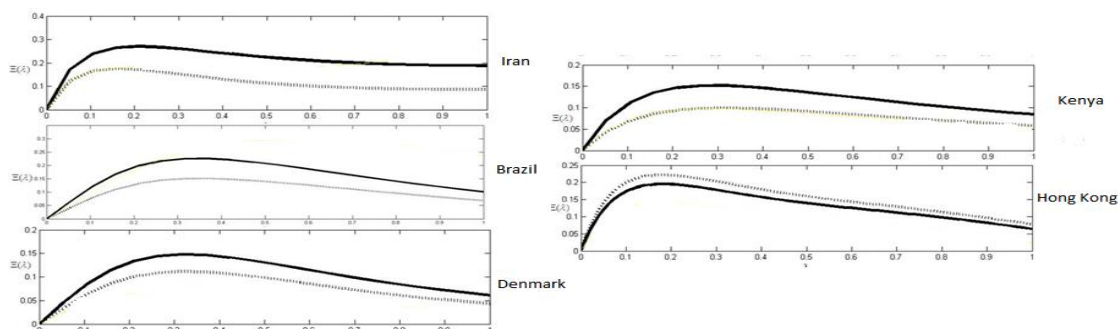Pricing and Industrial Statistics
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Figure 1: Efficiency of optimal linear (solid line) and Bayesian (dotted line) relativities.

Figure 1 shows the efficiency evolution for the BMS in the mentioned countries in a non-asymptotic approach. In all cases, except for Hong Kong, for the number of claims less than 0.25 , the efficiency of the optimal linear relativity is better than Bayesian relativity.

In every case, if the expected number of claims is smaller than 0.25, the efficiency of the optimal linear relativity is better than the Bayesian relativity. For claim distributions, several papers have been published recently have used distributions which have means between 0.07 and 0.12 with an average of 0.10 (Park et al. (2010)). Therefore, in practice, the optimal linear relativity presented is practical and better than the Bayesian relativity.

4- Conclusion

This article, using the maximum entropy approach, derives an optimal linear relativity for a given bonus-malus system. Such linear relativity has been determined such that our estimator is closed to the Bayesian relativity, an optimal condition. This linear relativity retains logical order $r_1 < r_2 < \cdots < r_s$ with very easy computation. Practical applications of our findings have been given for five BMS.

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Pricing Life Annuities By Using Fuzzy Technical Interest Rate

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Abstract

Life insurance future financial commitments depend on variables such as the age of the insured, lifetime, and technical (guaranteed) interest rate. Life annuity insurance is one type of life insurance. Life annuity pricing models must consider number of variabales, some of which are subject to change in the future and uncertain presently, including demographic events and financial variables. In standard life insurance mathematics, the uncertainty of demographic phenomena is considered as stochastic and the corresponding probabilities are obtained from life tables. Recent research in actuarial science has focused on formalizing uncertainties related to the economic parameters by means of random variables and stochastic processes. The most important of those parameters is, undoubtedly, the discount rates used to price policies. This paper has developed life annuity pricing models with stochastic representation of mortality and fuzzy quantification of interest rates, as a measurement of uncertain parameters such as interest rate. Therefore, the present value of life annuities is modeled using fuzzy random variables that can explain certain related measures: mathematical expectation, variance, standard deviation, distribution function, and quantiles.

Keywords

Life Annuities, Pricing, Interest Rate, Fuzzy.

1- Introduction

Actuaries traditionally have used a deterministic approach in most of their calculations. In particular it is usual to assume a constant rate of interest. The proposals to introduce variable life insurance generated a number of papers in the Transactions of the Society of Actuaries which used statistical models of investment returns (Boyle, 1976). The papers published by Andrés and Terceño (2003), Betzuen et al. (1997), Lemaire (1990) and Ostaszewski (1993) are particularly noteworthy in a life-insurance context as are those of Andrés and Terceño (2003), Cummins and Derrig (1997) and Derrig and Ostaszewki (1997) clarified the applications of FST in actuarial science is actuarial pricing of insurance contracts with fuzzy interest rates. In many real situations the uncertainty is the result any one of numerous different causes: randomness, hazard, inaccuracy, incomplete information, etc. As Kaufmann and Gil-Aluja (1990) points out stochastic variability is described by the use of probability theory and other types of uncertainties such as incomplete information or imprecision can be captured with the use of fuzzy subsets (Sanches and Puchades, 2012). In this

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article, we try to solve the question "how Life annuities can be priced by using fuzzy technical (garenteed) interest rates?".

2- Results

In Iran insurance industry, life products are planned based on bylaw 68. Technical Interest Rate (18 percent for the first 5 years; 15 percent for the next 5 years and 10 percent for the periods more than 10) and death probability (TD88-90) as the main variables effecting life products prices are used (Bylaw 68, Iran Bimeh Markazi, 2011).

We suppose a life annuity for a 62 male. Premium period is 3 years (m=3) and annuity period is 10 years (n=10). For fuzzing technical interest rates according to central values 18, 15, 10, interval 2± are considered. So, we have fuzzy value $\tilde{i}_1 = (0.16, 0.18, .20)$ for the first 5 years, $\tilde{i}_2 = (0.13, 0.15, .17)$ for the next 5 years and $\tilde{i}_3 = (0.08, 0.10, .12)$ for the periods more than 10. Then, we have:

$$\forall \alpha \in [0,1], i_{1\alpha} = \left[\underline{i_1}(\alpha), \overline{i_1}(\alpha)\right] = [0.16 + 0.02\alpha, 0.2 - 0.02\alpha]$$
$$\forall \alpha \in [0,1], i_{2\alpha} = \left[\underline{i_2}(\alpha), \overline{i_2}(\alpha)\right] = [0.13 + 0.02\alpha, 0.17 - 0.02\alpha]$$
$$\forall \alpha \in [0,1], i_{3\alpha} = \left[\underline{i_3}(\alpha), \overline{i_3}(\alpha)\right] = [0.08 + 0.02\alpha, 0.12 - 0.02\alpha]$$

And also for $\tilde{d}_t = (1+\tilde{i})^{-t}$ we have:

$$d_{1t_\alpha} = \left[\underline{d_{1t}}_\alpha, \overline{d_{1t}}_\alpha\right] = [(1.2 - 0.02\alpha)^{-t}, (1.16 + 0.02\alpha)^{-t}]$$
$$d_{2t_\alpha} = \left[\underline{d_{2t}}_\alpha, \overline{d_{2t}}_\alpha\right] = [(1.17 - 0.02\alpha)^{-t}, (1.13 + 0.02\alpha)^{-t}]$$
$$d_{3t_\alpha} = \left[\underline{d_{3t}}_\alpha, \overline{d_{3t}}_\alpha\right] = [(1.12 - 0.02\alpha)^{-t}, (1.08 + 0.02\alpha)^{-t}]$$

Fuzzy variable of annuity present value $_{3|10}\tilde{a}_{62}$, takes fuzzy numbers with below probabilities

Fuzzy variable of annuity present value

Fuzzy outcomes	Deterministic outcomes	Alpha cuts of outcomes	Probability (TD88-90)
0	0	0=[0,0]	$_3q_{62}$=0.0571
$(1 + \tilde{i}_1)^{-3}$	0.6086	$[(1.2 - 0.02\alpha)^{-3}, (1.16 + 0.02\alpha)^{-3}]$	$_{3\mid}q_{62}$=0.0208
. .	. .	. .	. .
$\sum_{t=3}^5 (1+\tilde{i}_1)^{-t} +$ $\sum_{t=6}^{10}(1+\tilde{i}_2)^{-t}+$ $\sum_{t=11}^{12}(1+\tilde{i}_3)^{-t}$	4.3563	$\sum_{t=3}^5 (1.2 - 0.02\alpha)^{-t}, \sum_{t=3}^5(1.16 + 0.02\alpha)^{-t}]+$ $[\sum_{t=6}^{10}(1.17 - 0.02\alpha)^{-t}, \sum_{t=6}^{10}(1.13 + 0.02\alpha)^{-t}]+$ $[\sum_{t=11}^{12}(1.12 - 0.02\alpha)^{-t}, \sum_{t=11}^{12}(1.08 + 0.02\alpha)^{-t}]$	$_{11}p_{62}$=0.743

This is clear that for α=1, deterministic values are calculated.

According to calculations, this case should pay 1.1615 (deterministic and net) by the beginning of every 3 years to satisfy 1 unit annuity by the beginning of every 10 years.

We have also:

$$E(_{m|n}\widetilde{a}_x)_\alpha = E(_{m|n}\underline{\widetilde{a}_{x\alpha}}) = \sum_{t=3}^{12}\underline{d_{t\alpha}} \cdot {}_t p_x = (1.2-0.02\alpha)^{-3} \times 0.943 + \cdots + (1.12-0.02\alpha)^{-11} \times 0.743 +$$

$$\underline{(1.12-0.02\alpha)^{-12} \times 0.712}$$

$$\overline{E(_{m|n}\widetilde{a}_x)}_\alpha = E(_{m|n}\overline{\widetilde{a}_{x\alpha}}) = \sum_{t=3}^{12}\overline{d_{t\alpha}} \cdot {}_t p_x = (1.16+0.02\alpha)^{-3} \times 0.943 + \cdots + (1.08+0.02\alpha)^{-11} \times 0.743 +$$

$$(1.08+0.02\alpha)^{-12} \times 0.712$$

$$Var(_{m|n}\underline{\widetilde{a}_{x\alpha}}) = \{(1.2-0.02\alpha)^{-6} \times 0.0208 + [\sum_{t=3}^{4}(1.2-0.02\alpha)^{-t}]^2 \times 0.0216 + \cdots + ([\sum_{t=3}^{5}(1.2-0.02\alpha)^{-t}]^2 +$$

$$[\sum_{t=6}^{10}(1.17-0.02\alpha)^{-t}]^2 + (1.12-0.02\alpha)^{-22}) \times 0.0315 + ([\sum_{t=3}^{5}(1.2-0.02\alpha)^{-t}]^2 + [\sum_{t=6}^{10}(1.17-0.02\alpha)^{-t}]^2 +$$

$$[\sum_{t=11}^{12}(1.12-0.02\alpha)^{-t}]^2) \times 0.743\} - \{(1.2-0.02\alpha)^{-3} \times 0.943 + \cdots + (1.17-0.02\alpha)^{-10} \times 0.773 + (1.12-$$

$$0.02\alpha)^{-11} \times 0.743 + (1.12-0.02\alpha)^{-12} \times 0.712\}^2$$

$$Var(_{m|n}\overline{\widetilde{a}_{x\alpha}}) = \{(1.16+0.02\alpha)^{-6} \times 0.0208 + \cdots + ([\sum_{t=3}^{5}(1.16+0.02\alpha)^{-t}]^2 + [\sum_{t=6}^{10}(1.13+0.02\alpha)^{-t}]^2 +$$

$$[\sum_{t=11}^{12}(1.08+0.02\alpha)^{-t}]^2) \times 0.743\} - \{(1.16+0.02\alpha)^{-3} \times 0.943 + \cdots + (1.08+0.02\alpha)^{-11} \times 0.743 + (1.08+$$

$$0.02\alpha)^{-12} \times 0.712\}^2$$

And also (*r=1,…,n-1*):

$$\overline{F_{_{m|n}\widetilde{a}_x}(\text{y})}_\alpha = \begin{cases} 0 & \textit{if } y<0 \\[4pt] 0.0571 & \textit{if } 0 \le y < (1.16+0.02\alpha)^{-3} \\[4pt] 0.0779 & \textit{if } (1.16+0.02\alpha)^{-3} \le y < \sum_{t=3}^{4}(1.16+0.02\alpha)^{-t} \\[4pt] \vdots & \vdots \\[4pt] 0.2882 & \textit{if } \sum_{t=3}^{5}(1.16+0.02\alpha)^{-t}+\sum_{t=6}^{10}(1.13+0.02\alpha)^{-t}+(1.08+0.02\alpha)^{-11} \le y < \\ & \sum_{t=3}^{5}(1.16+0.02\alpha)^{-t}+\sum_{t=6}^{10}(1.13+0.02\alpha)^{-t}+\sum_{t=11}^{12}(1.08+0.02\alpha)^{-t} \\[4pt] 1 & \textit{if } y > \sum_{t=3}^{5}(1.16+0.02\alpha)^{-t}+\sum_{t=6}^{10}(1.13+0.02\alpha)^{-t}+\sum_{t=11}^{12}(1.08+0.02\alpha)^{-t} \end{cases}$$

$$\underline{F_{_{m|n}\widetilde{a}_x}(\text{y})}_\alpha = \begin{cases} 0 & \textit{if } y<0 \\[4pt] 0.0571 & \textit{if } 0 \le y < (1.2-0.02\alpha)^{-3} \\[4pt] 0.0779 & \textit{if } (1.2-0.02\alpha)^{-3} \le y < \sum_{t=3}^{4}(1.2-0.02\alpha)^{-t} \\[4pt] \vdots & \vdots \\[4pt] 0.2882 & \textit{if } \sum_{t=3}^{5}(1.2-0.02\alpha)^{-t}+\sum_{t=6}^{10}(1.17-0.02\alpha)^{-t}+(1.12-0.02\alpha)^{-11} \le y < \\ & \sum_{t=3}^{5}(1.2-0.02\alpha)^{-t}+\sum_{t=6}^{10}(1.17-0.02\alpha)^{-t}+\sum_{t=11}^{12}(1.12-0.02\alpha)^{-t} \\[4pt] 1 & \textit{if } y \ge \sum_{t=3}^{5}(1.2-0.02\alpha)^{-t}+\sum_{t=6}^{10}(1.17-0.02\alpha)^{-t}+\sum_{t=11}^{12}(1.12-0.02\alpha)^{-t} \end{cases}$$

- If $0 < \varepsilon \le 0.0571$ ،then. $Q_{_{m|n}\widetilde{a}_x\alpha}{}^\varepsilon = 0$

- If $0.0571 < \varepsilon \le 0.0779$ ،then. $Q_{_{m|n}\widetilde{a}_x\alpha}{}^\varepsilon \in \{(1.2-0.02\alpha)^{-3}, (1.16+0.02\alpha)^{-3}\}$

- If $0.2567 < \varepsilon < 0.2882$ ،then

$$Q_{_{m|n}\widetilde{a}_x\alpha}{}^\varepsilon = \{[\sum_{t=3}^{5}(1.2-0.02\alpha)^{-t} + \sum_{t=6}^{10}(1.17-0.02\alpha)^{-t} + (1.12-0.02\alpha)^{-11}], [\sum_{t=3}^{5}(1.16+$$

$$0.02\alpha)^{-t}+\sum_{t=6}^{10}(1.13+0.02\alpha)^{-t}+(1.08+0.02\alpha)^{-11}]\}$$

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And $r \epsilon\{1,2,\dots,n-2\}$ و n-2<m و $\{1,2,\dots,n-2\}=\Theta$.

- If $0.2882 < \varepsilon \leq 1$ ،then

$$Q_{\underset{m|n}{\tilde{a}_x}\alpha}{}^{\varepsilon}=\{[\sum_{t=3}^{5}(1.2-0.02\alpha)^{-t}+\sum_{t=6}^{10}(1.17-0.02\alpha)^{-t}\sum_{t=11}^{12}+(1.12-0.02\alpha)^{-t}],[\sum_{t=3}^{5}(1.16+0.02\alpha)^{-t}+\sum_{t=6}^{10}(1.13+0.02\alpha)^{-t}+\sum_{t=11}^{12}(1.08+0.02\alpha)^{-t}]\}$$

It should be considered that expected value, distribution function and quantiles of a fuzzy numbers are fuzzy. We can consider fuzzy numbers as upper and lower limit for diterministic actuarial judgments.

3- Conclusion

Life insurance future financial commitments depend on variables such as the age of the insured, lifetime, and technical (guaranteed) interest rate. Life annuity insurance is one type of life insurance. Life annuity pricing models must consider number of variabales, some of which are subject to change in the future and uncertain presently, including demographic events and financial variables. In standard life insurance mathematics, the uncertainty of demographic phenomena is considered as stochastic and the corresponding probabilities are obtained from life tables. Recent research in actuarial science has focused on formalizing uncertainties related to the economic parameters by means of random variables and stochastic processes. The most important of those parameters is, undoubtedly, the discount rates used to price policies. This paper has developed life annuity pricing models with stochastic representation of mortality and fuzzy quantification of interest rates, as a measurement of uncertain parameters such as interest rate. Therefore, the present value of life annuities is modeled using fuzzy random variables that can explain certain related measures: mathematical expectation, variance, standard deviation, distribution function, and quantiles.

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Predict Failure Time in a Mechanical System While the System Is Running

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Abstract

The application of condition monitoring techniques within industry to manage maintenance actions has increased swiftly over recent years. Knowing when a special component fails is very important because by replacing it before the failure time, many troubles such as loss production, injuries because of failures, damaging other components, endangering the safety of factories can be avoided. In this paper, a decision making model based on condition monitoring big data is proposed. This model uses condition monitoring big data as covariates and finds their effects on the life time of a component. Proposed model is first trained offline using previously recorded logfile data, and is then used to predict failures online – while the system is running.

Keywords

Reliability, Condition monitoring, Failure time, Proportional hazard model.

1- Introduction

The failure of engineering systems is a developing process involving the load action and damage reposition. There are two types of data that are often applied to forecast failure time: condition monitoring data and lifetime data. For most common reliability analysis methods, the reliability function is estimated by historical lifetime data, and the failure probability at any given time can be computed. However, the lifetime data, which is the length of run time prior to failure, only provide the final outcome from the failure and are not appropriate for modeling the failure process under various operating situations. If condition monitoring is done through the lifetime of a mechanical system, a trend model for failure features can be built to predict the failure time in which the reliability value would reach a predefined threshold. To attain proper failure prediction, a more effective strategy that combines lifetime data and monitoring data is needed.

Many researchers attempt to relate the failure probability to both lifetime data and condition monitoring variables. The proportional hazard model (PHM) is the most widely used in these researches [1–4]. In the PHM proposed by Jardine et al. [2], the Weibull distribution parameters are estimated using lifetime data from aircraft engines and marine gas turbines, and the metal particle level is applied as the monitoring variable to provide the condition information. Results show that the PHM well denotes the actual influence of monitoring variable on system residual life. In

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addition, the PHM is also remodeled to proportional intensity model (PIM) and proportional covariate model (PCM). Volk et al. [3] studied the application of the PIM to analyze the inter-arrival failure times and vibration data obtained from bearings. Sun et al. [4] introduced the PCM, applying the data from accelerated life tests to estimate mechanical system hazards for the case of sparse or even no historical failure data.

In this article, we present a Reduced modified Weibull proportional hazard model (RWPHM) for mechanical system failure prediction. It combines lifetime and monitoring data of multiple failure modes to predict failures online – while the system is running.

2- The model

The PHM, which was first introduced by Cox [5], has become an important statistical regression model and has been applied in many studies of mechanical system reliability. The basic assumption of the PHM is that the hazard rate of a system consists of two multiplicative factors baseline hazard rate and an exponential function including the effects of the monitoring variables. The hazard rate at time t is written as:

$$h(t, \mathbf{z}_t) = h_0(t)e^{\mathbf{\gamma} \cdot \mathbf{z}_t} \tag{1}$$

where is the $h_0(t)$ baseline hazard rate that is dependent on the service time, $\mathbf{z}_t$ is a row vector composed of monitoring values at time t, and $\mathbf{\gamma}$ is a column vector composed of the regression parameters corresponding to the monitoring variables. In the PHM, $\mathbf{z}_t$ is regarded as a vector of covariates that increases or decreases the system hazard rate proportionally, and the coefficient vector γ defines the influence of the monitoring variables on the failure process.

The RNMW distribution by [6] is used to model the failure time of mechanical systems with a bathtub shaped hazard rate function. The hazard rate function of the RNMW distribution is selected as the baseline hazard rate of the PHM. The PHM with the RNMW baseline function is called the RW proportional hazard model (RWPHM). Then, the hazard function of the RWPHM is defined as:

$$h(t, \mathbf{z}_t) = \frac{1}{2\sqrt{t}}[\lambda_1 + \lambda_2(1 + 2kt)e^{kt}]e^{\mathbf{\gamma} \cdot \mathbf{z}_t} \tag{2}$$

where $\lambda_1 > 0, \lambda_2 > 0, k > 0$. According to the principle of reliability analysis, the reliability function is estimated as:

$$R(t, \mathbf{z}_t) = \exp[-\int_0^t h(t, \mathbf{z}_t)dt] = e^{-[\lambda_1\sqrt{t}+\lambda_2\sqrt{t}e^{kt}]e^{\mathbf{\gamma} \cdot \mathbf{z}_t}} \tag{3}$$

The maximum likelihood method is commonly applied to estimate the unknown parameters of the RWPHM. In practice, a mechanical system will sometimes run to failure, and at other times the system will be repaired prior to failure. Therefore, the lifetime data usually contains the failure times and the suspension times. To deal with both types of data, a likelihood function is defined as:

$$L = \prod_{i=1}^{n} f(t_i, \mathbf{z}_{t_i}) \prod_{j=1}^{m} R\left(t_j, \mathbf{z}_{t_j}\right) = \prod_{i=1}^{n} (\{[\frac{1}{2\sqrt{t_i}}[\lambda_1 + \lambda_2(1 + 2kt_i)e^{kt_i}]]e^{\mathbf{\gamma} \cdot \mathbf{z}_{t_i}}\}$$

$$\times \{e^{-\left[\lambda_1\sqrt{t_i}+\lambda_2\sqrt{t_i}e^{kt_i}\right]e^{\mathbf{\gamma} \cdot \mathbf{z}_{t_i}}}\}) \prod_{j=1}^{m} \exp[-\left[\lambda_1\sqrt{t_j} + \lambda_2\sqrt{t_j}e^{kt_j}\right]\exp(\mathbf{\gamma} \cdot \mathbf{z}_{t_j})] \tag{4}$$

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The PHM with the Weibull distribution baseline function is called the Weibull proportional hazard model (WPHM). Then, the hazard function of the WPHM is defined as:

$$h(t) = \alpha\beta t^{\beta-1} \exp(\boldsymbol{\gamma} \cdot \mathbf{z}_{t_j}) \tag{5}$$

3- Failure prediction based on RWPHM

For a working system, suppose the monitoring data at time t is $\mathbf{z}_t$. The system reliability function, $R(t, \mathbf{z}_t)$, can be obtained according to Eq. . If the determined value for falls below the reliability threshold R_0, the system is expected to fail. The failure time is predicted as follows:

$$T(R_0) = \inf\{t, R(t, \mathbf{z}_t) = R_0\} \tag{6}$$

A system failure may occur in less than one second. So, every second one or more value of the variables monitoring may be stored. During the running system, provides a large amount of data. The data are instantly analyzed to predict when the system is failed.

4- Case study

A high-pressure water descaling pump, was selected for a case study. It is equipped with an 11-level centrifugal pump to provide high pressure and large water flow rates for removing oxide scale on stainless steel surface. Due to working continuously in a high-pressure environment, the descaling pump frequently breaks down from two failure modes: sealing ring wear and thrust bearing damage. To ensure safe operation, the outlet pressure, input-end vibration and thrust bearing temperature are regularly monitored. The outlet pressure, input-end vibration and thrust bearing temperature are denoted as z_1, z_2 and z_3 respectively. For these monitoring variables, z_2 and z_3 increase as failures arise, but z_1 decreases when the descaling pump malfunctions. The monitoring data from the beginning of operation to failure is shown in Figure 1.
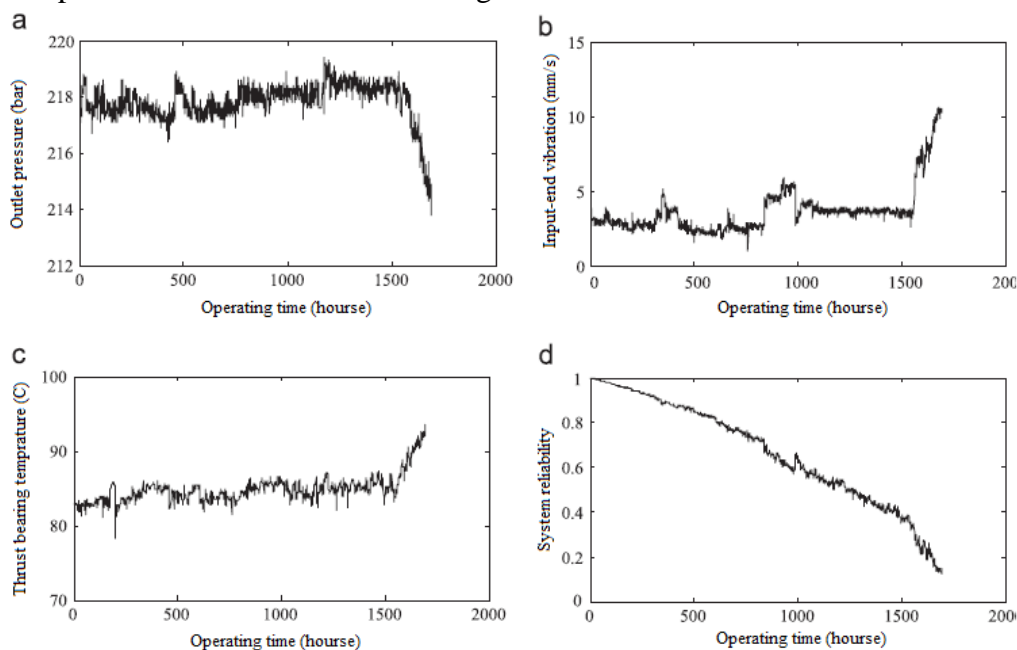

Figure 1: Monitoring data and system reliability of a failure process: (a) Outlet pressure; (b) Input-end vibration; (c) Thrust bearing temperature; and (d) System reliability.

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Table 1: ML estimates of parameters of the WPHM and RWPHM and goodness of fit statistics

	α	B		γ_1	γ_2	γ_3	Loglik
WPHM	0.005848	1.347713		-0.00909	0.105864	-0.03795	-113.46
	λ_1	λ_2	k	γ_1	γ_2	γ_3	
RWPHM	0.096432	0.879081	0.008412	-0.06696	0.727911	-0.07449	-103.94

Table 1, gives ML estimates of parameters of the WPHM and RWPHM and goodness of fit statistics. We find that the RWPHM distribution provides a better fit than the WPHM. It has the largest Log likelihood. Figure 2 show failure time predictions for the descaling pump using the RWPHM model for four pump with reliability threshold $R_0 = 0.2$.
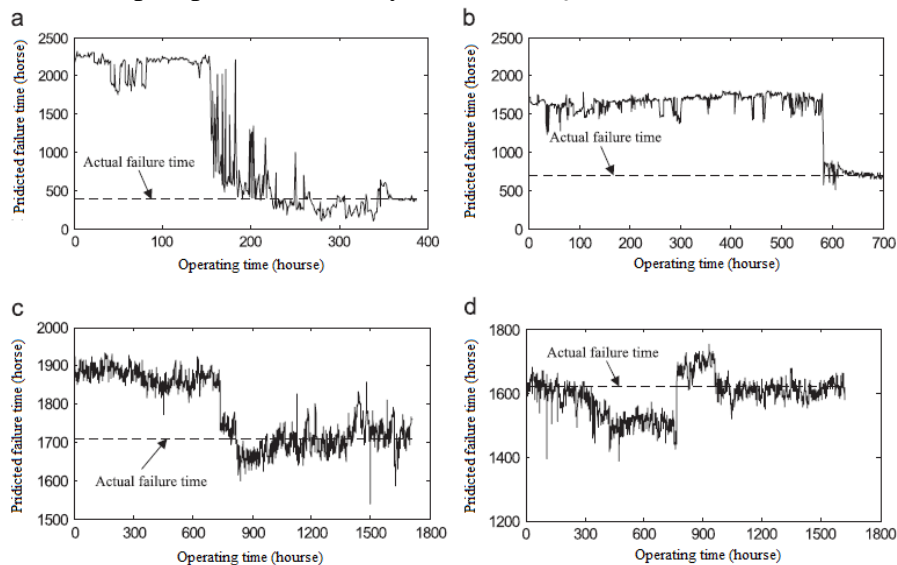

Figure 2: Failure predictions for the descaling pump using the RWPHM for four pump

5- Conclusion

In this paper, a mixture Weibull proportional hazard model is proposed to predict the failure time of a mechanical system with multiple failure modes. The model parameters of the RWPHM are estimated using the lifetime and monitoring data of all failure modes. Then, the monitoring data are input into the MWPHM to estimate the system reliability and predict system failure time. The experimental results demonstrate that our method can provide satisfactory failure distribution estimation and lifetime prediction.

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Pricing Financial Options Embedded in Insurance Contracts

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Abstract

Variable Annuities represent a major part of the present annuity market, which has undergone enormous change in the last two to three decades. There are optional riders provided by insurance companies in variable annuities. Two main types of Variable Annuities' guarantees can be distinguished: the guarantees in case of death (GMDB) and the guarantees in case of survival (GMAB, GMIB, GMWB and GLWB).

The report is composed of three parts. First, each GMxB guarantee will be described with some of its variants. We introduce the fundamental assumptions of option theory and Black- Scholes and Merton model. We apply option theory to the embedded options of financial guarantees in insurance products. Later, we employ a special approach which is derived from Yan-Liu (2010) to price GMWBs. Under the constant static withdrawal assumption, the approach is to decompose the GMWB and the variable annuity into a call option and an annuity certain. We will present some basic results for each of GMWB and GMMB guarantees, illustrated by numerical examples. Once our main pricing framework is presented we will profit from it to study the impact of interest rate, market volatility, mortality and longevity risk. Overall, we claim that the proposed model provides further useful information about pricing insurance products with regard to existing economic conditions in Iran. Using this model, we are able to provide new innovative products which can be very critical and useful.

Keywords

Variable Annuities, GMWB, European Options, Black-Scholes and Merton Model.

1- The Black–Scholes–Merton option pricing formula

Under the Black–Scholes–Merton model assumptions, we have the following important result:

- There is a unique risk neutral distribution, or Q-measure, for the stock price process, under which the stock price process$\{S_t\}_{t\geq0}$, is a lognormal process with parameters $r - \sigma^2/2$ and σ^2.

- For any European option on the stock, with payoff function $h(S_T)$ at maturity date T, the value of the option at time t ≤ T denoted v(t), can be found as the expected present value of the payoff under the risk neutral distribution (Q-measure)

$$v(t) = E_t^Q\big[e^{-r(T-t)}h(S_T)\big]$$

Now consider the particular case of a European call option with strike price K.

The option price at time t is $c(t)$, where

$$c(t) = E_t^Q\big[e^{-r(T-t)}(S_T - K)^+\big].$$

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which we usually write as

$$c(t) = S_t \Phi(d_1(t)) - Ke^{-r(T-t)} \Phi(d_2(t)),$$

where

$$d_1(t) = \frac{log(S_t/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}} \quad and \quad d_2(t) = d_1(t) - \sigma\sqrt{T - t}.$$

1-1- Pricing GMWB in Terms of a Call Option

The account value is denoted by A_t at time t. The initial value of the account balance is A_0. The GMWB guaranteed withdrawal rate is denoted by g, and the guaranteed maximum annual withdrawal amount is denoted by $w = g \times A_0$. The actual annual withdrawal amount is assumed to be equal to w and the withdrawals last for T years from time 0.

The present value of the annuity certain is equal to

$$wa_{\overline{N}|}^{(1/h)} = \sum_{i=1}^{N} whe^{-r_i h} = wh\frac{1-e^{-rT}}{e^{rh}-1}$$

All the cash flows from the annuity certain and the call option are financed by the initial premium. From the no-arbitrage pricing theory, the present value of the annuity certain, plus the expected present value of the option payoff, should be equal to the initial account value. We have the following equation:

$$wa_{\overline{N}|}^{(1/h)} + E_Q[e^{-rT}max(B_T, 0)] = A_0$$

The charge rate q affects the account value A_t, so the expected present value of the call option V_C is a function of q. The fair value of charge q^* must satisfy the following equation:

$$wa_{\overline{N}|}^{(1/h)} + V_C(q^*) - A_0 = 0$$

2- Pricing of GMMB

Throughout this section, we consider equity-linked contracts paid for by a single premium, P, which, after the deduction of any initial charges, is invested in the policyholder's fund. This fund, before allowing for the deduction of any management charges, earns returns following the underlying stock price process, $\{S_t\}_{t\geq0}$. We make all the assumptions in relating to the Black–Scholes–Merton framework. In particular, we assume the stock price process is a lognormal process with volatility σ per year, and also that there is a risk-free rate of interest, r per year, continuously compounded.

Consider the situation at the issue of the contract. If the policyholder does not survive n years, the GMMB does not apply at time n, and so the insurer does not need to fund the guarantee in this case. If the policyholder does survive n years, the GMMB does apply at time n. Thus, the expected amount (with respect to mortality and lapses) required by the insurer at the time of issue per contract issued is π(0), where

$$\pi(0) = \ _np_x E_0^Q[e^{-rn}(P - F_n)^+].$$

Then the price at the issue date of a GMMB, guaranteeing a return of at least the premium P, is

$$\pi(0) = P \ _np_x(e^{-rn} \Phi(-d_2(0)) - \varepsilon\Phi(-d_1(0))),$$

Where

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$$d_1(0) = \frac{log(\varepsilon) + (r + \sigma^2/2)n}{\sigma\sqrt{n}}$$
$$d_2(0) = d_1(0) - \sigma\sqrt{n}$$

3- Numerical Examples

3-1- Example I

Consider that an insurer offers a contract with a GMWB guarantee under which the policyholder invests a net single premium. At first, we suppose that the insurer keeps any percent of the premium to cover expenses, so the market value of the option is equal to the initial account value and we do not subtract any charge rate. The entire premium is invested in different ways and comprised a diverse portfolio, so we just investigate the net pricing of the product.

The investment value is assumed to follow a lognormal process. The risk free rate of interest per year is compounded continuously. The insurer guarantees the policyholder's ability to get the premium back by making periodic withdrawals regardless of the impact of poor market performance on the account value.

Now the following assumptions are made and Table 4.1 gives the results based on them:

Initial premium $A_0 = S_0 = 10,000,000$;

Risk-free interest rate $= 10\%$;

Volatility of fund return $= 20\%$;

Guaranteed annual withdrawal rate $= 5\%; 8\%; 10\%$;

Maturity $= 20; 15; 10$;

Length of time interval $h = 1; 1/4; 1/12$:

For more frequent withdrawals, we divide the maximum annual withdrawal amount into equal amounts according to the length of time interval and then deduct that amount from the account value at the end of each time interval.

GMWB rate	Maturity	The length of the time period					
		h=1		h=1/4		h=1/12	
g	T(yrs)	a	c(0)	a	c(0)	a	c(0)
5	20	4,110,760	8,659,315	4,269,507	8,659,315	4,305,335	8,659,315
8	15	5,909,389	7,801,915	6,137,595	7,801,915	6,189,099	7,801,915
10	10	6,010,412	6,409,443	6,242,520	6,409,443	6,294,904	6,409,443

Table 4.1: The price of the call option and annuity with market volatility of 20%

As we can see in table 4.1, the present value of annuity becomes a little higher as the frequency increases. This is because the present value of the guaranteed withdrawal benefit becomes larger as more values are withdrawn earlier. This is the same when the guaranteed withdrawal rate g rises and the maturity time decreases.

As the guaranteed withdrawal rate g rises, the GMWB benefit value increases, and therefore the charge rate q should goes up too. But as previous said, we suppose any expense loading, so by increasing the rate of withdrawal, you can see that the price of the call option decreases which

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shows that we should load more charge rate. In the real world of business, by assuming a suitable charge rate, we can obtain the fair price of the call option.

Now for obtaining the fee rate, as it was previously said, we suppose that the total initial expenses are a proportion q of the single premium. In the case of above assumptions with annual rate of withdrawal 5% and maturity of 20 years, Charge rate of approximately 29% of the initial premium at the beginning of the contract which is equal to2, 900,000 , is the fair charge rate obtained based on equation (20) and it is shown in the figure 4.1. The equivalent continuously compound rate, can be obtained too.
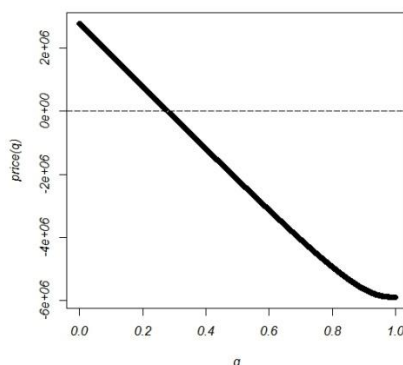
Fig. 4.1: The fair fee rate of the call option

3-2- Example II

Here we investigate an account that can be desirable for the policyholders if all the Iranian insurance companies decide to concentrate on designing and selling different kinds of life insurance products with considering variable interest rates, and as a result encounter with the risk of economic conditions that is great and non ignorable

Now assume that the insurance company presents an account which makes triple the net single premium of the policyholder at the first of the contract. Under the assumptions below, we have:

Initial premium $A_0 = 10,000,000$;

Value of the account $S_0 = 30,000,000$;

Risk-free interest rate $r = 8\%$;

Volatility of fund return $\sigma = 20\%$;

Guaranteed annual withdrawal rate $= 5\%$;

Maturity $= 10$;

Length of time interval $h = 1$:

The present value of the withdrawals as a certain annuity is 9,917,585 which is less than the net single premium. Also the price of the option is 17,113,537, which can be lower after loading a fair fee rate. By considering the charge rate in the model, approximately 40% of the initial premium is subtracted and the remaining will be invest in the market.

In this way, the company presents lower guaranteed interest rates than which is common nowadays under the 68 directive of the central insurance of Iran. Moreover the policyholder can benefit the return of investments.

   I.

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We want to price a GMMB and compare it with a pure endowment. We assume a life aged 32, who invests 10,000,000 for buying a variable annuity with GMMB which guarantees that payout at maturity, 20 years after the issue date of the contract, will be at least equal to the single premium.

After a deduction of 5% for initial expenses, the premium is invested. An annual management charge of 2% is deducted from the fund at the start of every year except the first. The risk free rate of interest is 10% per year, continuously compounded, and market volatility is 20% per year.

The cost of GMMB, by assuming no lapses, is 35,700 which is very lower than the price of a pure endowment that costs 1,379,780.

As it is shown, under a variable annuity with a GMMB, the policyholder can benefit from the interest rates of the market besides guarantee of not discarding initial premium at the end of contract's duration. This kind of insurance product is much cheaper than a pure endowment, and can be desirable for the policyholders.

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Some New Ordering Relations for Policy Limits and Deductibles with Homogeneous and Non-Homogeneous Risks

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Abstract

Policy limits and deductibles are two common insurance coverage that frequently used by the insurance company. In the policy limit coverage, the company indemnifies the policyholder's loss up to pre-specified value such as l, known as the limit amount and if the loss exceeds l, the remaining loss will be indemnified by the policyholder. In policy deductible coverage, the policyholder is self insured up to pre-specified value such as d, known as deductible amount and if the loss exceeds d, the remaining loss will be covered by the company.

Let $X_1,...,X_n$ be n risks faced by the policyholder. In policy limit coverage with limit amount l, the policyholder should divide l in to n non-negative values $l_1,...,l_n$ such that each l_i is a limit value, taking care of the risk X_i and $\sum_{i=1}^{n} l_i = l$. Parallelly, in the policy deductible coverage with deductible amount d, the policyholder should divide d in to n non-negative values $d_1,...,d_n$ such that each d_i is a deductible value, taking care of the risk X_i and $\sum_{i=1}^{n} d_i = d$. One main concern, considered in the literature, is how to find an allocation of limits or deductibles that maximizes the expectation of policyholder's utility or minimizes the remaining risk. Most of the previous works that had been done in this area only focused on the independent or comonotonic risks. In order to extend the previous results, we consider the dependent risks. We assume that the faced risks are exchangeable, arrangement increasing or arrangement decreasing and obtain the best and the worst allocation in each case, for both policy limits and deductibles.

Keywords

Arrangement increasing function, exchangeable risks, log-concave functions, majorization, stochastic ordering, risk theory, utility function.

Streaming Big Data Processing Using Improved Harmony Search

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Abstract

Streaming big data concerns large-volume, growing and complex data sets. Most attention on the data stream classification paid on non-evolutionary methods. In this paper, we introduce new incremental learning algorithms based on harmony search. We first propose a new classification algorithm for the classification of batch data called harmony-based classifier and then give its incremental version for classification of data streams called incremental harmony-based classifier. Finally, we improve it to reduce its computational overhead in absence of drifts and increase its robustness in presence of noise. This improved version is called improved incremental harmony-based classifier. The proposed methods are evaluated on some real world data sets. Experimental results show that the proposed incremental methods can effectively address the issues usually encountered in the streaming big data environments.

Keywords

Big Data, Classification, Harmony Search, Concept Drift.

1- Introduction

Big data has drawn huge attention in the recent years [1,2]. Examples of such applications include financial data analysis, large sets of web pages requested by HTTP calls, credit card transactions, and telephone call records, to mention a few. Such data has some distinguishing features such as huge volume, dynamic changes, and demanding fast response time. Because of these unique features, data mining algorithms for batch data are not suitable for big data mining. Classification is an important optimization task in data mining and has also many applications in big data as well. One of the main challenges in big data' classification is the inability to store and then use of all training data resulted from huge volume of such data.

In this paper, we give focus on stream processing of big data. Evolutionary algorithms are stochastic methods and have been applied successfully to a wide range of optimization problems. Harmony search (HS) is a new meta-heuristic algorithm that imitates the improvisation process of playing musical instruments and has been successfully applied in many optimization problems [3]. In this paper, we first propose a new classifier based on harmony search called harmony-based classifier (HC). In next, an incremental version of HC, called incremental harmony-based classifier (IHC), is

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proposed for classification of big data. This classifier combines harmony search and StreamMiner [4]. IHC tries to handle the problems of large amounts of data and changes in the distribution of data in big data. Finally an improvement of IHC is proposed that also detects and handles concept drifts. This improved version is called improved incremental harmony-based classifier (IIHC). The IIHC has the advantages of StreamMiner and tries to remove its limitations. The experimental results also show that IIHC can be used successfully for incremental classification as compared to other state-of-the-art methods.

2- The Proposed Algorithms

In this section, HC, IHC and IIHC are explained in the following, respectively.

2-1 The Harmony-based Classifier

In HS, the candidate solutions are improved iteration by iteration like as a musical harmony is improved gradually. For more details about harmony search, please refer to [5]. In this paper, we use the harmony search algorithm to optimize different parameters of classifier to apply it for big data classification. In HC, each harmony (candidate solution) is a rule based classifier in which the number of rules is a parameter specified by the user. A typical classification rule produced by HC has the following form:

$$\text{IF } L_1 \leq A_1 \leq U_1 \text{ and...and } L_n \leq A_n \leq U_n \text{Then class label is } C_i \text{ (for } 1 \leq i \leq m),$$

where $C = \{C_1, ..., C_m\}$ is a set of class labels, $A = \{A_i, ..., A_n\}$ is a set of attributes. The condition part of each rule consists a set of triples (F_i, L_i, U_i), where F_i is a flag showing the presence of the ith condition, L_i ($LB_i \leq L_i$) and U_i ($UB_i \geq U_i$) are the lower and upper bounds for the ith attribute in the rule, respectively. Each rule is evaluated using a fitness function, which is its accuracy. The classifier rules are grouped and ranked in such a way that rules with the same class label are included in one group. In the ranking algorithm, groups are ranked as the first group has fewest false positives on training samples; the second group has fewest false positives on remaining training samples and so on. At the end, the class which contains the most training samples and is not covered by any rule is selected as default class. In order to calculate the accuracy, the class label of the first group satisfying the test sample is predicted as the result. Whenever the sample is not satisfied by each group, default class is returned. HC is operated as follows: (1) Initializing the problem and harmony parameters: The triple (F_i, L_i, U_i) and pair (F_i, V_i) (for $1 \leq i \leq n$) initialized in the following way, are one harmony variable. Variables L_i and $U_i$are chosen randomly in the interval $[LB_i, UB_i]$, V_i is chosen randomly in the possible values of ith attribute and F_i is randomly selected from set $\{0,1\}$. (2) Improvising a new harmony: The value of each harmony variable is selected from harmony memory with probability HMCR and randomly with probability (1-HMCR). The selected value is adjusted with probability PAR; each three values in a harmony variable (F_i, L_i, U_i) can be increased or decreased by a user defined bandwidth. The fitness of the new harmony is its accuracy. (3) Updating the harmony memory: If the accuracy of the new classifier is better than the worst classifier in harmony memory, the worst classifier is replaced by new one. (4) Checking the stopping criterion: Steps 2-4 are done until NI improvision process has been done or the best classifier has the accuracy of 100 percent.

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The built classification model is used to predict the class label of the new arriving data; if the new data satisfies the conditions of at least one rule in the first group mentioned above, the class label of this group is predicted as the result. Otherwise, other groups are explored respectively and the class label of the first group that satisfies the new data is given as the result. If no groups satisfy the new data, the default class is given as the result.

2-2 The Incremental Harmony-based Classifier

HC is a batch classifier and can be used when all training data are available when the training begins. In some applications such as classification of streaming big data, all training data are not available at the same time. In such applications, the incremental learning must be used. In this section, we introduce an incremental version of HC, IHC, which is used for streaming big data classification. IHC is based on the combination of HC and StreamMiner framework in which three classifiers $FN_i(x), FN_i^+(x)$, and $FO_{i-1}^+(x)$) are trained with the same training algorithm and different training data (O: train on Old data, N: train on New data). The selection of suitable training data is very important when there is huge data. Some algorithms discard the old data and learn from the newly arriving data only, while some other algorithms update existing model learned on the old data. StreamMiner is an algorithm that selects the training data systematically. These classifiers are trained using different initial population and the accuracy is calculated using test data. For more details about StreamMiner, please refer to [4]. Hence, we only explain these two steps. In IHC, StreamMiner steps are adapted by using HC as follows:

1- Train a new classifier $FN_i(x)$: Generate the initial population randomly and then evaluate it on the new data block.

2- Find s_{i-1}: Find s_{i-1} using $s_{i-1} = \{\forall(x, y) \in D_{i-1} \,|\, (FN_i(x) = y) \text{and} (FO_{i-1}(x) = y))\}$.

3- Train classifier$FN_i^+(x)$: Generate the first half of initial population randomly and its second half with $FN_i(x)$ learned from S_i.

4- Train classifier $FO_{i-1}^+(x)$ from $D_{i-1} \cup S_i$: Generate the first half of initial population randomly and the second half of it with $FO_i(x)$ learned from D_{i-1}.

5- Compare the accuracy of $FN_i(x), FN_i^+(x), FO_{i-1}^+(x)$ and $FO_{i-1}(x)$: Compare the accuracy of these classifiers using n-fold cross validation by considering the assumption that when some examples are excluded from training set, learned rules remain the same and only the ranking of rules and default class are changed. So we only re-rank rules on this new data, instead of retraining it. Average accuracy of each classifier under n-fold cross validation is then calculated and the classifier with the best accuracy is chosen as the final classifier.

Let c be the maximum size of training data (maximum size of stored data blocks). In order to reduce the computational overhead of StreamMiner in dealing with stable concepts, hypothesis test is used to detect concept drift. If concept drift is detected, then steps 2 through 5 of IHC algorithm are repeated; otherwise, it is not necessary to train three classifiers $FN_i(x), FN_i^+(x)$ and $FO_{i-1}^+(x)$ and the most recent classifier $FO_{i-1}(x)$, is used for predicting class label of test data. IHC is based on StreamMiner algorithm and so has its drawbacks: computational overhead of learning stable concepts, computational overhead of learning recurring concepts, and computational overhead of

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learning noisy data. In order to reduce the computational overhead in presence of noisy data and recurring concepts, we introduce an extra memory called historical memory (HSM). The IHC which uses hypothesis test and HSM is called IIHC. HSM is a memory that can be viewed as a $(\frac{HMS}{2} \times$ NVAR) matrix. At step t, HSM is filled with the best classifier in the last $\frac{HMS}{2}$ time steps. When initializing harmony memory for training $FN_i(x)$, one half of HM is filled with the HSM values if exists. The effects of this initialization on occurring stable and noisy data are: 1) after arriving a new data block with the concept that occurs in $\frac{HMS}{2}$ last time steps, the learned model exists in HSM and is used in the initialization of HM, so it is improved instead of being learned, 2) if the new data block is noisy, algorithm interpret it as a concept drift and learns it. It also saves the current model in HSM. After arrival of the second new data block without changing concept and noise, the current model in HSM is used in the initialization of HM, so it is improved instead of being learned. Based on the discussion given in this section, IIHC have the following features: 1) when old concept recurs, IIHC immediately relearns (recalls) it by updating the corresponding classifier that exists in harmony memory by initializing HM from HSM. 2) IIHC automatically detects concept drift; hence it only updates the existing model when concept drift is detected. 3) IIHC is robust for noisy data by saving the best last results in the harmony memory. 4) IIHC is flexible in response time by controlling NI in harmony algorithm. 5) IIHC supports discrete and continuous attributes. 6) IIHC has systematic selection of training data by using StreamMiner.

3- Experimental Results

The purpose of this experiment is to compare the error of IIHC classifier with some related big data classifiers. All of these classifiers except for CVFDT are ensemble based classifiers and logically are comparable with the proposed method. We evaluated IIHC on two datasets: Forest CoverType and Yeast [6]. The specifications of parameters for each data set are given in Table 1 and the classification errors of these classifiers are given in Table 2.

Table 1: Parameter setting of IIHC for comparing it with related big data classifiers

Data Set	HMS	HMCR	PAR	# of Rules	NI
Forest CoverType	5	0.9	0.9	7	50
Yeast Database	50	**0.99**	0.9	20	500

Table 2: Classification error of IIHC and some other classifiers

Data Set	IIHC		OVA	CVFDT	SEA	UFFT	WCE
	Block size 1000	Block size 500					
Forest CoverType	28.72	**27.97**	34.12	36.97	39.72	36.63	35.79
Yeast Database	51.90	52.70	52.77	**46.94**	67.97	55.93	50.17

For the above experiment, the block size of IIHC is set to 500 and 1000. The block size of WCE is set to 1000 and block size of SEA is set to 500. The results show that IIHC outperforms all other classifiers on Forest CoverType and its error is less than OVA, SEA and UFFT on Yeast dataset.

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4- Conclusion

In this paper, we have shown potential power of harmony search for the classification of streaming big data. This paper proposed one classification algorithm using harmony search algorithm called harmony-based classifier, then presented an incremental approach to classifying big data called incremental harmony-based classifier and finally improved it to improved incremental harmony-based classifier. Experimental results show that improved incremental harmony-based classifier deals with streaming big data classification problem appropriately and it is comparable with non-evolutionary big data classifiers in term of accuracy. It also maintains an accurate and up-to-date classifier and can outperform some of popular existing streaming big data classifiers.

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Some Distributions in Terms of Fractional Polynomials

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Abstract

In this article, we try to find how fractional polynomials can be used to solve the problems in the probability distribution theory. In a rather straightforward manner, we develop the well-known formulas for the probability density function and cumulative distribution function of some distributions such as gamma and beta distributions in terms of the fractional Bell polynomials and fractional Laguerre polynomials, respectively.

Keywords

Fractional calculus, Laguerre polynomial, Bell polynomial.

1- Introduction

Bell and Laguerre polynomials are classical mathematical tolls for representation the nth derivatives of a composite functions. In [2], the probability function and the moments of the generalized random variable Z are expressed in terms of the Bell polynomials (cf. Bell (1927), Riordan (1968)). Moreover, definitions for these polynomials of arbitrary order (fractional order) in complex plan were defined in [3], which are suitable for representing the probability density function (p.d.f.) and cumulative distribution function (c.d.f.) of some distributions such as beta and gamma distributions. In this article, we present p.d.f. and c.d.f. of gamma distribution with shape parameter $\beta = 1$, in terms of fractional Bell polynomials and p.d.f. and also c.d.f. of beta distribution with scale parameter $\beta = 1$, in terms of fractional Laguerre polynomials. First, we quote the definitions from [1], [5] and [6].

Definition 1. The Bell polynomials take the form

$$B_n(x) = e^{-x} D^n e^x = \sum_{k=1}^{n} B_{n,k}, \quad n \in \mathbb{N}_0 \tag{1}$$

where D^n means that n^{th} derivative and

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$$B_{n,k} = \sum_{|m|=k, \|m\|=n} \frac{n!}{m!} \left(\frac{x_1}{1!}\right)^{m_1} \cdots \left(\frac{x_n}{1!}\right)^{m_n} \qquad (2)$$

Such that $|m| = m_1 + \cdots + m_n$, $\quad m_1 \geq 0, \ldots, m_n \geq 0$, $\quad m! = m_1! \ldots m_n!$ and $B_0 = 1$.
Also, the Laguerre polynomials take the form

$$L_n(x) = e^{-x} D^n e^{-x} x^n, \quad L_0 = 1, \quad n \in \mathbb{N}_0$$

The definitions of fractional operators are as following.

Definition 2. Let $f(x)$ be a function defined on the interval $[a, b]$, and q be a positive real number, then the right Riemann-Liouville fractional integral is defined as,

$$_aI_x^q f(x) = \frac{1}{\Gamma(q)} \int_a^x (x-t)^{q-1} f(t) dt, \qquad -\infty < a < x < \infty \qquad (2)$$

and the right Riemann–Liouville fractional derivative is

$$_aD_x^q f(x) = \left(\frac{d}{dx}\right)^n \left(_aI_x^{n-q} f(x)\right)$$

$$= \frac{1}{\Gamma(n-q)} \left(\frac{d}{dx}\right)^n \int_a^x \frac{f(t)}{(x-t)^{q-n+1}} dt, \qquad n = [q]+1, \qquad x > a. \qquad (3)$$

and we denote $_0I_x^q$ only with I_x^q.

Suppose that the function $f(z)$ is analytical in simply-connected region of the complex z-plane ($\mathbb{C}$) containing the origin and the multiplicity of $(z-t)^{\alpha-1}$ is removed by requiring $log(z-t)$ to be real when$(z-t) > 0$, then the expressions (2) and (3) in Definition 2, are valid, when $a = 0$. By using the operators, we have definition of fractional polynomials in complex z-plane. Also, for such functions, $I_z^\alpha f(z) = f(z) \times \frac{z^{\alpha-1}}{\Gamma(\alpha)}$, for $z > 0$ and 0 for ≤ 0.

Definition 3. Let $q \in [n-1, n)$ and $n = 1, 2, \ldots$. The fractional Bell polynomials of order q and $(-q)$ are

$$B_q(x) = e^{-x} D_x^q e^x,$$

and

$$B_{-q}(x) = e^{-x} I_x^q e^x,$$

respectively and the fractional Laguerre polynomials of order q and $(-q)$ are

$$L_q(x) = e^x D_x^q e^{-x} x^m,$$

and

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$L_{-q}(x) = e^x I_x^q e^{-x} x^m, \qquad m \in \mathbb{N}_0,$

respectively.

Lemma 1. Suppose that f be a continuous function and $\in [0,1)$, then

$$DI_x^q f(x) = \frac{x^{q-1}}{\Gamma(q)} f(0) + I_x^q Df(x); \quad D = \frac{d}{dx}.$$

(4)

2- Methods

2.1. Gamma Distribution in Terms of Fractional Bell Polynomials

Theorem 1. Suppose that $X \sim \text{Gamma}(\alpha, 1)$ and $0 < \alpha < 1$, then the c.d.f. of X is equal to

$F_X(x) = B_{-\alpha}(x),$

and the p.d.f. of X is equal to

$f_X(x) = B_{1-\alpha}(x) - B_{-\alpha}(x).$

Proof: We have,

$DB_{-\alpha}(x) = D[e^{-x} {}_0I_x^\alpha(e^x)] = e^{-x} D[{}_0I_x^\alpha(e^x)] - e^{-x} {}_0I_x^\alpha(e^x)$

$$= e^{-x} {}_0I_x^{\alpha-1}(e^x) - e^{-x} {}_0I_x^\alpha(e^x) = B_{1-\alpha}(x) - B_{-\alpha}(x),$$

On the other hand, by using Lemma 1, we obtain

$DB_{-\alpha}(x) = e^{-x} D[{}_0I_x^\alpha(e^x)] - e^{-x} {}_0I_x^\alpha(e^x)$

$$= e^{-x}\left[\frac{x^{\alpha-1}}{\Gamma(\alpha)} + {}_0I_x^\alpha(e^x)\right] - e^{-x} {}_0I_x^\alpha(e^x) = \frac{x^{\alpha-1}}{\Gamma(\alpha)} e^{-x}.$$

Obviously, it is equal to the gamma density function.

∎

Example1. Let $X \sim \text{Gamma}\left(\frac{1}{2}, 1\right)$. By considering recent result, we have

$F_X(x) = e^{-x} {}_0I_x^{0.5}(e^x)$

then

$f_X(x; 0.5; 1) = \left(\frac{d}{dx}\right)[e^{-x} {}_0I_x^{0.5}(e^x)] = -e^{-x} \cdot {}_0I_x^{0.5}(e^x) + \left(\frac{d}{dx}\right)[{}_0I_x^{0.5}(e^x)] \cdot e^{-x},$

so, we get,

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$$f_X(x; 0.5; 1) = -e^{-x} \cdot {}_0I_x^{0.5}(e^x) + {}_0D_x^{0.5}(e^x) \cdot e^{-x} = B_{0.5}(x) - B_{-0.5}(x).$$

2.2. Beta Distribution in Terms of Fractional Laguerre Polynomials

Theorem 2. Suppose that $X \sim \text{Beta}(\propto, 1)$ and $\alpha = m + \gamma - 1$ such that, $m \in \mathbb{N}$ and $0 < \gamma < 1$, then the c.d.f. of X is equal to

$$F_X(x) = c_{m,\gamma} L_{-\gamma}(x), \tag{5}$$

and the p.d.f. of X is equal to

$$f_X(x) = c_{m,\gamma}\Big(L_{1-\gamma}(x) - L_{-\gamma}(x)\Big),$$

which the coefficient $c_{m,\gamma}$ is equal to $\Gamma(\gamma)\left(1 - \frac{1-\gamma}{m}\right)$.

Proof: By derivation of equation (5), we have

$$Dc_{m,\gamma}L_{-\gamma}(x) = c_{m,\gamma}D\big[e^x {}_0I_x^{\gamma}(e^{-x}x^m)\big] = c_{m,\gamma}[e^x D\big[{}_0I_x^{\gamma}(e^{-x}x^m)\big] + e^x {}_0I_x^{\gamma}(e^{-x}x^m)]$$

$$= c_{m,\gamma}\big[e^x {}_0I_x^{\gamma-1}(e^{-x}x^m) - e^x {}_0I_x^{\gamma}(e^{-x}x^m)\big] = c_{m,\gamma}\Big(L_{1-\gamma}(x) - L_{-\gamma}(x)\Big),$$

On the other hand, by equation (4), for $z > 0$, we get

$$Dc_{m,\gamma}L_{-\gamma}(x) = c_{m,\gamma}[e^x D\big[{}_0I_x^{\gamma}(e^{-x}x^m)\big] + e^x {}_0I_x^{\gamma}(e^{-x}x^m)]$$

$$= e^x\left[(mx^{m-1}e^{-x} - e^{-x}x^m)\frac{x^{\gamma-1}}{\Gamma(\gamma)} + e^{-x}x^m\frac{x^{\gamma-1}}{\Gamma(\gamma)}\right] = (m + \gamma - 1)x^{m+\gamma-2}.$$

3- Conclusion

In this work, we discuss the use of fractional Bell and Laguerre polynomials in statistics. The Future works in the use of fractional polynomials in relation with statistics can be in the fields of asymptotic theory, software development and extensions to multivariate distributions.

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Some Improvements on Minimum Information Bivariate Copula to Analyze Liability Claims: the Indemnity Payment and the Allocated Loss Adjustment Expense

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Abstract

Minimum information copula provides parametric class of copula which have good levels of approximation, to any required degree of precision. It can be quantified, either in terms of expert judgment or data itself. The only technical assumptions should be made are that the multivariate copula density under study is continuous and non-zero, and no other assumptions are needed. The purpose of the presented paper is two folds: from one sides, doing minimum information copula is required to discretize or grid desired variables that has always be considered as a uniform grid. Here, we are going to investigate influence of different grids such as Chebyshev and Halton on the approximation accuracy. Since, for example, Chebyshev points allow more points in the tail or boundaries of our approximation which are very important especially in financial and actuarial applications. On the other, Lagrange multiplayers in minimum information copula should be determined which depends on expected values of the constraint in a nonlinear way. One of the possible solvers for this task would be FMINSEARCH - MATLAB's optimization routine, which implemented using the Nelder-Mead simplex method. In this paper, another root finding techniques FMINUNC is used which is extremely more useful/powerful as compared to FMINSEARCH in finding minimum information copula. The results gained show that our considered changes lead to better copula approximation. Finally, we apply our improved minimum information method to analyze general liability claims that randomly chosen from late settlement lags and were supplied by Insurance Services Office, Inc. For each claim, the indemnity payment (LOSS) and the allocated loss adjustment expense (ALAE) were recorded.

Keywords

Copula, grid, Optimization, General liability claim.

Spatial Point Processes in the Eye-Tracking Data Analysis

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Abstract

Stochastic Spatial Point Processes are useful but challenging to be developed and implemented tools in analyzing the events with random locations in the d-dimensional space where more than one attribute are assigned to each event. As an example, as the early diagnosis of some quickly growing neuro-developmental mental disorders, like autism, might be of high importance in cognitive science, we try to analyze the data taken from subjects exposed to these kinds of disorders at the Eye-tracking Laboratory at the Research Institute for Cognitive and Brain Sciences in Tehran. Hopefully, useful information about the hidden mechanism of some structural and behavioral functions can be obtained using this approach.

Keywords

Stochastic Spatial Point Process, Random location, Cognitive Science.

Strategies for Application of Directional Statistics in Probabilistic Machine Learning

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Abstract

Statistical models are playing an important role in probabilistic machine learning applications. In this paper, we focus on a special type of data that is normalized, so that its "direction" is more important than its magnitude. Directional Statistics is a branch of statistics that deals with this type of data. Specifically, we consider protein backbone angles, Text Documents, and fMRI time series that lie either on the surface of the unit hypersphere, torus or on the real projective plane. For such data, we briefly review common mathematical models prevalent in machine learning, while also describing some technical algorithms for model-based clustering algorithms and some other challenges.

Keywords

Directional Statistics, Dihedral Angles, Protein, Circular Distribution, Non-parametric models, Document Clustering, fMRI Data Analysis, Neuroimaging.

1- Introduction

Selecting the appropriate model for a problem in machine learning depends on the type of data being investigated. Data can be characterized as vectors in an Euclidean space , but some data may have constraints that change the data space. A simple example of this is when the data are normalized to a unit norm, and thereby put on the surface of the unit hypersphere S_{p-1}. Such data are better viewed as objects on a manifold instead of ; when building mathematical models for such data it is often advantageous to exploit the geometry of the manifold (here S_{p-1}). Sometimes the data are originally represented in a manifold, e.g. the dihedral angles of protein backbone are intrinsically in S_2. Hence, the appropriate model should be selected for this type of data as well. p p

In this paper, we will introduce some popular statistical models in directional statistics that are very useful in the application of probabilistic machine learning. One application of these models is in studying the protein structures using angular representations, and this has recently attracted much attention among structural biologists. The main challenge is how to efficiently model the continuous conformational space of the protein structures based on the differences and similarities between different Ramachandran plots. Despite the presence of statistical methods for modeling angular data of proteins, there is still a substantial need for more sophisticated and efficient statistical tools to model the large-scale circular datasets. To address this need, we have developed a Page 2

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Penalized Spline Collective Density Estimator (PSCDE) method that can be used as a model-based clustering approach for this type of data [1-2].

We also introduce Directional Statistics in group analysis of fMRI data. Standard fMRI experiments investigate the functional organization of the brain by contrasting the responses to two or more sets of stimuli or tasks that are hypothesized to be treated differently by the brain. An activation map is generated by statistically comparing the fMRI responses of each voxel to a distinct set of tasks or stimuli. The consistency of these activation maps across subjects is commonly evaluated by aligning the brain data across multiple subjects in a common anatomical space using spatial normalization. This type of normalization evolves the data to a complex form that classical clustering algorithms do not handle well. In this paper, we also discuss the recent challenges in this area of science.

The rest of the paper is organized in three different sections. In Section 2, we introduce some novel models that apply Directional Statistics to Biotechnology. In Section 3, we discuss the advantages of a directional view of normalized data in fMRI data analysis and its effects on the results of group analysis and finally in Section 4, we review the effect of Directional Statistics in the measure of document clustering.

2- Directional Statistics in Protein Backbone Angles

Since the seminal work of Ramachandran et al. [3], the two-dimensional plot of dihedral angles has been commonly used to determine accessible regions and validate new protein structures. The histogram is a rough non-parametric density estimation where the number of parameters is equal to the number of data points. Furthermore, because of the circular nature of protein angles, traditional parametric or non-parametric density estimation methods cannot be used for estimating Ramachandran distributions. In the last decade, novel parametric and non-parametric methods have been introduced to address this problem. The parametric methods propose to use directional distributions such as von Mises distribution or short Fourier series that are naturally designed for periodic data. On the other hand, the non-parametric techniques use kernel density estimates with periodic kernels, a Dirichlet process with boundary modification, or a mixture of directional distributions [1-2,4].

In some cases of learning the model of angular representation, the number of residues (data points) is too small, and that makes it challenging to obtain reliable bivariate densities using techniques that estimate each distribution separately. An intuitive solution to this problem is to borrow information from a group of plots that has some common features. To this end, Lennox et al. [5] proposed a hierarchical Dirichlet process technique based on bivariate von Mises distributions that can simultaneously model angle pairs at multiple sequence positions. This method is typically used for predicting highly variable loops and turn regions.. In another approach, Maadooliat et al. [1] and Najibi et al. [2] proposed a PSCDE to represent the log-densities based on some shared basis functions. This method showed significant improvements for loop modeling of hard cases in a benchmark dataset where existing methods do not work well [2]. Furthermore, the score coefficient Page 3 of this model can be imported into the clustering algorithms to identify the structure of the proteins [1].

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3- Directional Statistics in Neuroimaging

In the last twenty years, functional magnetic resonance imaging (fMRI) has become a popular tool for understanding the human brain [6-7]. Neuroscientists and physicians carry out fMRI experiments to precisely determine which parts of the brain is active for critical functions such as thinking, speech, motion, sensation, and/or attention. This is usually done to assess the effects of stroke, trauma or degenerative diseases (such as Alzheimer's Disease) on brain function, and to monitor the growth and the activity of brain tumor regions for pre-surgical planning, radiotherapy or other surgical treatments of the brain.

After designing an fMRI paradigm, running the experiment, and collecting the data, various analysis steps must be applied on the resulting data before answering the questions of neuroscientists and physicians about the brain activities corresponding to the stimuli. The goal of computer-based analysis is to automatically determine those parts of the brain that respond to the stimuli presented to the subjects.

In the conventional univariate approach of fMRI data analysis, the responses of the entire population of voxels are not considered as a whole; tests are performed separately on the single voxels to examine the significance of *a priori* hypothesized activations. In contrast, the modern models explain the selectivity profiles of all voxels by grouping them into a number of systems (clusters), each with a distinct and representative selectivity profile across the stimuli/tasks in the experiment. Once a small set of robust systems is discovered, we can map the location of the voxels that corresponds to each system to find out where they are in the brain, if they are spatially contiguous, and if they are in similar locations across subjects. In the directional approach, first we need to normalize different subjects into a common anatomical space. Since brain structure is highly variable across subjects, establishing accurate relationships among anatomical images of different subjects is intrinsically challenging [8].

4- Directional Statistics in Document Clustering

In classical information retrieval, the cosine similarity is a more effective measure of similarity for analyzing and clustering text documents than Just Euclidean distances. This clearly shows that normalizing the data vectors helps to remove the biases induced by the length of a document and to provide more robust results. In another view, the spherical k-means (spkmeans) algorithm [8] that runs k-means with cosine similarity for clustering unit norm vectors has been found to work well for text clustering and a variety of other data. Another widely used similarity measure is Pearson correlation which is exactly the cosine similarity between the two vectors. Moreover, in the document clustering, similarity measures such as cosine, Jaccard or Dice methods [9] are more effective than measures derived from Mahalanobis type distances. Hence, because of the intrinsic "directional" characteristics of this type of data, directional statistics can be better used compared to classical models [10]. Page 4

5- Conclusions

In this paper, we aimed to introduce a few advantage of the machine learning classification and clustering methods that used the directional statistics. Today, in many fields of science, statistics is

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being used as a valuable tool; however, the people of the other fields, such as a neuroscientist or a computer programmer, are only able to utilize the currently available methods, regardless of their robustness. This is a vital job of the statisticians to continually work on the development of more reliable and more accurate analysis methods, to help a great number of fields of science to achieve more accurate inferences about their findings. Directional statistics is one of this type of view that statistician and mathematicians needs to take in to account and showed their contribution.

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The ORSA Process in Action

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Abstract

The presentation will be on the ORSA (own risk and solvency assessment) process which is a very important part of Solvency II in Europe. In essence, the ORSA is about decision-making and planning within an insurance company taking risks into consideration. In Europe, all insurance companies need to do an ORSA and submit to local Financial Supervisory Authorities. Solvency II came into effect from January 1, 2016.

We shall include the concept of ORSA but more importantly – how can it be implemented and how is it being implemented with the insurance companies. Furthermore, we shall include:

•   What kind of data and information is needed to do ORSA and do you have the needed quality of data?

•   How should the board work with ORSA? They need to rely on actuarial information combined with underwriting, investment and other risk data and should be considered as a year around activity.

•   We shall include some examples of a part of an ORSA process and risk culture development.

As an important byproduct, we shall talk about how the actuaries can make use of their expertise of capital modeling, reserving, assets risks etc. The key to get the benefits of ORSA is people and data – and we will talk more about how to get the internal processes right.

The presentation is viewed from a company angle but we include some important aspects laid down in the framework of Solvency II.

Keywords

ORSA, Risk Management, Solvency II, capital requirement.

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The SASSC Algorithm: an Application to Insurance Data

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Abstract

A classification technique is used to distinguish or discriminate between two (or more) groups of individuals. Frequently if the number of groups is large, the algorithm could have some problem to build a model that reaches a satisfying accuracy level. In order to overcome this problem we propose an approach called Sequential Automatic Search of a Subset of Classifiers (SASSC) that splits a classification problem among C classes into K < C less complex two-classes sub-problems. SASSC focuses both on the interpretation aspects trying to improve the comprehension of the relationships among predictors, and on classification accuracy. The main contributions of SASSC concern the new criteria for the aggregation of classes and super-classes and the alternative criteria for the estimation of the response class for unseen (test-set) observation. An application of SASSC Algorithm to Insurance data is illustrated.

Keywords

SASSC, insurance, classification, super-classes.

Testing for More DMRL Order by Non-parametric Kernel Method

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Abstract

This paper presents test of equality of two unknown distributions against the alternative that one is more decreasing mean residual life (DMRL) than the other and propose a new non-parametric test based on kernel method and also the asymptotic normality of the proposed statistic is presented. Next, the empirical power of the proposed test is simulated for some specific families of distributions that are ordered with respect to more DMRL order.

Keywords

More DMRL order; Asymptotic normality; Kernel method; Power estimate; Eempirical processes.

1- Introduction

Let X and Y be two lifetime random variables with continuous distribution functions F and G and survival functions $\overline{F} = 1 - F$ and $\overline{G} = 1 - G$, respectively. Assume that X and Y have the finite means. For these two lifetime random variables, the residual life times at age t are denoted by

$X_t \overset{st}{=} [X - t \mid X > t]$ and $Y_t \overset{st}{=} [Y - t \mid Y > t]$, respectively. The functions $\mu_{X_t}(t) = \dfrac{\int_t^\infty \overline{F}(u)\,du}{\overline{F}(t)}$ and

$\mu_{Y_t}(t) = \dfrac{\int_t^\infty \overline{G}(u)\,du}{\overline{G}(t)}$ called mean residual life (MRL) functions of X and Y, respectively, which evaluate stochastic properties of X and Y over the area after t. For more details on MRL function the reader is referred to Shaked and Shanthikumar (2007). A random variable X is said to be decreasing mean residual life ($DMRL$) if $\mu_{X_t}(t)$ is decreasing in $t \geq 0$. The class of $DMRL$ distributions is very useful in reliability studies. Kochar and Wiens (1987) defined and studied the following univariate order based on the mean residual life function.

Definition 1. *The random variable X is said to be smaller than the random variable Y in more DMRL order (denoted by $X \leq_{DMRL} Y$) if $\dfrac{\mu_{X_t}(F^{-1}(u))}{\mu_{Y_t}(G^{-1}(u))}$ is decreasing in $u \in (0,1)$.*

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Noting that in this stochastic order, if $\overline{G}(t) = e^{-t}$ then $X \leq_{DMRL} Y$ is equivalent to that the random variable X is DMRL. It should be pointed out here that the more IFR ordering implies that the more DMRL ordering (see, Kochar and Wiens (1987)), recalled that, the random variable X is more IFR than the random variable Y and denoted by, $X \leq_{IFR} Y$, if $G^{-1}oF(t)$ is a convex function. Also, if the failure rates exist, then the more IFR property is equivalent to that $\frac{r_{X_t}(F^{-1}(u))}{r_{Y_t}(G^{-1}(u))}$ is increasing function in $u \in (0,1)$, where r_X and r_Y are the failure rate functions of X and Y, respectively. For studying the properties of the IFR ordering, please see Kochar and Wiens (1987). In this paper, we propose a new non-parametric procedure for testing the lifetime random variables X and Y have the same distribution functions against the alternative that the lifetime random variable X is smaller than Y in more DMRL order.

The paper is organized as follows: In section 2, we present the test statistic for testing H_0, that indicates X and Y have the same distributions versus H_1, that indicates $X \leq_{DMRL} Y$. In section 3, we estimate the proposed test in section 2 based on kernel function and establish the asymptotic distribution of the test statistic. Throughout this paper decreasing means ``non-increasing''.

2-Test statistic

Let X and Y be the life times of two systems with continuous distribution functions F and G respectively. In this section we develop a test statistic for testing the null hypothesis versus $H_0 : F \overset{D}{=} G$ the alternative hypothesis $H_1 : X \leq_{DMRL} Y$ based on two random samples arising from the distribution functions F and G, respectively. It is well known that the DMRL order is equivalent to that

$$\frac{\mu_{X_t}(F^{-1}(u))}{\mu_{Y_t}(G^{-1}(u))}$$

is decreasing in $u \in (0,1)$, that is equivalent to that

$$\frac{v_F(F^{-1}(u))}{v_G(G^{-1}(u))}, \quad (1)$$

is decreasing in $u \in (0,1)$, where $v_F(t) = \int_t^\infty \overline{F}(u)du$.

With taken derivative with respect to u in (1) for all $u \in (0,1)$, it can be shown that, (1) is equivalent to that

$$\psi_{F,G}(u) = g(G^{-1}(u))v_G(G^{-1}(u)) - f(F^{-1}(u))v_F(F^{-1}(u)) \geq 0.$$

Now, the measure of deviation from H_0 to H_1 is given in the following. Define

$$\Delta(F,G) = \int_0^1 \psi_{F,G}(u)du = \Lambda(G) - \Lambda(F),$$

where

$$\Lambda(F) = \int_0^1 f(F^{-1}(u))v_F(F^{-1}(u))du \text{ and } \Lambda(G) = \int_0^1 g(G^{-1}(u))v_G(G^{-1}(u))du$$

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We use $\Delta(F,G)$ to establish a test statistic to test the null hypothesis (H_0) against alternative hypothesis (H_1). Under H_0, $\Delta(F,G) = 0$ and under H_1, $\Delta(F,G) > 0$.

3-Empirical Test Statistic and Asymptotic Distribution of the Test Statistic

Let $X_1,...,X_n$ and $Y_1,...,Y_m$, be two random samples which taken from distribution functions F and G, respectively. We assume that X_i's and Y_i's are independent. Let F_n and G_m, denote the empirical functions formed by random samples X_i's and Y_i's, respectively. The kernel method is simple to implement and it is widely used to estimate the non-parametric functional forms and distribution functions, for a review please see Boente and Fraiman (1995). In the following, it is established the asymptotic distribution of $\Lambda(F_n)$.

Theorem1. *Let $na_n \to \infty$ and $na_n^4 \to \infty$, as $n \to \infty$ and also assume that the density function f has bounded first and second derivatives. Then*

$$\sqrt{n}\left(\frac{\Lambda(F_n) - \Lambda(F)}{\sigma_F^2}\right) \xrightarrow{D} N(0,1),$$

where

$$\sigma_F^2 = Var\left(X\int_0^X f(x)dF(x) - \int_0^X xf(x)dF(x) + 2\left[f(X)\int_X^\infty xdF(x) - Xf(X)\overline{F}(X)\right]\right).$$

Now, the limit distribution of $\Delta(F_n,G_m)$ can be established by applying Theorem 1.

Theorem2 *.Let a_n and b_m be tow sequences of positive real numbers such that $na_n \to \infty$ and $na_n^4 \to \infty$, as $n \to \infty$ and also $mb_m \to \infty$ and $mb_m^4 \to \infty$, as $m \to \infty$. Assume X and Y be two continuous non-negative random variables, with distribution functions F and G respectively, such that $\sigma_X^2, \sigma_Y^2 > 0$. Suppose that*

$$\sigma_{F,G}^2 = \frac{m\sigma_F^2 + n\sigma_G^2}{n+m}$$

then, it holds that

$$\sqrt{\frac{nm}{n+m}}\left(\frac{\Delta(F_n,G_m) - \Lambda(F,G)}{\sigma_{F,G}^2}\right) \xrightarrow{D} N(0,1),$$

if $\min(n,m) \to \infty$ *and for any* $w \in (0,\frac{1}{2})$, $(n,m) \in \left((n,m) \mid w \leq \frac{m}{n+m} \leq 1-w\right)$.

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Using Machine Learning to Estimate Some Anisotropy Indices, Application to Brownian Textures and Breast Images

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Abstract

In this paper, we analyze image textures with help of anisotropic fractional Brownian fields. We also use some anisotropy indices characterizing the anisotropy of these textures. Multi-oriented quadratic variations form the basis of mentioned indices. Anisotropy indices are invariant to some image transforms. Furthermore they can be estimated from the observed data. An application of these indices, combining with a measure of texture roughness, is in lesion detection in mammograms.

Keywords

Texture Analysis, Machine Learning, Anisotropy, Breast Cancer, Fractional Brownian Fields, Lesion Detection.

1- Introduction

Medical imaging techniques have widely improved and revolutionized health care delivery all around the world. Melding the medical imaging and the power of Mathematics and Statistics offers highly personalized and targeted means of diagnosis or prognosis generation and is fostering higher quality and efficiency in health care.

One important aspect of a medical image is its texture which delivers information about the spatial arrangement of intensities in that image and is essential for processing the image. From a study to another, the definition of texture varies, depending mainly on analysis approaches. In such approaches, texture features are usually derived from the estimation of model parameters.

Brownian textures refer to a large class of irregular and non-stationary image textures described by Gaussian random field models. Kolmogorov(1940), Mandelbrot and Van Ness (1968) defined Fractional Brownian field (FBf) as the unique Statistical Tests of Anisotropy for Fractional

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Brownian Textures centered Gaussian field, with stationary increments, isotropic and self-similar of an order H, changing in the range of (0,1). Elements of the textures are satisfied in Fractional Brownian Field having the so-called Hurst index, H, which is directly related to a degree of texture roughness.

However, according to Richard (2016), this index is not always sufficient to characterize the texture aspect. In another words, since it is orientation independent, it does not account for directional properties of textures. To explain this shortcoming, consider Figure 1 as an example with two different holder indices. If we consider this issue from a regularity viewpoint, textures of the two different rows can be distinguished while those of a same row cannot. Differences between textures of a same row are only due to variations of their directional properties.

We define some new indices in order to overcome this problem. By the way, there are some requirements for defining them:

1. Indices should be an intrinsic quantity: These quantities are invariant to some image transforms such as rotations, rescaling, or linear changes of intensities.
2. We should be able to estimate these indices from an observed image: This second requirement is probably rigid. In the sense that their related Hurst index should be efficiently estimated using quadratic variations.

The rest of this paper is organized as follows. In section 2, there is a brief overview of application of anisotropy indices in mammograms. In the last section, we share the conclusion and the results.
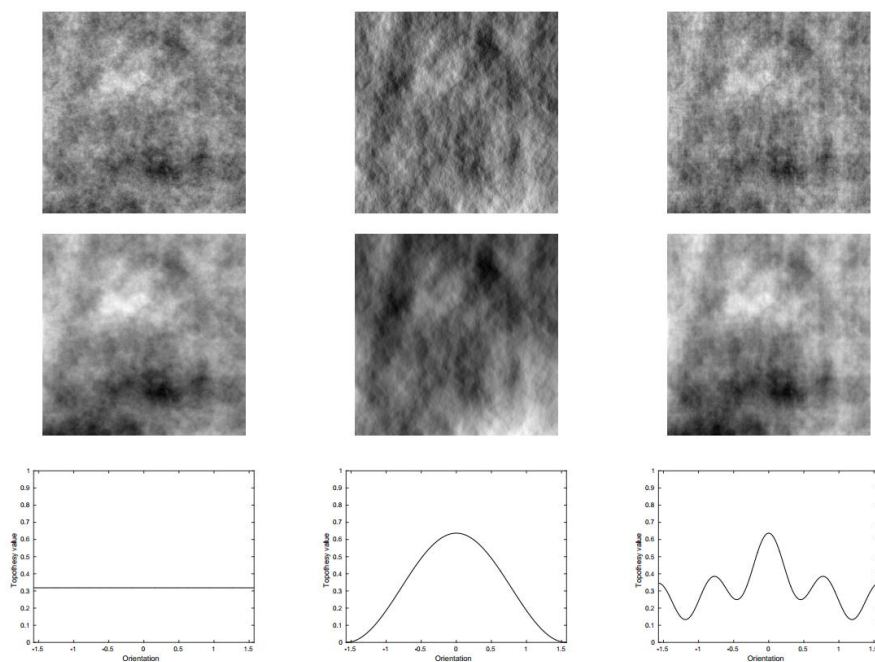

Figure1. In first row, three different ROIs with Holder indices of 0.3 are illustrated and in the second row, you can see the demonstration of three ROIs with Holder indices of 0.6. Each column has its own topothesy function which is represented in the bottom of that column.

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2- Application to Mammograms

Breast cancer is one of three first reasons of women mortality all around the world. The early detection of this disease, raises the possibility of remedy and survival rate. Richard (2016) claims that today, mammography acts as the most efficient imaging for early detection and it has become a contractual tool of imaging.

We applied our methodology on a set of data. ROIs of size 100*100 are extracted from the dataset. These data contains 92 pathological mammograms. We also included 358 random ROIs from normal cases. To build our method, we computed increments, resolved the trend by fitting a polynomial of order 1, and finally we computed the anisotropy indices H and A, where H stands for the Hurst index and A is the Anisotropy index respectively. Values of H and A are illustrated in Figure 2.
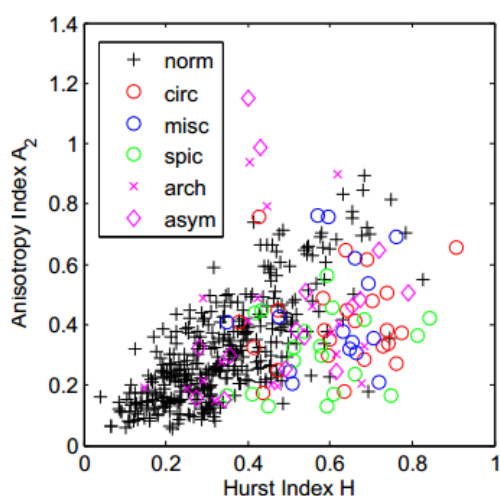
Figure2. Values of Hurst index and Anisotropy index of each ROI (for p=2).

On average, abnormal ROIs seems to be smoother and have higher values of H, which can be explained with the density of the mammogram in the regions with presence of malignancy (pathology). Furthermore, the Anisotropy of these ROIs are lower than the normal ones. This fact confirms the reflection of orientation of the tissues into the nipple, in other words, a normal breast mammogram texture would rather to be anisotropic.

3- Discussion

We applied some indices to characterize the anisotropy of an image texture. The calculations show that these indices are intrinsic and invariant to number of image transforms such as rotation and linearity. Results shows that the estimation mean square error is lower than 2% for selected indices. Combining anisotropy indices with the Hurst index, made us able to describe irregularity and anisotropy of mammogram textures at the same time. And this lead us to detect lesions in the regions of interests. The most important advantage of anisotropy indices is detecting lesions that were the most difficult to detect.

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Tail variance risk measure under univariate type III generalized Logistic distribution

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Abstract

During the last decades, the quantile-based downside risk measures have received much attention from practitioners and researchers. In this paper, we derive the explicit formulae of the tail vairance of portfolio under assumption of type III geberalized logistic distribution.

Keyword

Risk Measure, Gerelized Logistic Distribution.

1- Introduction and preliminaries

During the last decades, the quantile-based downside risk measures have received much attention from practitioners and researchers. One of risk measures is the Value at risk(VaR) that has been become a standard risk measure written into industry regulations due to its conceptual simplicity, ease of computation and ready applicability. Although, VaR has undoubtedly become a standard risk measure and has been written into industry regulation, VaR ignores the magnitude of extreme or rare losses. While insurance loss is often characterized as having a long right tail risk. The right-tail risk represents events with low frequency but large loss amounts. In addition, it has been shown in [2] that the problem of minimizing VaR of a portfolio can have multiple local minimizes. As response to these deficiencies of VaR, the notation of coherent risk measure was introduced in [3], which is an important breakthrough for a comprehensive theory in financial risk measurement. In fact, VaR is also severely criticized that it not a coherent measure of risk due to its lack of subadditivity. That is, VaR associated with a combination of two portfolios may larger than the sum of the VaRs of the individual portfolio (see [1, 11, 7])

Several types of measures of right-tail insurance risk have been proposed in the literature and used in practice. The tail conditional expectation (TCE) which measures the "average" along the right-tail, has gained popularity over the recent years both in insurance and finance.

Consider the random variable (r.v.) X representing the claims related to an insurance policy or portfolio over a given insurance period. The cumulative distribution function and the probability density function of X are denoted by $F_X(x)$ and $f_X(x)$, respectively. A premium principle transforms the r.v. X into a real number. It expresses the amount to be paid by the policyholder (or

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insurer) as compensation for the transfer of his risk to the insurer (or reinsurer). Now we recall some downside risks based on quantile such as value-at-risk (VaR), tail conditional expectation (TCE).

Definition 1.1 *The VaR and TCE of the portfolio at confidence level* $1-p$ *are defined as*

$$VaR(X;p) = inf\{x \mid P(X \geq x) \leq 1-p\} \equiv x_p,$$

$$TCE(X;p) = E[X \mid X > VaR(X;p)]$$
$$= x_p + E(X - x_p \mid X > x_p),$$

[4] observed that in many cases the TCE does not provide adequate information about the risks on the right tail. They illustrated their opinion by numerical examples. In particular, TCE is a conditional expectation and hence, does not includes information about deviation of the risk from its expectation in the upper tail. However, we note that these risk measures only describe the mean of tail loss beyond VaR, but pay no attention to the deviation of such tail loss. This is obviously inadequate for measuring tail risk, especially for the loss of extreme event with small probability but large loss amounts. In order to overcome this problem and to consider the variability of the tail loss and consequently better control the extreme risk, [12] proposed a tail conditional variance (TCV) defined as

$$TCV(X;p) = Var(X - \mu_X \mid X > x_p).$$

However the well-known tail conditional expectation risk measure provides a risk manager with information about the average of the tail of the loss distribution, [4] introduced the new risk measure that estimates the variability along such a tail. Therefore, they suggest use of the tail variance (TV) as a risk measure of the right-tail variability defined to be

$$TV(X;p) = Var(X \mid x > VaR(X;p))$$
$$= E[(X - TCE(X;p))^2 \mid X > VaR(X;p)] \tag{1.1}$$

[4, 10] derived formulae for the tail variance and the tail variance premium (TVP) of risks in a multivariate elliptical and log-elliptical setting. [5] investigated TCE and TVP for the family of symmetric generalized hyperbolic distribution. [8, 9],[6] in which the author derived the closed-form expression of VaR, ES tail variance of linear portfolio under generalized elliptical distribution, generalized Laplace distribution and their mixture stributions.

Now, in this paper, we derive the explicit formulae of the tail vairance of portfolio under assumption of type III geberalized logistic distribution.

2- Main results

Let X be a random variable with a p.d.f of univariate type III geberalized logistic (UGLD) distribution given by

$$f(x) = \frac{1}{B(\alpha,\alpha)} \frac{exp(-\alpha(x-\mu)/\sigma)}{(1 + exp(-(x-\mu)/\sigma))^{2\alpha}}, \ x,\mu \in P \ \ \alpha,\sigma \in P^+$$

where $B(.,.)$ is complete beta function and μ, σ and α are location, scale and shape parameters.

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Theorem 2.1 *Let* $X : UGLD(\mu,\sigma,\alpha)$. *Then under confidence level* $1-p$, *VaR and TCE are given by*

$$Var(X;p) = -\mu + q(p,\alpha)\sigma \quad TCE(x;p) = -\mu + K(p,\alpha)\sigma,$$

where $q(p,\alpha)$ is the positive solution of the following transcendental equation $\overline{F}_Z(q(p,\alpha)) = p$ and

$$K(p,\alpha) = \frac{1}{\alpha B(\alpha,\alpha)(1+exp(-\frac{1}{2}z_p^2))} \,_2F_1(\alpha,1-\alpha,\alpha+1,\frac{1}{1+exp(-\frac{1}{2}z_p^2)}),$$

where $_2F_1(.,.,.,.)$ is hypergeometric function and $Z = (x-\mu)/\sigma$ is the standared type III geberalized logistic random variable.

Theorem 2.2 *Suppose* $X : UGLD(\mu,\sigma,\alpha)$. *Then under conï¬• dence level* $1-p$, *the TV risk is given by*

$$TV(X;p) = \sigma^2\big(T(p,\alpha) - (K(p,\alpha)^2))\big),$$

where $T(p,\alpha) = \frac{1}{B^2(\alpha,\alpha)} \,_2F_1(\alpha,1-\alpha,\alpha+3,\frac{1}{1+exp(-\frac{1}{2}z_p^2)})$.

In compelete version of the paper, by using Matlab software, we give some numerical results when applying Theorem 2.1 and Theorem 2.2. For computing tail variance of the portfolio not only need to estimate the parameters μ and σ, but also need to evaluate some important parameters such as $q(p,\alpha), K(p,\alpha)$. It is a valuable note that the key in numerical computation is to obtain the value of $q(p,\alpha)$. The values of $K(p,\alpha)$ and $T(p,\alpha)$ can be computed from Theorem 2.1 with the help of numerical software.

Remark 2.3 *An application of this paper can be refered to the optimal portfolio selection which is an important issue in financial engineering. The primitive measure of risk, the variance of returns of a portfolio, is employed in the classic mean-variance model by Markowitz. After that many new risk measures were proposed and applied in portfolio optimization.*

The classic mean-variance criterion for portfolio selection does not pay any special attention to the variability of the tail loss beyond some critical level. For solvimg this problem, we can use tail variance premium defined by

$$TVP(X;p) = TCE(X;p) + \lambda TV(X;p)$$

where $0 < p < 1$ and $\lambda > 0$ is a constant dependent on the investors preference.

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Determining and Classifying the Factors Affecting Risk in Automobile Hull Insurance Using Fuzzy Delphi Method and Factor Analysis

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Abstract

Automobile hull insurance is a line of business which has attracted much attention due to the current lifestyles and the high rate of vehicle applications in the daily life transportations. These policies generated more than 13.7% of the total premium in automobile insurances in 1394 (2015) [1]. Since buying this policy for the vehicles is optional, and the rates in this line of business is competitive, a competitive atmosphere for attracting low risk drivers has been created for the insurance companies. However, most of the insurers still use comparative rates and pay no or less attention to the factors affecting risk in their premium calculations. Considering the importance of fair ratemaking in attracting and maintaining good risks and encouraging bad risks to repent or leave the portfolio, and also taking into account the shortcomings of the databases in insurance companies and the central insurance of Iran regarding the available policies in this field, this paper focuses on determining the risk factors affecting premium calculation in the automobile hull insurance from the viewpoint of the experts. In this regard, fuzzy Delphi method is utilized and the effective factors are reduced by their dimension and classified by SmartPLS.

Keywords

Automobile hull insurance, risk, fuzzy Delphi method, confirmatory factor analysis.

1- Introduction

Automobile insurance is an important line of business which comprises a huge share of the insurance industry portfolio. Based on the most recent information from the insurance statistical

yearbook, more than 62.65% and 54.28% of the total premium in nonlife insurance policies issued in the years 1393 (2014) and 1394 (2015), respectively, were gathered from automobile insurance policies; and the share of hull insurance from the premium in automobile insurances in these two years were 12.62% and 12.48%, respectively [1].

Beside the premium, the loss ratio attracts attention in this line of business. Loss ratios of 65.1% and 61.8% for the years 1393 (2014) and 1394 (2015) in automobile hull insurance [1] highlights the importance of fair ratemaking based on the proposed risk to the insurance companies by the proposers. Fair premium rates lead to risk control and reducing the abnormal behaviors of the high risk drivers. Therefore, they play a vital role in reducing the loss ratio in automobile insurance.

As the tariffs in hull insurance is competitive between the insurance companies, and buying this type of policy is optional for the drivers, calculating fair premium rates in this line of business would help the insurance companies to attract good risks and push away bad risks. In the current rate making system in governmental and private insurance companies in Iran, not a big difference is being considered between high risk and low risk drivers, as human factors and personal characteristics of the drivers are not being taken into account in the ratemaking. The available data regarding the personal characteristics of the drivers are also non-reliable; because the policy holder's information in captured by the insurance companies, while this person is not necessarily the driver. In addition, more than one driver may use a single car. Therefore, there is a need for creating a comprehensive and coherent database, which captures the necessary information for the insurance companies to calculate fair rates and helps making suitable systems of policy issuance in the insurance industry.

2- Methodology

The data in this research is gathered through library searches and also questionnaires and interviews. The designed questionnaires are filled out by a group of automobile insurance department managers and their deputies, technical managers of the insurance companies and some members of the insurance syndicate in automobile insurance workgroup. The interviews are also done with the same people.

4 steps are taken to do this research, which are shown in Figure 1.

STEP 1: Studying the available researches and initial dentification of the factors affecting risk in the vehicle (passenger cars) hull insurance

STEP 2: Determining the factors affecting risk in the hull insurance using Fuzzy Delphi method and takign the viewpoints of the experts in this field

STEP 3: classifying the initially identified factors and using SmartPLS software to do the confirmatory factor analysis

STEP 4: Omiting the factors, which have a high correlation with other factors, and identifying the factors affecting risk in the vehicle (passenger cars) hull insurance

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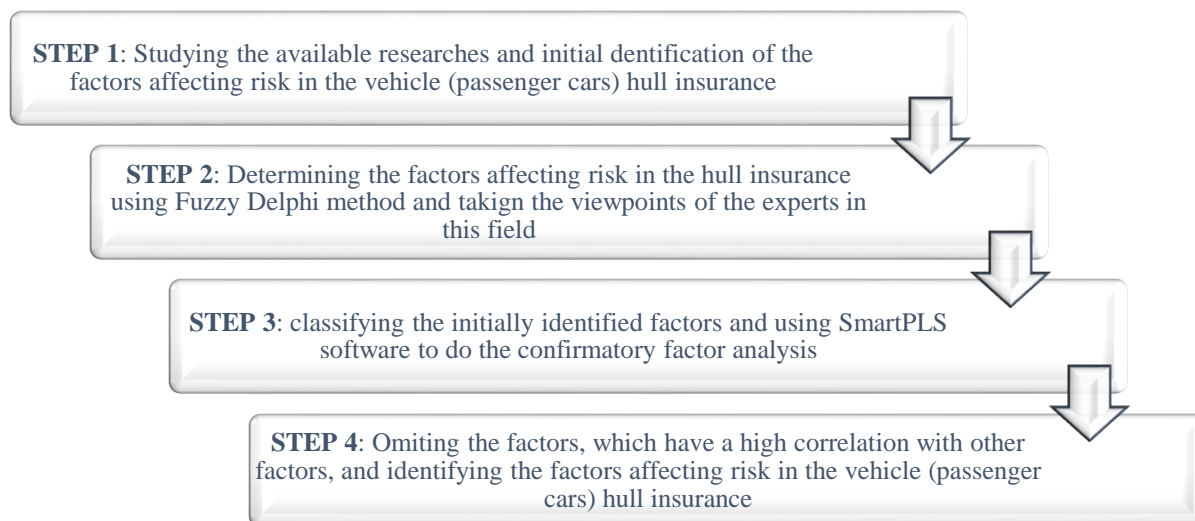3-5 September, 2017

Figure 1. The methodology framework

The stages to apply fuzzy Delphi method in Step 2 are as follows:

1. *Gathering the viewpoints of the decision makers group*: At this stage, the viewpoints of the experts regarding the importance of each factor is gathered through the designed questionnaires. In these questionnaires, factors are extracted from the literature; and the viewpoint of the experts are reflected through selecting an interval instead of a specific number for each of the questions. Experts can also add new factors to the list provided.

2. *Creating triangular fuzzy numbers*: The degrees of importance captured in the first stage are transformed into triangular fuzzy numbers. To do so, the method in [2] is used. In this method, the importance degree of factor j, which is determined by the ith expert among n experts, is set as $\widetilde{w_{ij}} = (a_{ij}, b_{ij}, c_{ij})$, where $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$. Note that a, b and c are the vertices of the fuzzy triangular number. Then, the fuzzy weight of the jth factor is calculated as $\widetilde{w_j} = (a_j, b_j, c_j)$, such that:

$$a_j = Min_i\{a_{ij}\} \qquad\qquad b_j = \frac{1}{n}\sum_{i=1}^{n} b_{ij} \qquad\qquad c_j = Max_i\{c_{ij}\}$$

3. *Defuzzifying*: In order to defuzzify the fuzzy weight $\widetilde{w_j}$ of each factor and transform it into the crisp amount of S_j, the approximation of the three-parameter Beta distribution with the following formula is used.

$$S_j = \frac{a_j + 4b_j + c_j}{6} \qquad\qquad j = 1, 2, \ldots, m$$

4. *Screening by evaluation criteria and providing the results*: At this stage, considering the threshold α, and taking into account the following 2 rules, the effective factors and almost non-effective factors are segregated.

- If $S_j \geq \alpha$, then the jth factor is considered as an effective one, and
- If $S_j < \alpha$, then the jth factor is omitted.

$\alpha = 0.25$ is considered in this paper. The schema of this threshold is illustrated in figure 2.
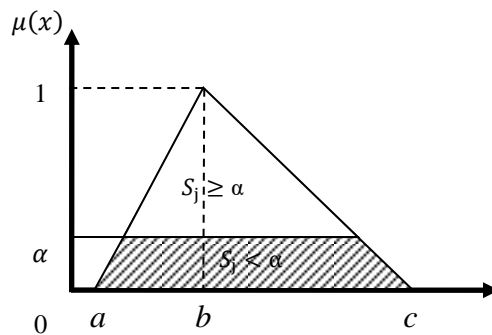
Figure 2. The schema of threshold α

3- Results and Discussion

In this research, after reviewing the related papers and researches, Fuzzy Delphi method was used to determine the factors affecting risk in the automobile hull insurance in Iran. More than 60 factors were extracted from the related scientific and experimental sources, out of which, 40 factors that were logically applicable for Iran were selected. Later on, 2 other factors were added by the experts; and the number of factors reached 42. Of these factors, 35 in 4 groups of "the driver's characteristics", "the vehicle properties", "vehicle usage" and "Policy details" were recognized by the experts as the effective ones.

Based on the obtained results, the importance of the factors related to the usage of the vehicle is higher than the other 3 groups. These factors include number of drivers and their personal characteristics, transportation and parking area, and having more than 1 car.

Besides, the average importance of the driver's characteristics group and the vehicle properties group are very near to each other (0.77 and 0.75, respectively). This highlights the significance of considering the personal characteristics of the driver(s) in the ratemaking. Specifically, among the factors in this group, violations and driving offenses (including high speed) needs to be seriously considered when assessing and predicting the risk.

In the next step, to ease data gathering in proposal forms at the beginning of policy issuance process, these factors are classified by applying confirmatory factor analysis and their number is reduced through omitting the factors which have a high correlation with other factors.

Using these factors in the ratemakings, provided that they are correctly assessed, can lead insurance companies towards a fair ratemaking. Besides, these factors can be used for designing intelligent expert systems to evaluate the level of risk for the proposers and determining the initial premium rate in the hull insurance. In the next years, the insurance company can increase or decrease the premium rate based on the bonus-malus system applied in the company. The results obtained in this research, can help the insurance companies to better assess the risk in automobile hull insurance and calculate more accurate and fair premium rates.

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Determine the Difference between Bias and Unbias Comparison in Economic Statistical Design of T²–FRS and T²-VSS Control Charts

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Abstract

The Hotelling's T^2 control chart, is the most widely used multivariate procedure for monitoring two or more related quality characteristics. Recently, the variable sampling sizes (VSS) control scheme has been proved to have a very good performance on detecting small to moderate shifts when it is compared to the fixed ratio sampling (FRS) T^2 control scheme. Moreover, it is shown that the VSS scheme is more economical than the FRS scheme. It is applied the cost model proposed by Lorenzen and vance (1986). Furthermore, it is assumed that the length of time that the process remains in control is exponentially distributed which allows applied the Markov chain approach for developing the cost model. It is applied genetic algorithm to determine the optimal values of model parameters by minimizing the cost function. This paper studies unbias comparison between Economic Statistical design $T^2 - FRS$ and $T^2 - VSS$ control charts with respect to the expected cost per unit time.

Keywords

T^2 **Control Chart, Economic Statistical Design (ESD), Fixed Ratio Sampling (FRS), Variable Sampling Sizes (VSS), Unbias, Genetic Algorithm(GA).**

1- Introduction

The processes are characterized by several, usually correlated, variables indicating the need for the use of a multivariate control chart such as that due to Hotelling [1]. Lowry and Montgomery [2] mentioned the popularity of the Hotelling's T^2 chart in industrial applications leading to the development of control chart software for its application. One procedure to improve the statistical performance of FRS control schemes is a variable sampling sizes scheme that varies the sampling sizes between successive samples as a function of prior sample results. Aparisi [3] and Faraz and Moghadam statistically designed the $T^2 - VSS$ control chart. They showed that the Hotelling's T^2 control chart with a VSS scheme significantly improves the efficiency of the standard Hotelling's T^2 control chart in detecting small or moderate changes in the process mean. Nevertheless, none of these studies took into account the economic aspect of the design. Montgomery and Klatt were the first researchers who developed a method for choosing the approximate minimum cost test

parameters for the $T^2 - FRS$ control chart. As Woodall [6] mentioned, the main drawback of the ED's is that they typically have a high Type I error probability, which can lead to unnecessary process adjustments or a loss of trust in the control system. Sanig developed an approach called Economic Statistical Design (ESD) of control charts that places statistical constraints on optimal ED. Taylor [8] noted that economic control charts using FRS schemes are non-optimal designs.

Seif et al. [9] presented $T^2 - VSSCL$ base on the costa and rahim's model.

We study in this paper unbias comparison between Economic Statistical design $T^2 - VSS$ and $T^2 - VSS$ control charts. This paper is organized as follows: In section 2, $T^2 - VSS$ control scheme are reviewed. In section 3 unbias comparison between ESD $T^2 - FRS$ and $T^2 - VSS$ is discussed. In section 4, we have industrial example and final section provides some concluding remarks.

2- The $T^2 - VSS$ control scheme and markov chain approach

Consider a process in which p correlated characteristics are being measured simultaneously and is jointly. It is assumed that the joint probability distribution of the p quality characteristics is a p −variate normal distribution with in-control mean vector $\mu_0' = (\mu_{01}, ..., \mu_{0p})$ and variance-covariance matrix Σ. The T^2 control chart requires computing the sampling means for each of the p quality characteristics from a sample of size n. Then the subgroup statistic $T_i^2 = n(\bar{x}_i - \mu_0)'\Sigma^{-1}(\bar{x}_i - \mu_0)$ is plotted on a control chart in sequential order. The chart signals as soon as $T_i^2 \geq k$. We assume the parameters μ_0 and Σ are known. In this case k is given by the upper α percentage point of a chi-square variable with p degrees of freedom.i.e., $k = \chi_\alpha^2(p)$.

The following function summarizes the control scheme of the $T^2 - VSS$ control chart:

$$n_i = \begin{cases} n_1 & if \quad 0 \leq T_{i-1}^2 < w \\ n_2 & if \quad w \leq T_{i-1}^2 < k \end{cases} \qquad (1)$$

The *AATS* is the average time from the process mean shift until the chart produces a signal. If the assignable cause occurs according to an exponential distribution with parameter λ then the expected time interval that the process remains in-control is $\frac{1}{\lambda}$. Therefore, $AATS = ATC - \frac{1}{\lambda}$. The control chart produces a signal when $T^2 \geq k$. If the current state is 3, the signal is a false alarm and absorbing state (state 6) is reached when the true alarm occurs. Therefore, the transition probability matrix is given by

$$p = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ 0 & 0 & 0 & p_{44} & p_{45} & p_{46} \\ 0 & 0 & 0 & p_{54} & p_{55} & p_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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where p_{ij} denotes the transition probability that i is the prior state and j is the current state. In what follows, $F(x, p, \eta)$ will denote the cumulative probability distribution function of a non-control chi-square distribution with p degrees of freedom and non-centrality parameter, $\eta = nd^2$, where $d^2 = (\mu_1 - \mu_0)\Sigma^{-1}(\mu_1 - \mu_0)$. In the at state, the expected number of trials is each state to reach the absorbing state can be obtained from $b'(I - Q)^{-1}$ where Q is the 5×5 matrix obtained from p on deleting the elements corresponding to the absorbing state, I is the identity matrix of order 5 and $b' = (p_1, p_2, p_3, 0, 0)$ is a vector of initial probabilities, with $\sum_{i=1}^{3} p_i = 1$. Hence, $ATC = b'(I - Q)^{-1}h$

Where $h' = (h, h, h, h, h)$ is the vector of sampling time intervals. In this paper assumed $b' = (0, 1, 0, 0, 0)$. In-control period $= \frac{1}{\lambda} + (1 - \gamma_1)T_0 \times ANF$ Where $\gamma_1 = 1$ if the process is not shut down during false alarms and 0 otherwise. T_0 stands for the expected time spent searching for a false alarm and ANF is the expected number of false alarms in each quality cycle and is calculated as follows $ANF = b'(I - Q)^{-1}f$ Where in that $f' = (0, 0, 1, 0, 0)$.

The expected total cycle time is given as the sum of the in control and out of control cycle times as follows

$$E[T] = \frac{1}{\lambda} + (1 - \gamma_1)T_0 ANF + AATS + \bar{n}E + T_1 + T_2 = ATC + (1 - \gamma_1)T_0 ANF + \bar{n}E + T_1 + T_2 \quad (2)$$

where the expected time to discover the assignable cause, which is given as T_1 and the expected time to repair the assignable cause, which is given as T_2. The expected cost of producing non-conformities while the process is in control and out of control is given by the following $\frac{C_0}{\lambda} + C_1[AATS + \bar{n}E + \gamma_1 T_1 + \gamma_2 T_2]$ where γ_2 is an indicator function for if production continues during the repair of the process, C_0 and C_1 stand for the cost of producing non-conformities while the process is in control and out of control, respectively. The expected cost of evaluating false alarms and repairing the process is given by $a'_3 ANF + a_3$ where a'_3 is the cost of investigating false alarms and a_3 stands for the cost of locating and repairing an assignable cause. The expected sampling cost per cycle is given by $(a_1 ANS + a_2 ANI) + \frac{(a_1 + a_2 n_2)(\bar{n}E + \gamma_1 T_1 + \gamma_2 T_2)}{h}$ where a_1 and a_2 are the fixed and variable cost components of sampling and testing, respectively. the ANI and ANS stand for the expected number of inspected items and samples taken during the quality cycle and are calculated using follow equations. $ANI = b'(I - Q)^{-1}n$, $ANS = b'(I - Q)^{-1}1$ where $n' = (n_1, n_2, n_2, n_1, n_2)$ and $1' = (1, 1, 1, 1, 1)$. Now the expected cost per cycle is

$$E(C) = \frac{C_0}{\lambda} + C_1[AATS + \bar{n}E + \gamma_1 T_1 + \gamma_2 T_2] + a'_3 ANF + a_3$$
$$+ (a_1 ANS + a_2 ANI) + \frac{(a_1 + a_2 n_2)(\bar{n}E + \gamma_1 T_1 + \gamma_2 T_2)}{h} \quad (3)$$

Due to the renewal reward assumption, the expected cost per hour is just as follows

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$$E(A) = \frac{E(C)}{E(T)} \qquad (4)$$

Therefore, the general optimization problem is defined as follows:

$$\min E(A)$$

$$s.t : 0 \leq w < k, \ 1 \leq n_1 < n_2, \ n_1, n_2 \in Z^+, \ 0 < h \leq 8, ANF \leq ANF_0 \ and / or \ AATS \leq AATS_1 \qquad (5)$$

3- An unbias comparison between ESD $T^2 - FRS$ and $T^2 - VSS$ scheme

In order to make a unbias comparison between ESD $T^2 - FRS$ and $T^2 - VSS$ scheme, it should be noted that both economic FRS and VSS schemes should have the same in-control time and cost to guarantee a meaningful comparison between these two schemes. Therefore, the two charts then require the same ANF, ANS and ANI values to be inspected during the in-control period.

4- An industrial example

In this section the proposed approach to the ESD of the $T^2 - VSS$ control chart is illustrated through on industrial example concerning the GM casting operation as presented by Lorenzen and Vance [10]. 5. It's solved the optimization problem (5) with the constraint $ANF \leq 0.5$ to obtain the ESD of the $T^2 - VSS$ control chart scheme.

5- Concluding remarks

In this paper we have presented unbias comparison between Economic Statistical design $T^2 - FRS$ and $T^2 - VSS$ control charts when the in control process mean vector and process covariance matrix are known. The cost model adopted in the presented study in that of Lorenzen and Vance (1986) and derived by the markov chain approach. We applied the genetic algorithm to find the optimal chart parameters. The numerical comparison between the both ESD FRS and VSS schemes has shown that when we use unbias design , results show that mean percentage decrease cost per unit time in $T^2 - VSS$ scheme with respect to $T^2 - FRS$ is 0.09, while the if we use bias design, it is 0.11, so this will lead to 2 percent error.

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Economic-Statistical design of Adaptive T² control charts: Markov Chain Approach

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Abstract

This paper presents an economic-statistical design of T² control charts with variable parameters (VP). We optimize this model using a genetic algorithm (GA) approach. Furthermore, variable sample size and control limits (VSSC) and variable sample size (VSS), variable sampling interval (VSI), variable sampling interval and control limits (VSIC) and VP T² charts are compared with respect to the expected cost per unit time. The Hotelling's T² control chart with variable parameters (VP T²) has been proved to have a very good performance on detecting small shifts when it is compared to the other T² control chart.

Keywords

Hotelling's T² control chart; Economic-Statistical Design (ESD); Markov chain.

1- Introduction

Control chart technique plays an important role in production process monitoring. The Economic Design (ED) of control charts involves the optimal determination of charts parameters by minimizing the overall costs associated with maintaining current control of a process. Unfortunately, EDs also have weaknesses. Saniga [1] developed an approach called Economic Statistical Design (ESD) of control charts that places statistical constraints on optimal ED. Traditional T² control charts are static, that is, samples with fixed sample size are taken at regular sampling interval. Aparisi proposed three types of modified T² charts with variable sample size (VSS), variable sampling interval (VSI), and variable sample size and sampling interval (VSSI) [2-4]. Recently, Chen and Hsieh [5] statistically designed the T² control chart with variable sample sizes and control limits (VSSC).

2- VP T² control scheme and Markov chain approach

Consider p correlated quality characteristics are to be monitored simultaneously using the T² scheme. First, it is assumed that the joint probability distribution of the quality characteristics is a p-variate normal distribution with in-control mean vector $\mu_0' = (\mu_{01},...,\mu_{0p})$ and variance-covariance matrix Σ. Then the subgroups, statistic $T_i^2 = n(\overline{X}_i - \mu_0)' \Sigma^{-1} (\overline{X}_i - \mu_0)$ (each of size n) is plotted in sequential order and the chart signals as soon as $T_i^2 \geq k$. For the sake of simplicity, we assume here that μ_0 and Σ are known or are estimated from large enough samples. Let h_1 and h_2 be maximum and minimum sampling intervals, k_1 and k_2 be maximum and minimum control limits, and n_1 and n_2 be maximum and minimum sample size

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respectively, such that $0 < h_2 < h_1$, $0 < k_2 < k_1$ and $n_1 < n_2$. The following function summarizes the control scheme of the VP T^2 control chart:

$$(k_i, h_i, n_i) = \begin{cases} (k_1, h_1, n_1) & \text{if} \quad 0 \leq T_{i-1}^2 < w \\ (k_2, h_2, n_2) & \text{if} \quad w \leq T_{i-1}^2 < k_{i-1} \end{cases}$$

The average time from the process mean shifts until the chart produces a signal is used to measure its statistical efficiency. This statistical measure is called AATS and determines the speed with which a control chart detects a process mean shift. The average time of the cycle (ATC) is the average time from the start of the production until the first signal after the process shift. If the assignable cause occurs according to an exponential distribution with parameter λ then the expected time interval that the process remains in control is $1/\lambda$. Therefore, $AATS = ATC - \frac{1}{\lambda}$. The memoryless property of the exponential distribution allows the computation of the ATC using the Markov chain approach. Here, at each sampling stage, one of the following transient states is reached according to the status of the process (in or out of control), length of the sampling interval (short or long) and quantity of the control limit (k_1 or k_2):

State 1: $0 < T^2 < w$ and the process is in control;

State 2: $w \leq T^2 < k_j$ and the process is in control;

State 3: $T^2 \geq k_j$ and the process is in control;

State 4: $0 < T^2 < w$ and the process is out of control;

State 5: $w \leq T^2 < k_j$ and the process is out of control;

State 6 (absorbing state): $T^2 \geq k_j$ and the process is out of control. The transition probability matrix is given as follows:

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ 0 & 0 & 0 & p_{44} & p_{45} & p_{46} \\ 0 & 0 & 0 & p_{54} & p_{55} & p_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where p_{ij} denotes the transition probability that i is the prior state and j is the current state. In what follows, $F(x, p, \eta)$ will denote the cumulative probability distribution function of a non-central chi-square distribution with p degrees of freedom and non-centrality parameter $\eta_i = n_i d^2$ and $q_i = \exp(-\lambda h_i); \quad i = 1,2$. Then, p_{ij}'s are

$p_{11} = F(w, p, \eta_1 = 0) \times q_1$, $\qquad p_{12} = [F(k_1, p, \eta_1 = 0) - F(w, p, \eta_1 = 0)] \times q_1$,

$p_{13} = [1 - F(k_1, p, \eta_1 = 0)] \times q_1$, $\qquad p_{14} = F(w, p, \eta_1 = n_1 d^2) \times (1 - q_1)$,

$p_{15} = [F(k_1, p, \eta_1 = n_1 d^2) - F(w, p, \eta_1 = n_1 d^2)] \times (1 - q_1)$,

$p_{16} = [1 - F(k_1, p, \eta_1 = n_1 d^2)] \times (1 - q_1)$,

$p_{21} = p_{31} = F(w, p, \eta_2 = 0) \times q_2$, $\qquad p_{22} = p_{32} = [F(k_2, p, \eta_2 = 0) - F(w, p, \eta_2 = 0)] \times q_2$,

$p_{23} = p_{33} = [1 - F(k_2, p, \eta_2 = 0)] \times q_2$, $\qquad p_{24} = p_{34} = F(w, p, \eta_2 = n_2 d^2) \times (1 - q_2)$

$p_{25} = p_{35} = [F(k_2, p, \eta_2 = n_2 d^2) - F(w, p, \eta_2 = n_2 d^2)] \times (1 - q_2)$,

$p_{26} = p_{36} = [1 - F(k_2, p, \eta_2 = n_2 d^2)] \times (1 - q_2)$,

$p_{44} = F(w, p, \eta_1 = n_1 d^2)$ $\qquad p_{45} = F(k_1, p, \eta_1 = n_1 d^2) - F(w, p, \eta_1 = n_1 d^2)$

$p_{46} = 1 - F(k_1, p, \eta_1 = n_1 d^2)$, $\qquad p_{54} = F(w, p, \eta_2 = n_2 d^2)$,

$p_{55} = F(k_2, p, \eta_2 = n_2 d^2) - F(w, p, \eta_2 = n_2 d^2)$, $\qquad p_{56} = 1 - F(k_2, p, \eta_2 = n_2 d^2)$,

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Now, ATC is calculated as follows:

$$ATC = b'(I-Q)^{-1}(h_1, h_2, h_2, h_1, h_2)$$

(1)

Where, Q is is the 5×5 matrix obtained from **P** on deleting the elements corresponding to the absorbing state, **I** is the identity matrix of order 5 and $b' = (0,1,0,0,0)$ is a vector of initial probabilities.

3- The cost model

In the Lorenzen−Vance cost function, we assume that the process starts from an in-control state and that the time to occurrence of an assignable cause is exponentially distributed with a mean of $\frac{1}{\lambda}$. Due to the renewal reward assumption, the expected cost per hour is just as follows:

$$E(A) = \frac{E(C)}{E(T)}$$

(2)

where:

$$E(C) = \frac{C_0}{\lambda} + C_1[AATS + \bar{n}E + \gamma_1 T_1 + \gamma_2 T_2] + a_3' ANF + a_3 + (a_1 ANS + a_2 ANI) + \frac{(a_1 + a_2 n_2)(\bar{n}E + \gamma_1 T_1 + \gamma_2 T_2)}{h}$$

$$E[T] = ATC + (1-\gamma_1)T_0 ANF + \bar{n}E + T_1 + T_2$$

ANF is the average number of false alarms and can be obtained as follows:

$$ANF = \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}(0,0,1,0)$$

(3)

It is assumed that the time it takes to interpret the sample is proportional to the sample size and has a proportionality constant E. T_0 stands for the expected time spent searching for a false alarm. T_1 and T_2 stand for the expected time to discover and repair the assignable cause. C_0 and C_1 stand for the cost of producing non-conformities while the process is in control and out of control, respectively. $\gamma_1 = 0$ if the process is stopped while searches for a cause goes on and 1 if the process is continued while searches proceed. $\gamma_2 = 0$ if the process is stopped during the correction or repair and 1 if it continues while correction or repair is performed. a_3' is the cost of investigating false alarms and a_3 stands for the cost of locating and repairing an assignable cause. a_1 and a_2 are the fixed and variable cost components of sampling and testing, respectively. The *ANI* and *ANS* stand for the expected number of inspected items and samples taken during the quality cycle and are calculated as follows:

(4)

$$ANI = \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}(n_1, n_2, n_2, n_1, n_2), \quad ANS = \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}(1,1,1,1,1)$$

4- Solution to the cost model

The goal of ED of the VPT[2] control chart is to find the seven chart parameters $(k_1, k_2, w, n_1, n_2, h_1, h_2)$ which minimize (2). Therefore, the general optimization problem is defined as follows:

$$\min E(A)$$
$$s.t:$$
$$0.1 \le h_2 \le h_0 \le h_1 \le 8$$
$$0 < w_2 < w_1 < k_2 \le k_0 \le k_1$$
$$n_1 \le n_0 \le n_2 \in Z^+$$

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To obtain an ESD, statistical constraints are added which may be expressed in terms of $ANF \leq ANF_0$ and/or $AATS \leq AATS_1$.

5- Comparison Approach

Different FRS and VP control schemes should have the same comparing measure when the process is in control. Hence, the two different schemes should have the same expected cost per hour when the process is in control. By considering that the two schemes have the same in-control time, then the two charts are fairly comparable if and only if they have the same in-control cycle cost. Then, the two charts should have the same ratio sampling (sampled items and sampling frequencies) and the same Type I error rate as long as the process is in control. Now, in the VP design vector $D = (k_1, k_2, w, n_1, n_2, h_1, h_2)$, the values of w, n_2, k_2 are obtained such that the both VP and FRS schemes have the same in-control ANS, ANI and ANF.

$$w = F^{-1}(\frac{q_2 - q_0}{q_0(q_2 - q_1)}, p, 0) \ , \ n_2 = \frac{n_0(1 - q_2 + a(q_2 - q_1) - n_1 aq_2(1 - q_0)}{(1 - q_0)(1 - aq_1)} \ , \ k_2 = \frac{b_1 aq_1 q_2(1 - q_0) + (1 - b_0)(1 - q_2 + a(q_2 - q_0)}{q_2(1 - q_0(aq_1 - 1)} \quad (5)$$

where, $a = F(w, p, , 0)$ $b_i = F(k_i, p, , 0)$; $i = 0, 1, 2$

Therefore, for a given process and cost parameters, the optimal design of the FRS T^2 control scheme (k_0, n_0, h_0) is first defined. Then for a given (k_1, n_1, h_1, h_2) parameter set, the parameters w, n_2 and k_2 take value from equations (9), (10) and (16), respectively. Then we proceed to find the three chart parameters (k_1, n_1, h_1, h_2) that minimize (3). This procedure ensures that the comparison of the different schemes is meaningful and unbiased because the two procedures have the same cost while the process is in control. In the next section, the proposed scheme is illustrated through an industrial application.

6- Illustrative example

Application of developed economic statistical model for the VP T^2 charts is demonstrated using the numerical example in Lorenzen and Vance [6]. The values of the input parameters are: $\lambda = 0.05$, $E = T_0 = T_1 = 0.0833$, $\gamma_1 = 1$, $\gamma_2 = 0$, $T_2 = 0.75$, $C_0 = 114.24$, $C_1 = 949.2$, $a_1 = 5$, $a_2 = 4.22$, $a_3 = a_3' = 977.4$, $d = 1.5$ and $p = 3$. Also, consider that the statistical constraint $ANF \leq 0.5$ is of concern. Table 1 gives the optimal design vector for different values of mean shift $d=0.25(0.25)1.5(0.5)3$ with a cost comparison to the corresponding optimal FRS, VSSC, VSIC, VSS, VSI and VP schemes. The improvements are more meaningful for small to moderate shifts in the process than the moderate to large shifts.

7- Concluding remarks

The Hotelling's T^2 control chart is a widely applied multivariate control chart for detecting moderate to large shifts in process mean. The control charts using VRS policy has been shown to give substantially faster detection of most process shifts than the conventional control schemes. In this paper we have improved the cost benefits of the VP scheme by developing a model called VP control scheme. The cost model adopted in the present study is that of Lorenzen and Vance [1] and the expected total cost per hour is minimized using a genetic algorithm approach. We have done extensive comparisons between VP and FRS, VSSC, VSIC, VSS, VSI charts and have shown that the VP chart is preferable to the FRS, VSSC, VSIC, VSS, VSI charts.

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Table 1. A Comparison between the VP, VSSC, VSIC, VSS, VSI and FRS schemes

NO.		E(A)					
	d	VP	VSIC	VSSC	VSI	VSS	FRS
1	.25	478.97	514.91	480.16	515.1	485.42	546.14
2	.5	355.88	375.02	356.67	375.72	359.89	412.81
3	.75	308.34	315.43	316.79	315.93	316.14	349.54
4	1	279.98	282.71	281.07	283.24	281.37	314.13
5	1.25	260.98	261.78	261.64	262.33	262.97	291.21
6	1.5	249.11	246.25	248.19	246.56	248.68	273.6
7	1.75	239.40	235.53	238.96	235.8	239.29	261.08
8	2	231.88	232.92	233.26	232.21	233.52	251.85
9	2.25	226.55	227.77	227.41	227.92	227.44	244.1
10	2.5	223.44	224.85	224.47	224.92	224.47	238.91
11	2.75	217.59	218.04	219.61	218.4	219.61	233.26
12	3	214.77	216.8	217.18	216.86	217.18	228.99

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POSTERS

Option Pricing in a Fractional Model

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Abstract

In this paper, option pricing is driven when the dynamic of underling asset price follows a fractional model with jump. Using the properties of fractional Taylor formula and fractional Ito formula for $H \in [0.5, 1)$ a new model named fractional jump double Heston is presented.

Keywords

Fractional Taylor formula, Fractional jump double Heston, Jump.

Option pricing with using Levy process and comparison it with Black-Scholes model in Hilbert space

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Abstract

The Black-Scholes model is based on smooth function in continuous time rang, not allowing jumps in stock movements. However in actuality, stock price does jumps, and some risks cannot be handled within continuous-path models. The Exponential Levy model is a choice to include jumps allowing more accurate representation of the market movements. Levy process tenders a more realistic model of price dynamics than Black-Scholes model. It's obvious that the Levy process model is more difficult to implement and involves more computations compared to the Black-Scholes model. Thus ,the question is whether it is worth to implement a Levy process model.We summarize that the Levy process model does have certain advantages over the Black-Scholes model.

Keywords

Levy process,Option pricing,Black-Scholes equation.

Review on Construction Confidence Interval for Difference Quantiles of Two Independent Normal Distributions

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Abstract

In this paper, the problem of making inferences on the difference between quantiles of two independent normal distributions is addressed. In this regard, we apply a parametric bootstrap (PB) approach, and a method of variance estimates recovery (MOVER) to construct confidence intervals for the difference between quantiles. The coverage probability and average length of the proposed approaches are compared through Monte Carlo simulations. Simulation results show that our proposed MOVER confidence interval can be recommended generally for different sample sizes and number of groups. The interval estimation methods are applied to a real world case study to show their performance in life data analysis

Keywords:

Reliability comparison; Confidence interval; Quantile; Coverage probability; Parametric bootstrap.

The Multiple Comparison Problem in Neuroimaging

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Abstract

Functional magnetic resonance imaging (fMRI) is a safe and non-invasive way to assess brain functions by using signal changes associated with the brain activity. Current scientific techniques in fMRI image processing routinely produce hypothesis testing problems with hundreds or thousands of voxels to consider simultaneously. This matter occurs when we want to make statistical inferences about activated voxels simultaneously. This poses new difficulties for the statistician, but also opens new opportunities. In this paper, we will review and discuss the methods that are using as solutions of the multiple comparison problems in neuroimaging.

Keywords

FMRI data, False discovery rate, Multiple comparisons, Bonferroni correction, cluster-based correction.

Usual Stochastic Order of Aggregate Claim Amounts from Two Marshall-Olkin Extended Weibull Portfolios

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Abstract

In this paper, we discuss the stochastic comparison of two classical surplus processes in an one-year insurance period. Under the Marshall-Olkin extended Weibull random aggregate claim amounts, we establish some sufficient conditions for the comparison of aggregate claim amounts in the sense of the usual stochastic order (which implies stop-loss order). Applications of our results to the Value-at-Risk and ruin probability are also given. The obtained results show that the heterogeneity of the risks in a given insurance portfolio tends to make the portfolio volatile, which in turn leads to requiring more capital.

Keywords

Aggregate Claim Amounts, Usual Stochastic Order, Multivariate Chain Majorization, Value-at Risk, Ruin Probability.

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